

EXPERIMENTAL GEOPHYSICS SENSORS

GRAVITATION WITH MATTER

AND WAVE

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INTRODUCTION : AIM OF THESE LECTURES

Wave - particle duality

“The only mystery” for R. Feynman (in his lectures on quantum mechanics), who later corrected his statement pointing Entanglement as the only mystery



Summary / Lecture (Quantum Behaviour)

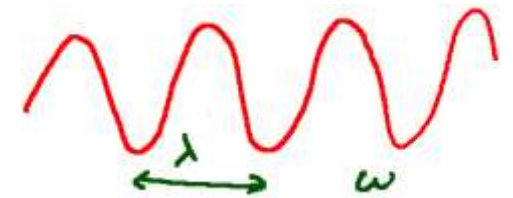
1. The probability of an event (in ideal experiment - no uncertain external disturbances) is absolute square of complex quantity called probability amplitude.
2. When event can occur in several alternative ways, the prob. amp. is the sum of a prob. amp. for each way considered separately.
3. If an experiment, capable of determining which alternative is actually taken, is performed, the interference is lost & the prob. becomes the sum of the prob. for each alternative.

Call Prob. Amp. ϕ :
Call Prob. P .

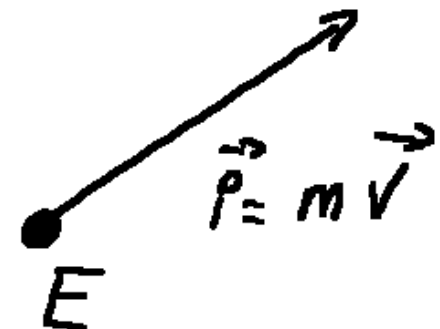
1. $Prob = |\phi|^2$
2. If two alternatives $\phi = \phi_1 + \phi_2$ so $P = |\phi_1 + \phi_2|^2$
3. In the case 3. above: $P = |\phi_1|^2 + |\phi_2|^2 = P_1 + P_2$

- Matter waves is a direct consequence of quantum mechanics and in particular the duality between

⇒ a wave described by *frequency* and *wavelength*



⇒ a particle described by *energy* and *momentum (mass.velocity)*



- Formalized by Louis de Broglie who extended to any particle the concept of coexistence of waves and particles discovered by Albert Einstein in 1905 in the case of light and photons.

⇒ introduced the *de Broglie wavelength*, matter analog of the photonic wavelength.

$$\lambda_{opt} \Rightarrow \lambda_{dB} = \frac{h}{m v_{rel.}}$$

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$$\lambda_{opt} \Rightarrow \lambda_{dB} = \frac{h}{m v_{rel}}$$

- ⇒ The wavelength decreases with mass and momentum
- ⇒ Wave properties (diffraction, interferences) is more prominent with large wavelength

	Mass (kg)	Speed (m/s)	Wavelength (m)
Human through door (1 m)	70	1	10^{-35}
Blood cells in capillary (100 μ m)	10^{-16}	0,1	10^{-15}
Atoms through slits(100 nm)	$10^{-27} - 10^{-25}$	500	$10^{-9} - 10^{-11}$

- ✓ Atom Interferometry consists in physical phenomena where the wave nature of atoms (and in particular the external degrees of freedom) is concerned.

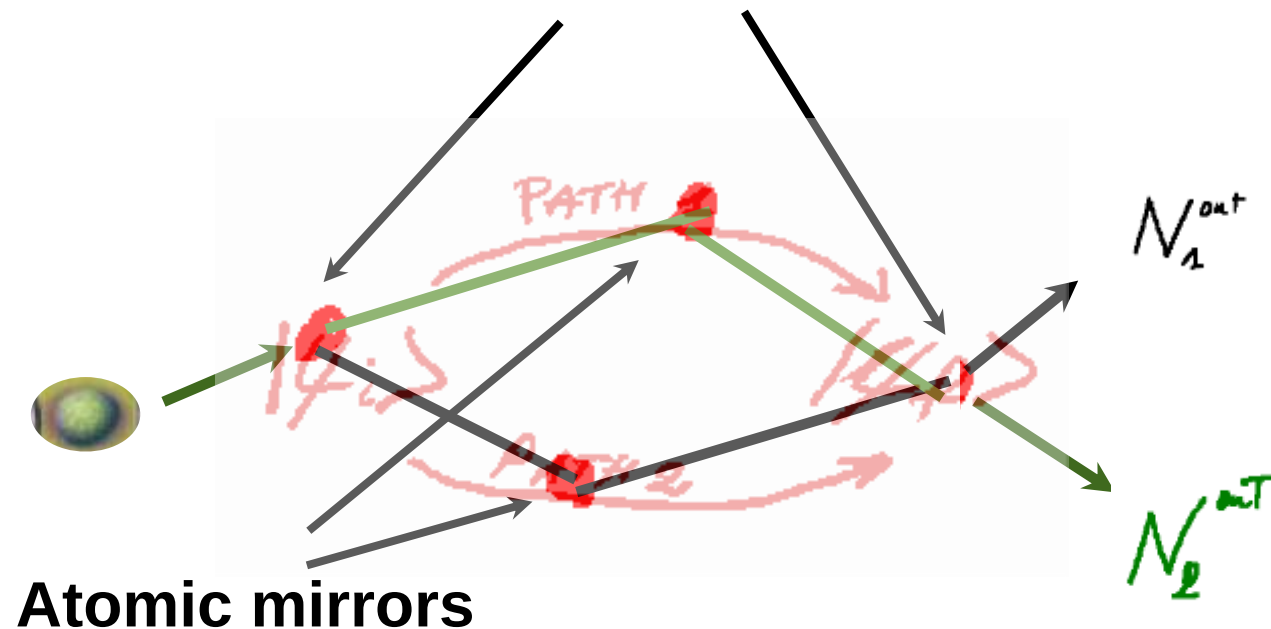
⇒ Analogous to light waves phenomena, but for de Broglie waves

$$\lambda_{opt} \Rightarrow \lambda_{dB} = \frac{h}{m v_{at}}$$

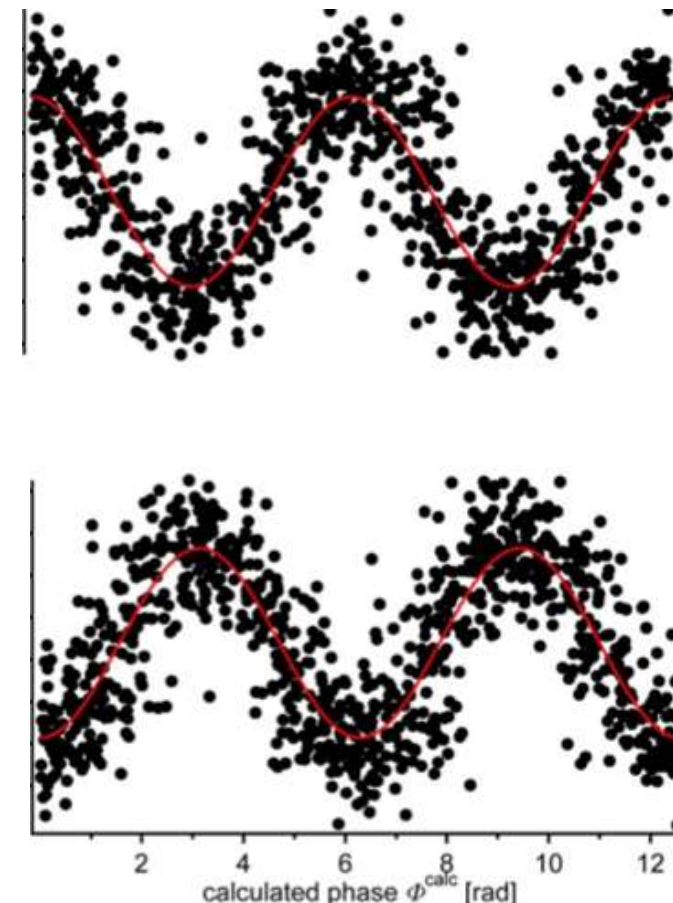
$$T \sim 1 \mu K \Rightarrow v_{at} \sim 1 \text{ mm/s} \Rightarrow \lambda_{dB} \sim 1 \mu m$$

- ✓ De Broglie wave associated to atoms.
- ✓ Interference process with atoms similar with light waves.
- ✓ Use coherent splitting to create multiple paths that will interfere
- ✓ Phase dependent on perturbations on different paths

Atomic beam splitter (using light)



$$N_1^{out} = 1 - N_2^{out} = 1 + \cos[\phi_{path_2} - \phi_{path_1}]$$



- ✓ 1979: First matter-wave gyroscope with neutron
 - *S. A. Werner, J.-L. Staudenmann, R. Colella, Phys. Rev. Lett., 42, p 1103, (1979)*
- ✓ 1991: Demonstration of atomic interferometers
- ✓ 1994: Demonstration of first molecular interferometer
 - *Ch.J. Bordé, et al. "Molecular interferometry experiments", Physics Letters A 188, 187 (1994)*
- ✓ 2001: Demonstration of interferometer with heavy molecules (C60)
 - *M. Arnd, et al. "High contrast interference with C-60 and C-70", Compt. Rend. Acad. Sci. Serie IV 2(4), 581.*

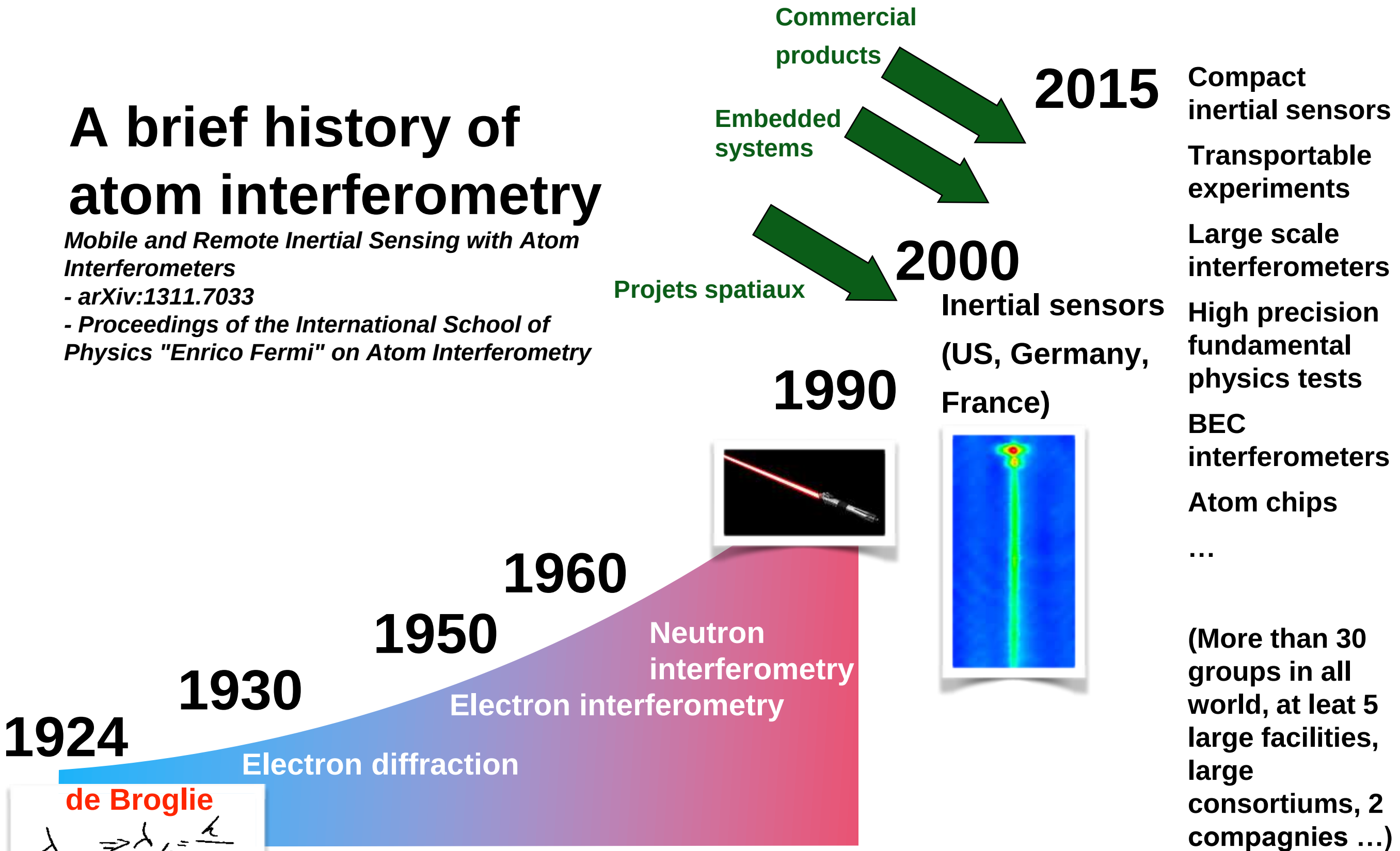
- ✓ 1991: Demonstration of atomic interferometers
- ✓ Atom interferometer using double-slit diffraction:
 - *O. Carnal, J. Mlynek, "Young's double-slit experiment with atoms : A simple atom interferometer", Phys. Rev. Lett., 66, 2689 (1991)*
- ✓ Atom interferometer gyroscope with atomic beam and light wave diffraction
 - *F. Riehle, Th. Kister, A. Witte, J. Helmcke, Ch. Bordé, Phys. Rev. Lett., 67, p 177 (1991)*
- ✓ Atom interferometer gyroscope with atomic beam and mechanical gratings diffraction
 - *D.W. Keith, C.R. Ekstrom, Q.A. Turchette, D.E. Pritchard, Phys. Rev. Lett., 67, p 2693 (1991)*
- ✓ Cold atom interferometer : accelerometer with cold atoms and light beam splitter (Diffraction using raman transitions)
 - *M. Kasevich, S. Chu, Phys. Rev. Lett., 67, p 177 (1991)*

A brief history of atom interferometry

Mobile and Remote Inertial Sensing with Atom Interferometers

- arXiv:1311.7033

- Proceedings of the International School of Physics "Enrico Fermi" on Atom Interferometry



Matter-wave interferometry, although a rather old subject, is nowadays at the leading edge of many new developments in geophysics and fundamental physics thanks to laser cooling

- ✓ Commercial development of transportable sensors
- ✓ Development of space interferometer and space missions
- ✓ Recent proposals of gravitational wave detectors using atom sensors.

The following lectures will aim to :

- ✓ Introduce the basics of atom inertial sensors.
- ✓ Introduce the recent developments in atom interferometry and inertial sensor.
- ✓ Give a general overview of the methods and the tools to be used with atom inertial sensors.
- ✓ Present some recent experimental achievements and recent project developments.

LECTURE 1 : INTRODUCTION TO ATOM INTERFEROMETRY AND INERTIAL SENSORS

- ✓ History of experimental developments
- ✓ Theoretical tools for atom interferometry
- ✓ How to build an inertial sensor

LECTURE 2 : MEASURING ACCELERATION AND ROTATIONS - METHODS, PRECISION, ACCURACY

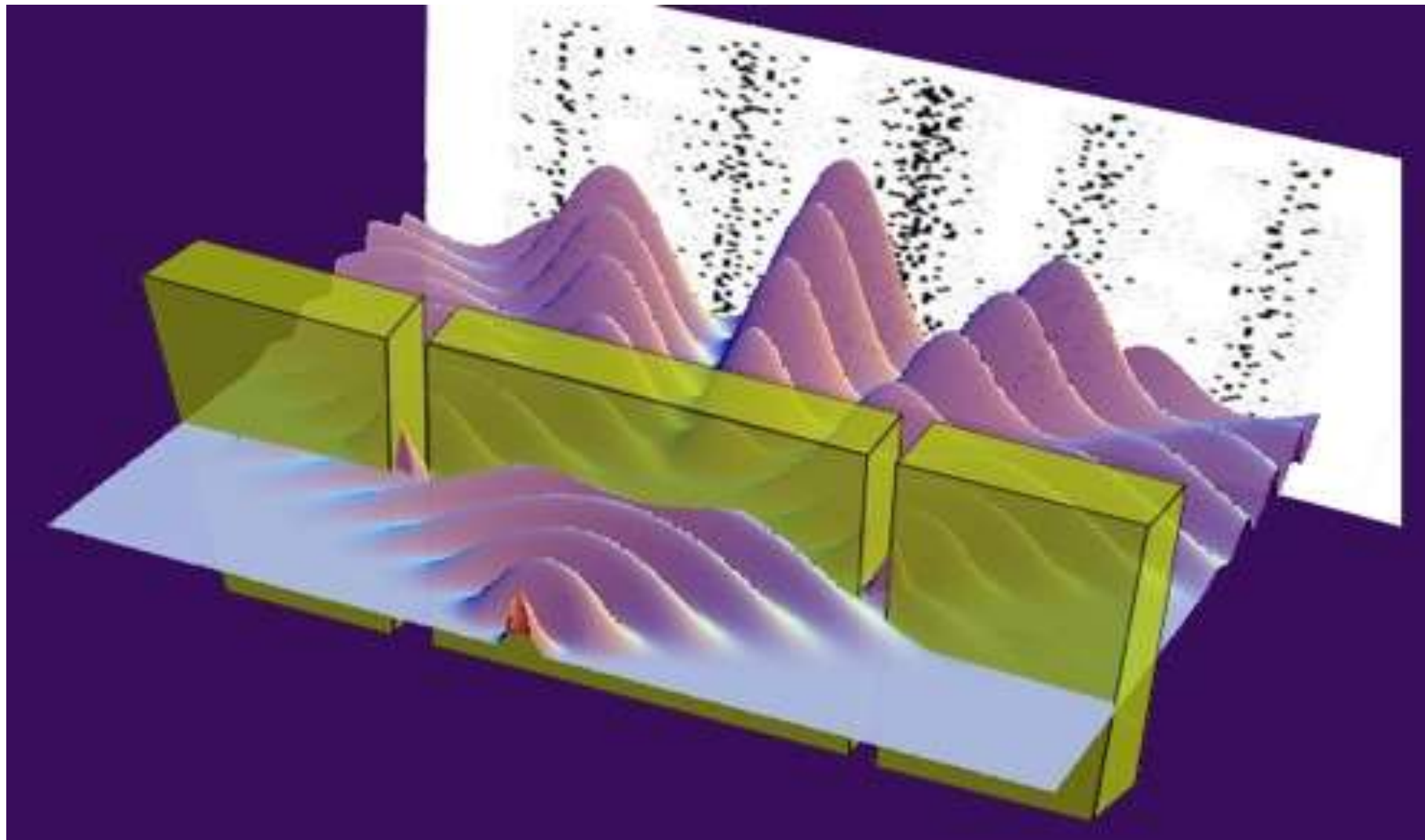
- ✓ Atom gyroscopes, atom accelerometers, atom gravi/gradiometers
- ✓ How to reach high precision and accuracy
- ✓ Some examples

LECTURE 3 : APPLICATION TO GEOPHYSICS AND FUNDAMENTAL PHYSICS - FROM GROUND TO SPACE

- ✓ Equivalence principle tests with atom interferometry
- ✓ Gravity antenna and gravitational wave detections

LECTURE 1 : INTRODUCTION TO ATOM INTERFEROMETRY AND INERTIAL SENSORS

- ✓ Atom Interferometry consists in physical phenomena where the wave nature of atoms (and in particular the external degrees of freedom) is concerned.
 - ⇒ Analogous to light waves phenomena, but for de Broglie waves



A monokinetic beam of particle can be described by wave function $\psi(\vec{r}, t)$

✓ The propagation is described by the Schrödinger equation

$$-\frac{\hbar^2}{2m} \Delta \psi(\vec{r}, t) = E \psi(\vec{r}, t) \quad \Rightarrow \quad \Delta \psi(\vec{r}, t) + k^2 \psi(\vec{r}, t) = 0 \quad k^2 = \frac{2mE}{\hbar^2}$$

✓ Analogy with electromagnetism : $\Delta E + k^2 E = 0, k = \frac{2\pi}{\lambda}$

✓ Analogy with optics :

$$\psi(x, y, z) = \psi_0 \exp \left[i \frac{m}{2\hbar T} (x^2 + y^2) \right]$$

A monokinetic beam of particle can be described by wave function $\psi(\vec{r}, t)$

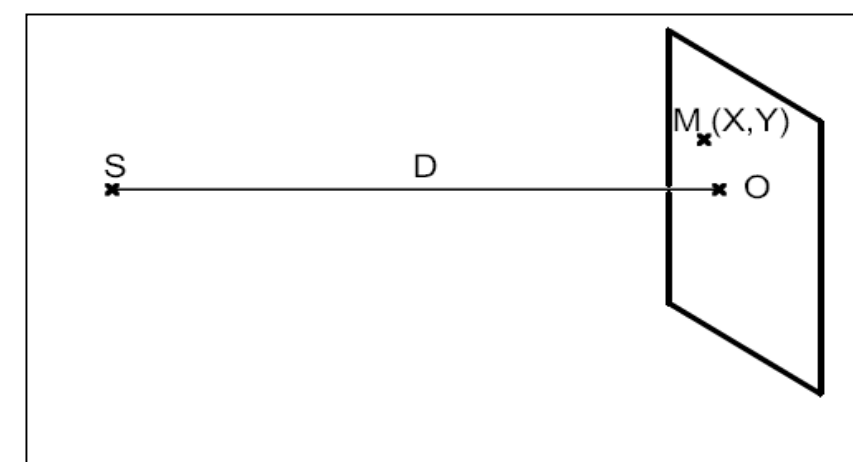
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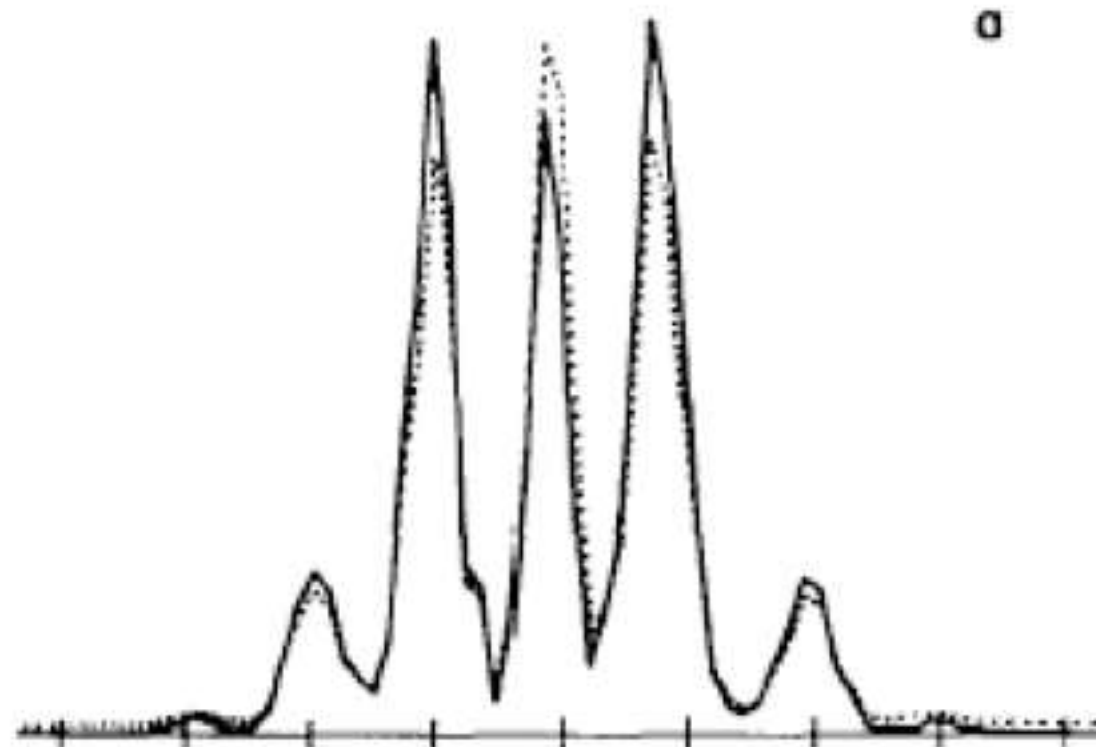
$$\psi(x, y, z) = \psi_0 \exp\left(\frac{ik(x^2 + y^2)}{2D}\right)$$



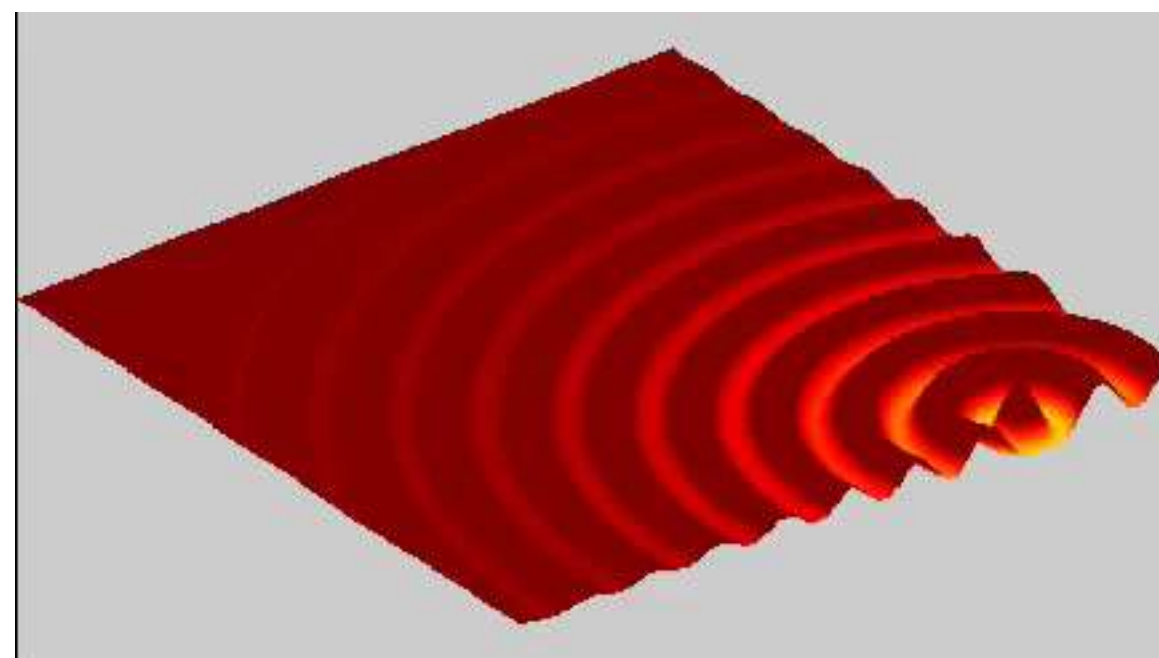
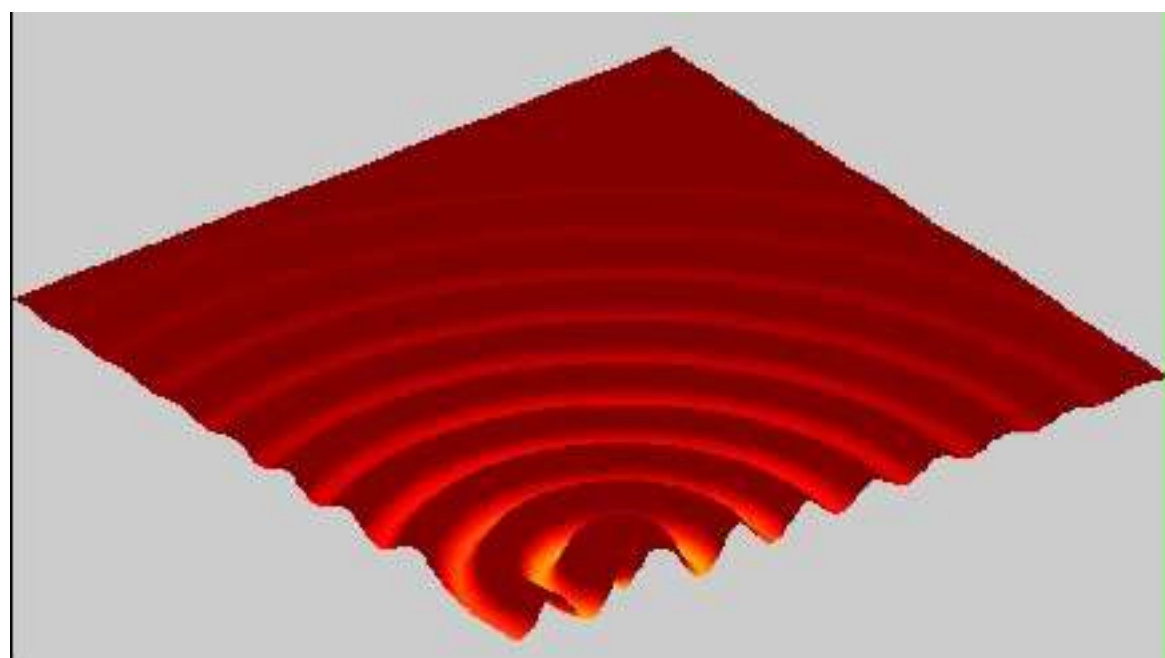
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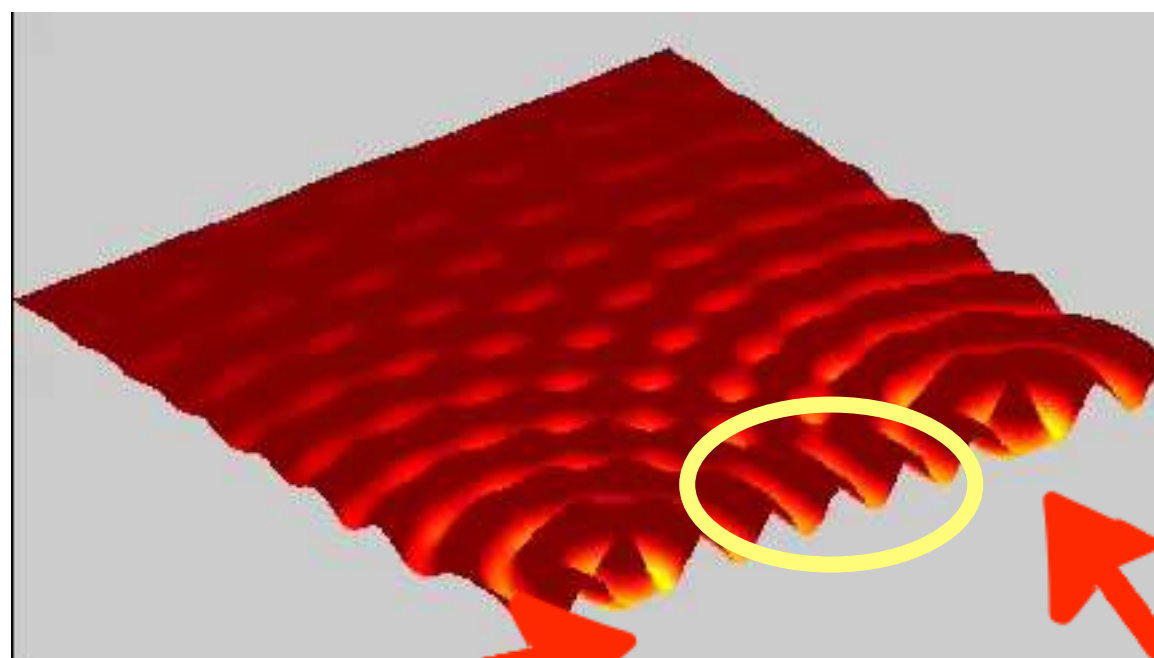


P. Gould et al. 1986



$$e^{-\frac{|\mathbf{r}+\mathbf{r}_0|}{l}} \cos(|\mathbf{r} + \mathbf{r}_0| + \omega t)$$

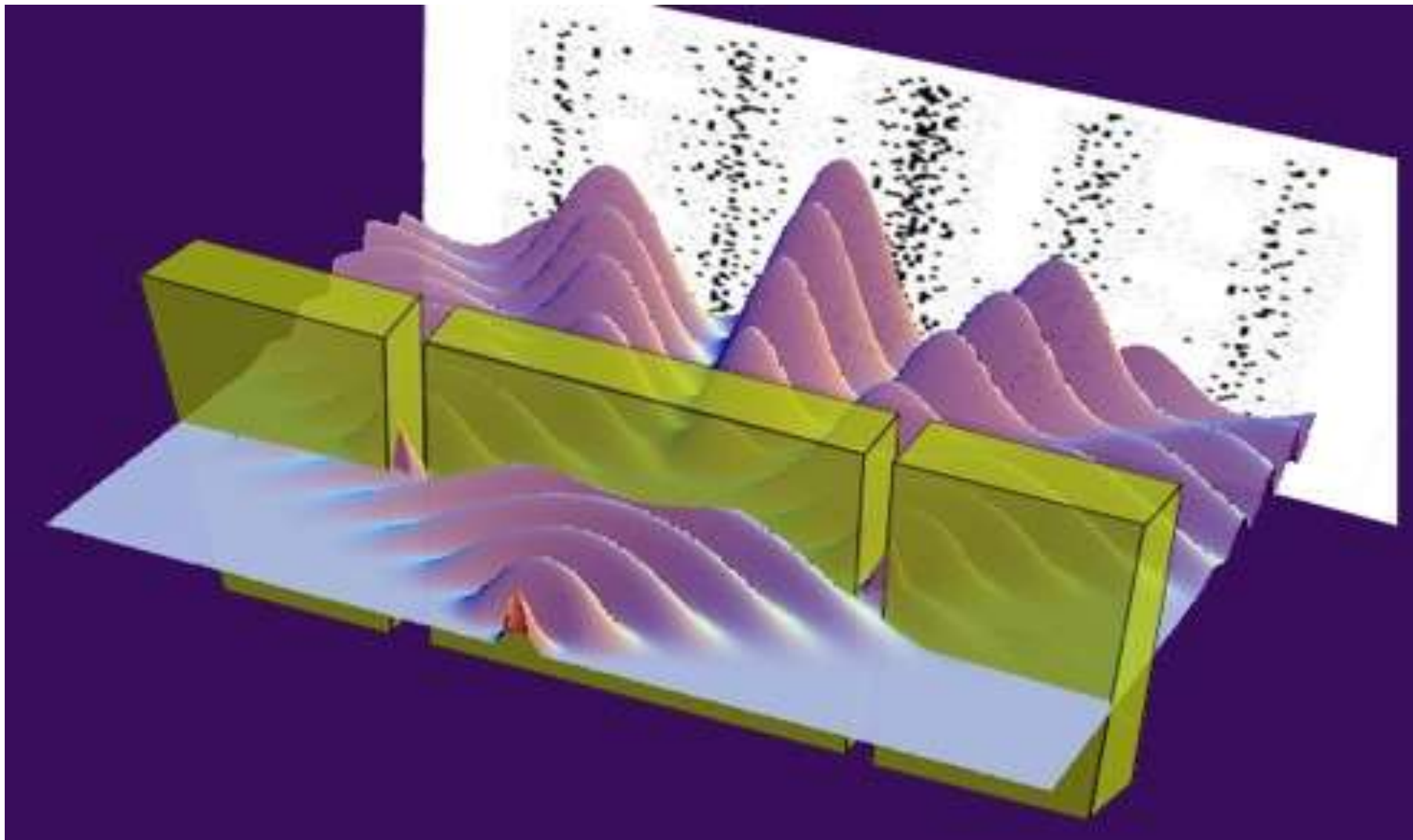
$$e^{-\frac{|\mathbf{r}-\mathbf{r}_0|}{l}} \cos(|\mathbf{r} - \mathbf{r}_0| + \omega t)$$

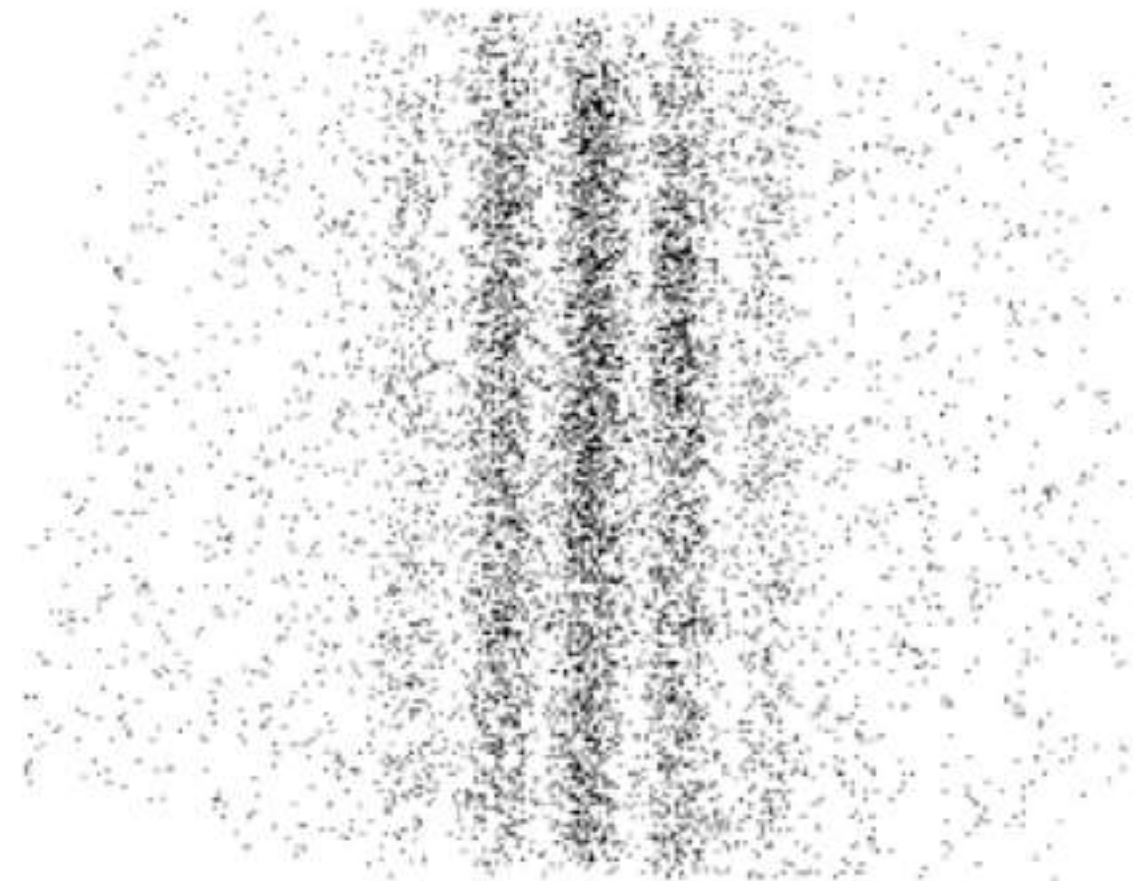
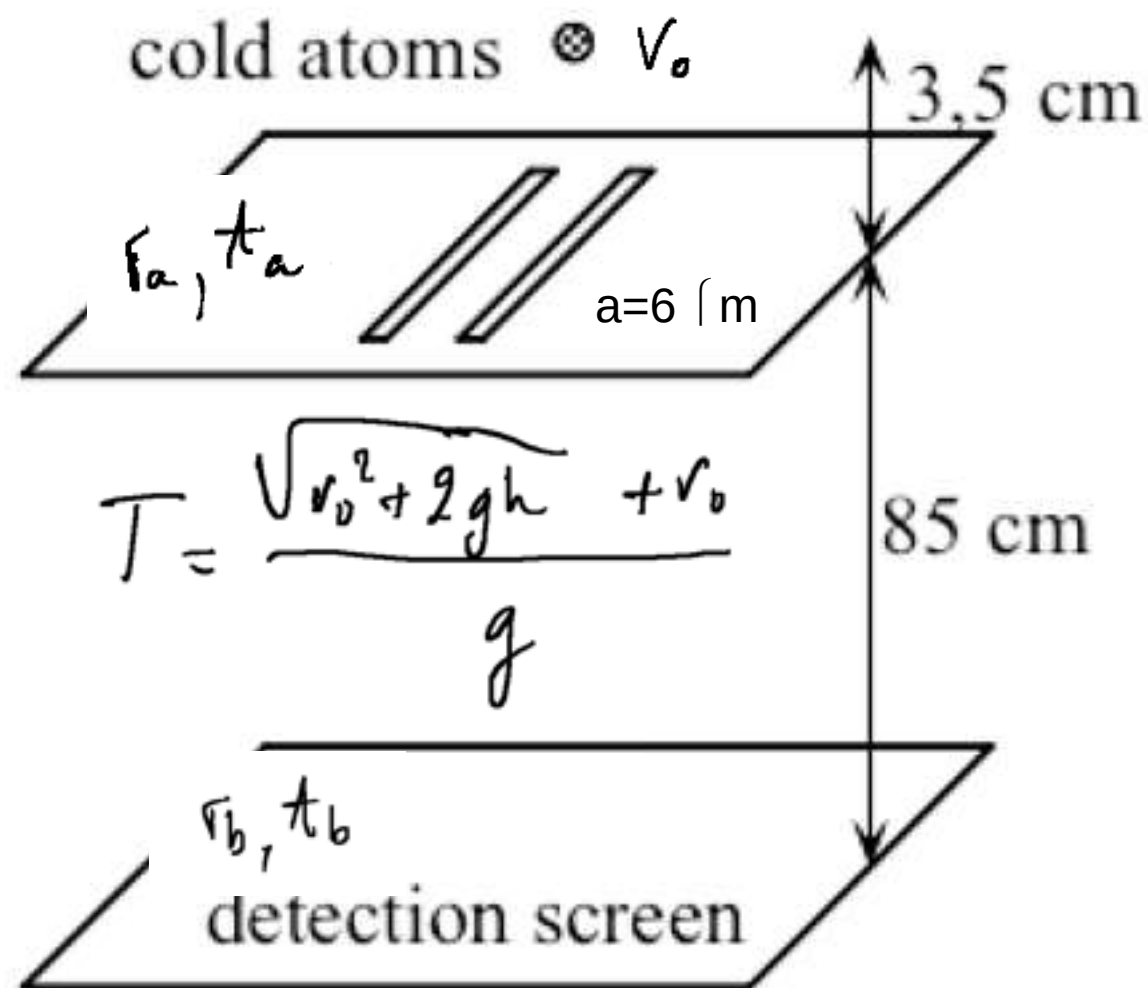


$$e^{-\frac{|\mathbf{r}+\mathbf{r}_0|}{l}} \cos(|\mathbf{r} + \mathbf{r}_0| + \omega t)$$

$$e^{-\frac{|\mathbf{r}-\mathbf{r}_0|}{l}} \cos(|\mathbf{r} - \mathbf{r}_0| + \omega t)$$

YOUNG DOUBLE SLITS EXPERIMENT



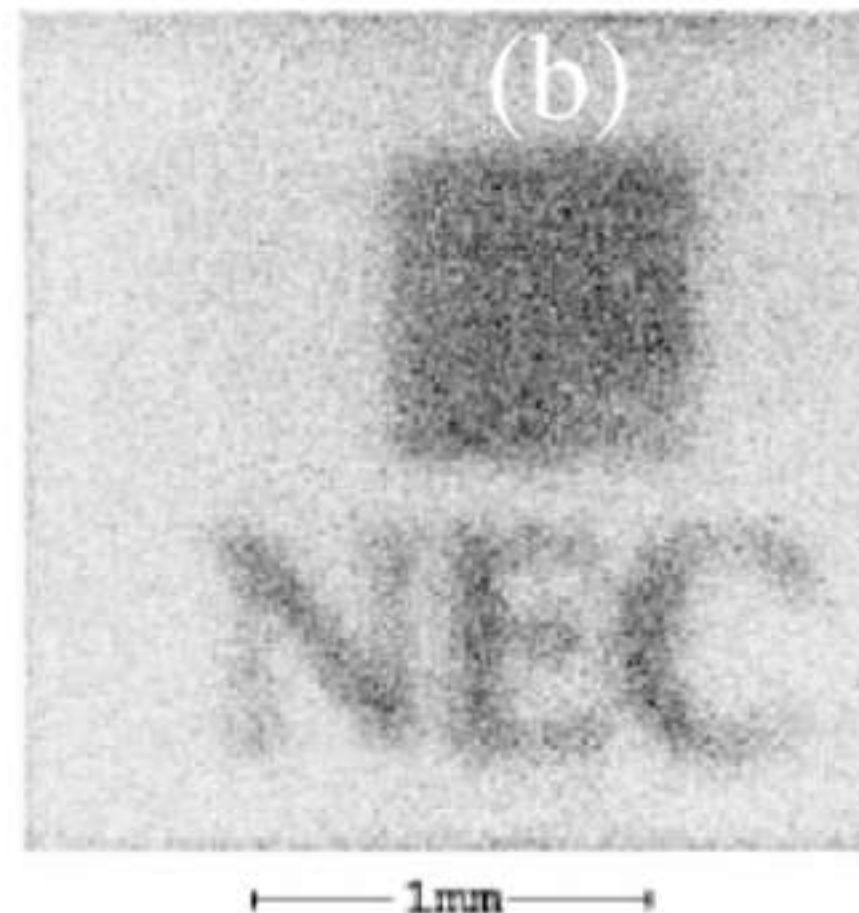
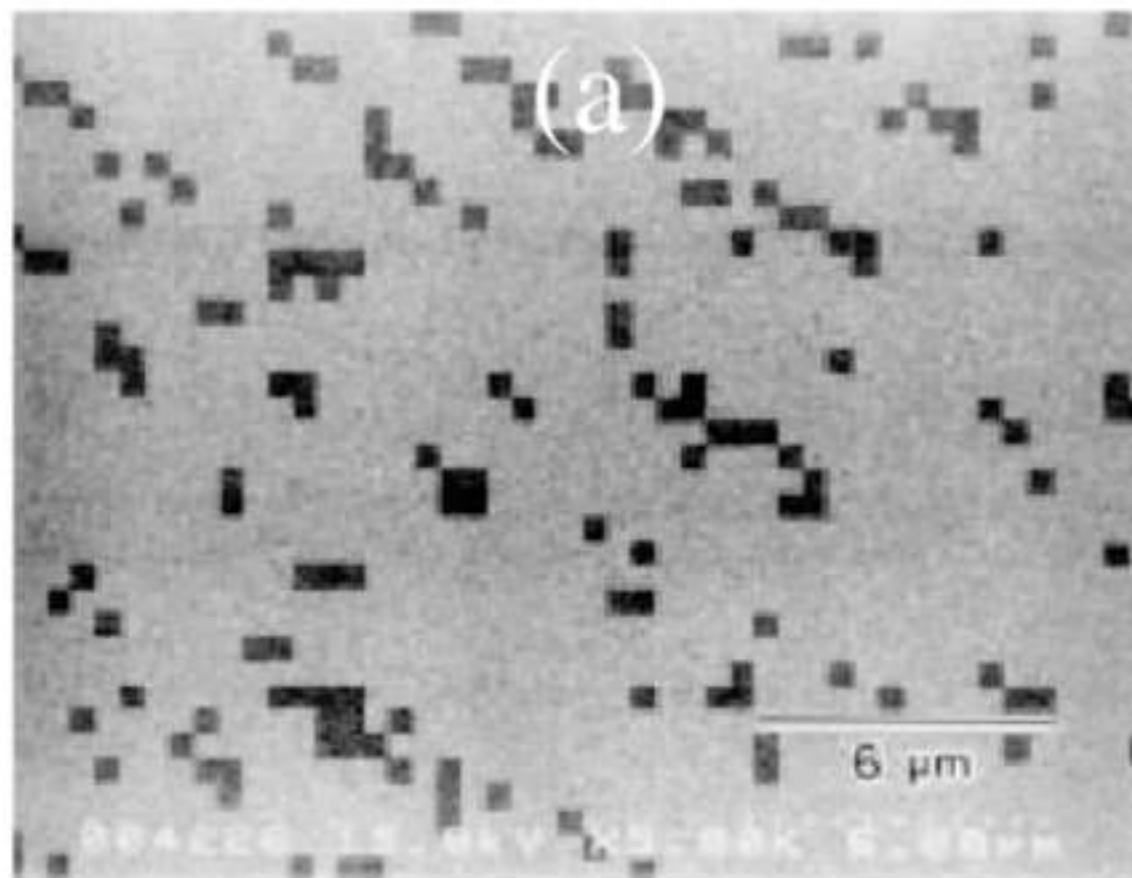


A scale bar indicates a length of 1 cm . Below the scale bar, the following equation is written:

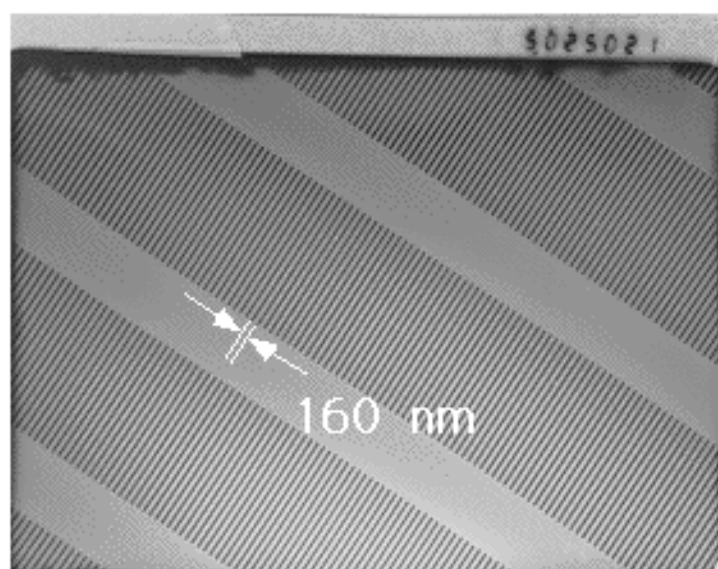
$$|\psi(r_b, t_b)|^2 \sim \cos^2 \left(\frac{m \lambda b a}{2 h T} \right)$$

F. Shimizu et al., Phys. Rev A, 46 R17 (1992)

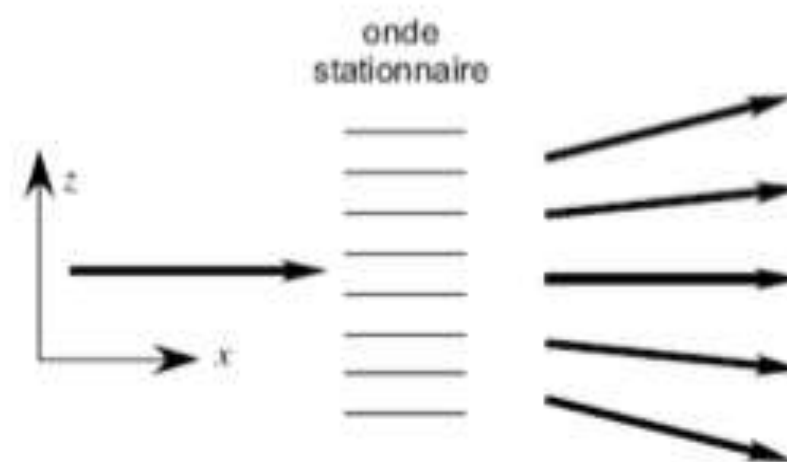
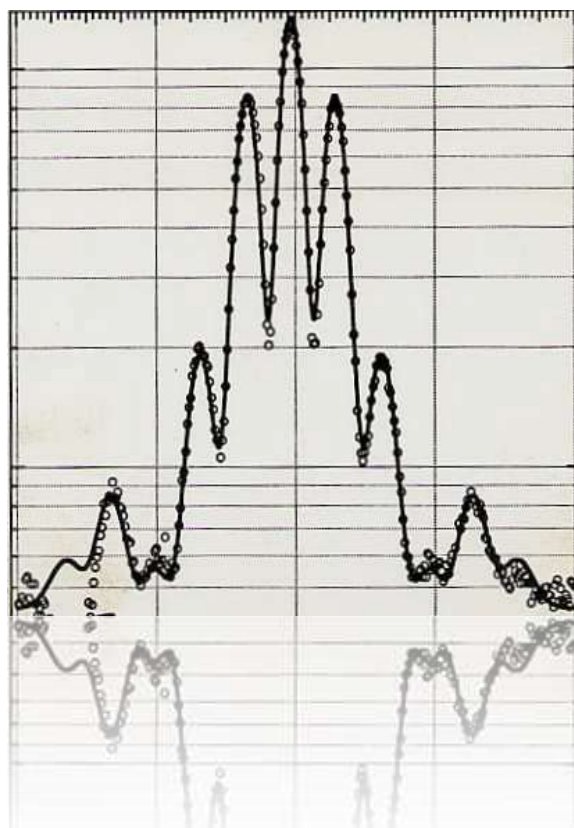
The extension to many holes gives **atom holography**, where an arbitrary pattern of matter is obtained $\Rightarrow$ Fresnel lenses or even more complex... (Shimizu)



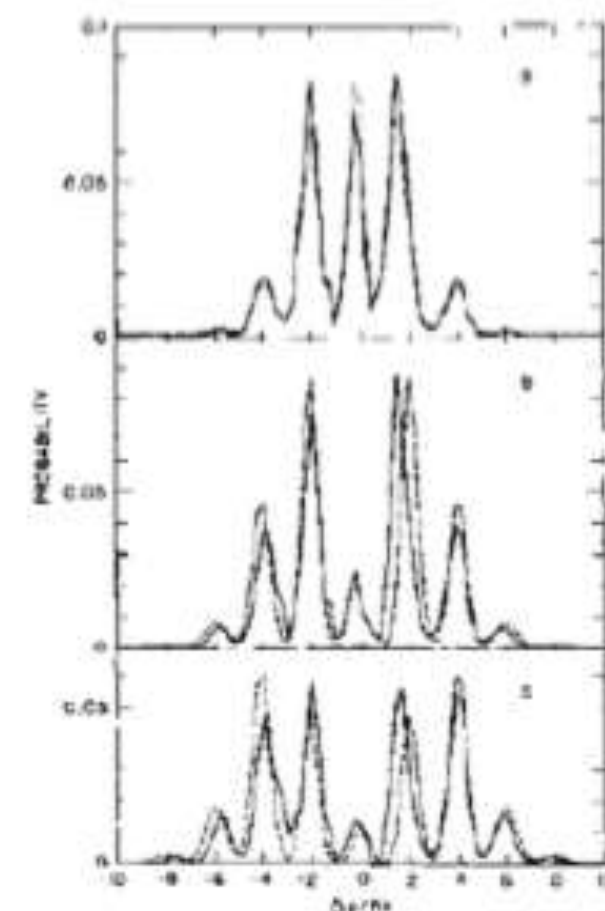
The atomic pattern (b) is the Fourier transform of the mask (a).



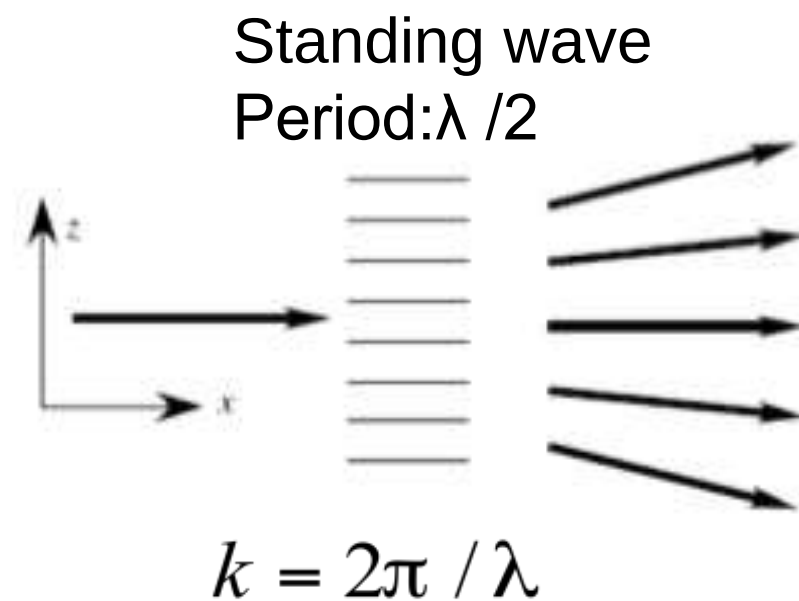
D. Keith et al (1988)



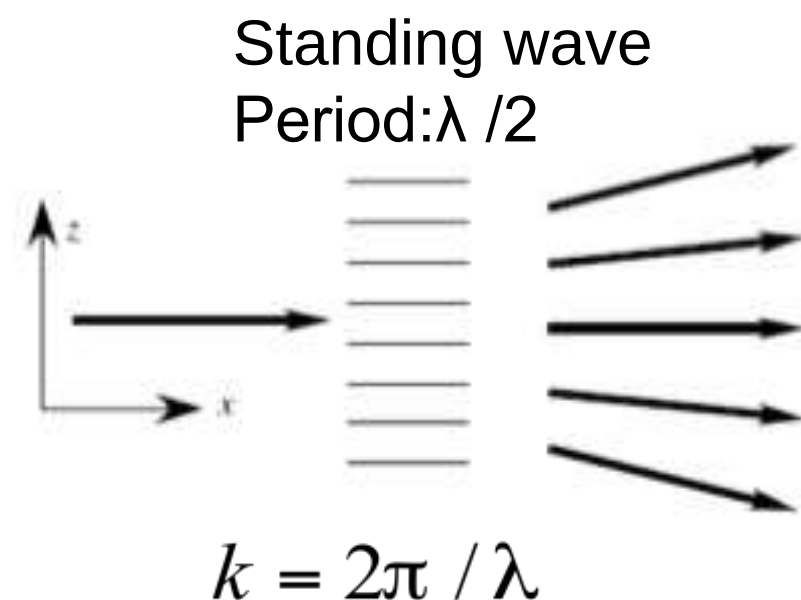
P. Gould et al. (1986)



- Material grating : amplitude modulation = loss
- Laser standing wave : phase modulation = 100 % transmission



- ✓ 1D along z , quasi monochromatic wave packet:
 - mean momentum p_0 ,
 - $\Delta p_z \ll \hbar k$

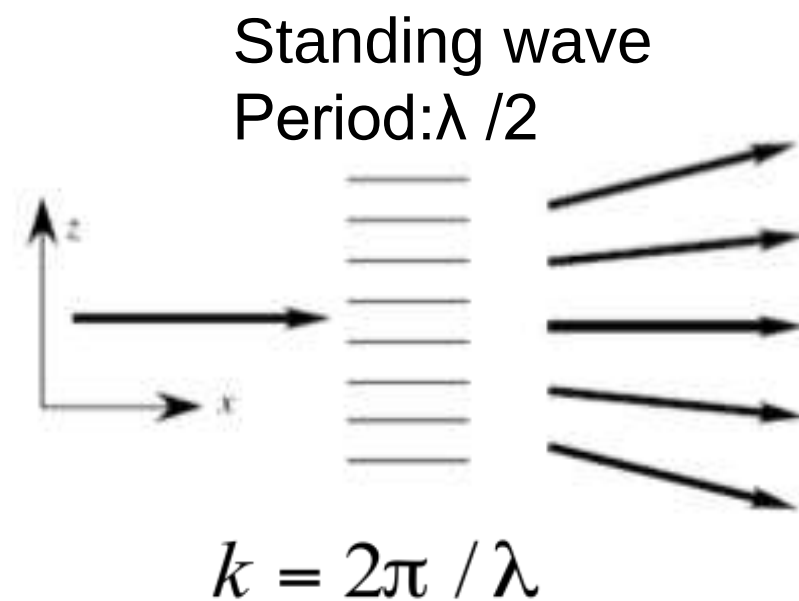


- ✓ 1D along z , quasi monochromatic wave packet:
 → mean momentum p_0 ,
 $\Delta p_z \ll \hbar k$

- ✓ Hamiltonian describing the grating :

$$H = \frac{p_z^2}{2m} + U_0 \sin^2 kz = \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{2} \cos 2kz$$

$$= \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{4} (e^{2ikz} + e^{-2ikz})$$



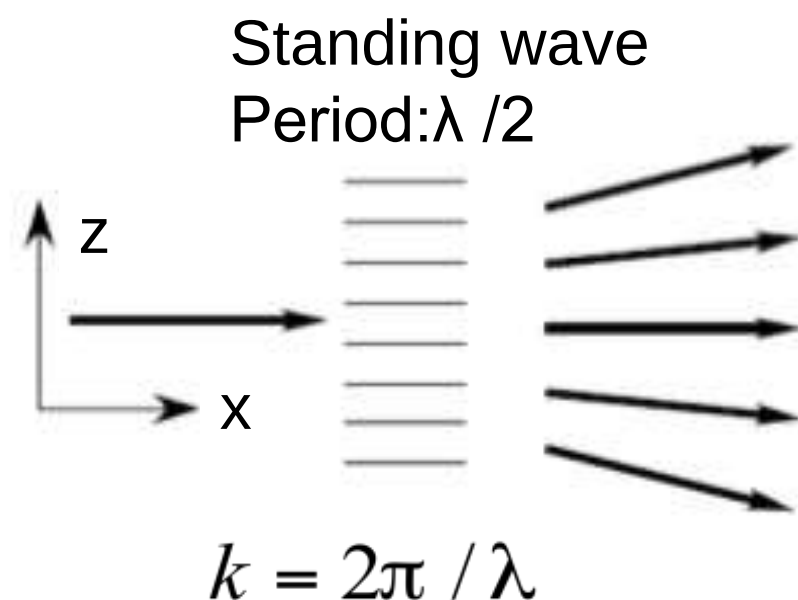
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$$= \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{4} (e^{2ikz} + e^{-2ikz})$$

- ✓ Evolution of wavefunction :

$$\psi(T) = e^{-iHT/\hbar} |\psi(0)\rangle \text{ with } |\psi(0)\rangle = |p_0\rangle$$



- ✓ 1D along z , quasi monochromatic wave packet:
→ mean momentum p_0 ,
 $\Delta p_z \ll \hbar k$

- ✓ Hamiltonian describing the evolution in grating :

$$H = \frac{p_z^2}{2m} + U_0 \sin^2 kz = \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{2} \cos 2kz$$

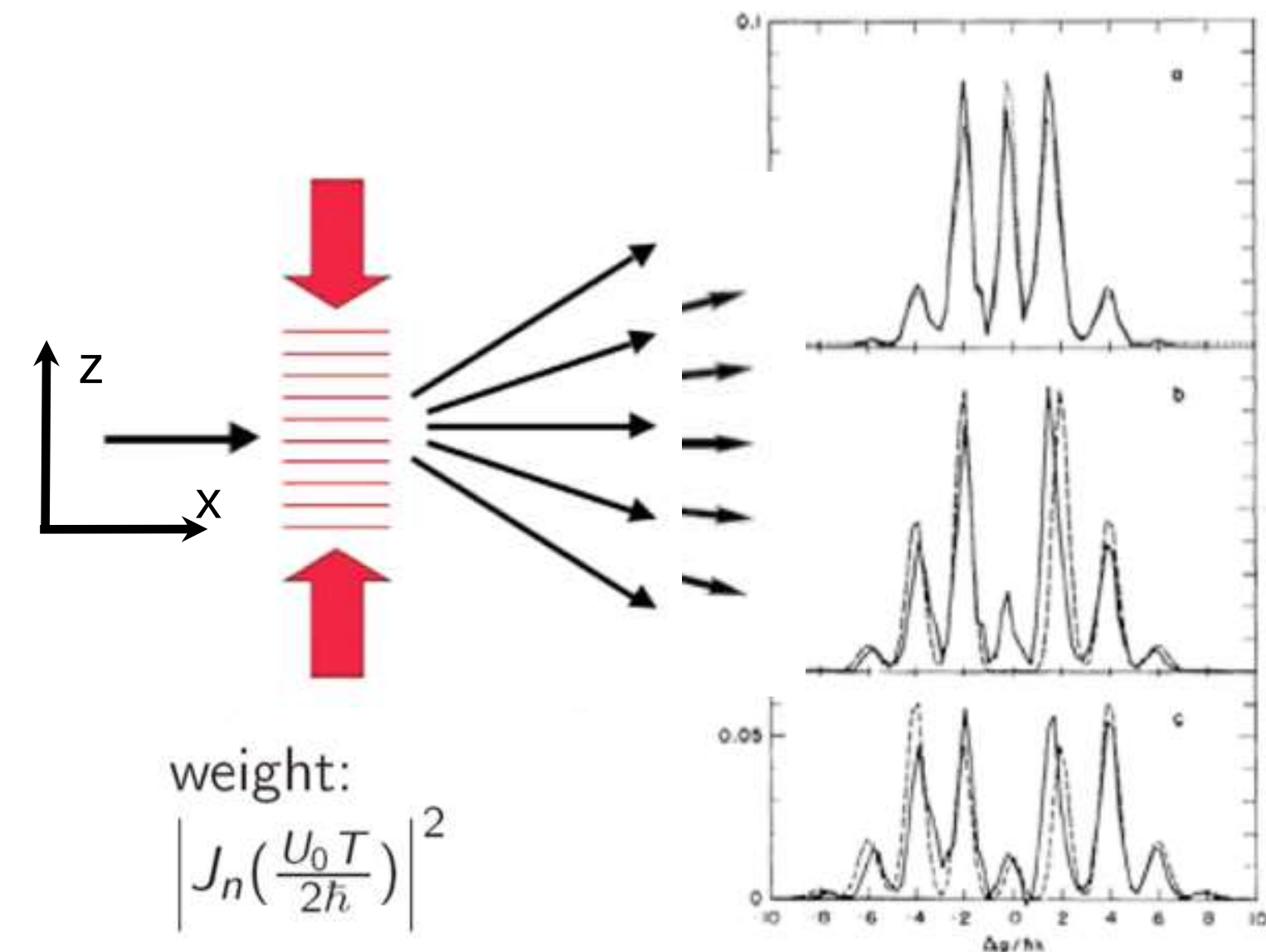
$$= \frac{p_z^2}{2m} + \frac{U_0}{2} - \frac{U_0}{4} (e^{2ikz} + e^{-2ikz})$$

- ✓ First we neglect motion along z during T

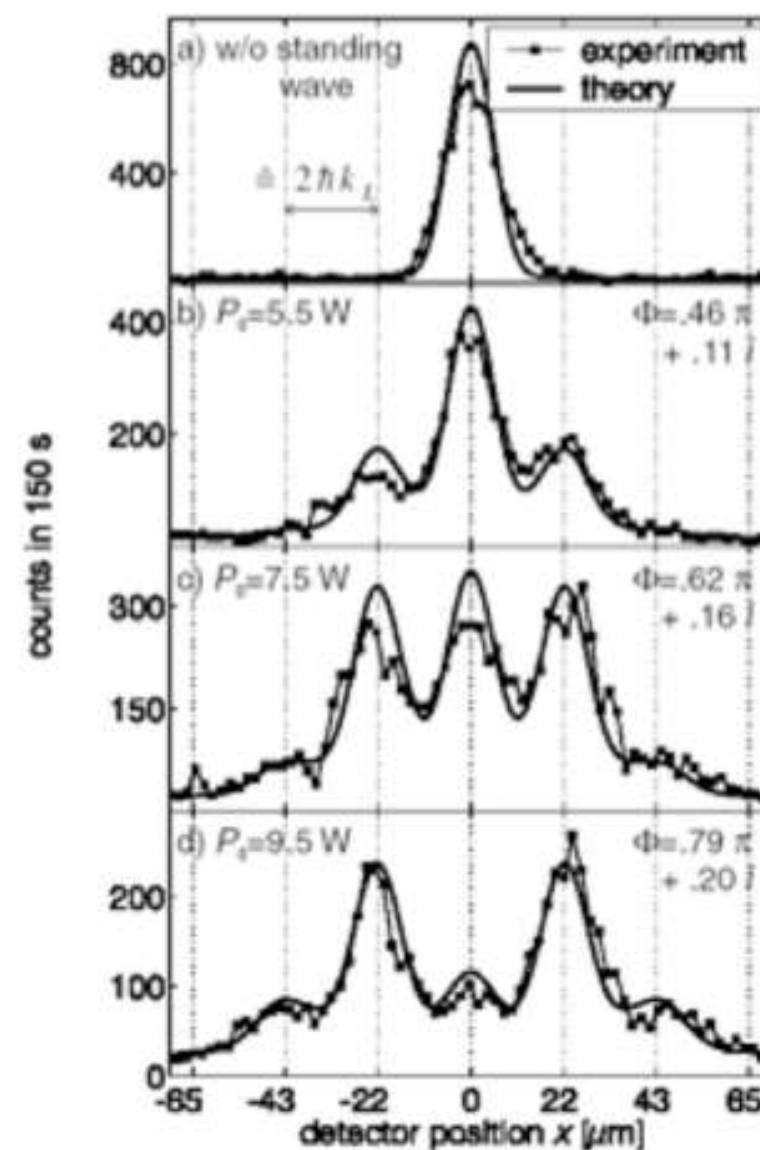
**Thin grating
approximation**

$$\psi(T) = e^{i(U_0 T / 2\hbar) \cos(2kz)} |p_0\rangle = \left(\sum_{n=-\infty}^{n=+\infty} (i)^n J_n \left(\frac{U_0 T}{2\hbar} \right) e^{-2inkz} \right) |p_0\rangle$$

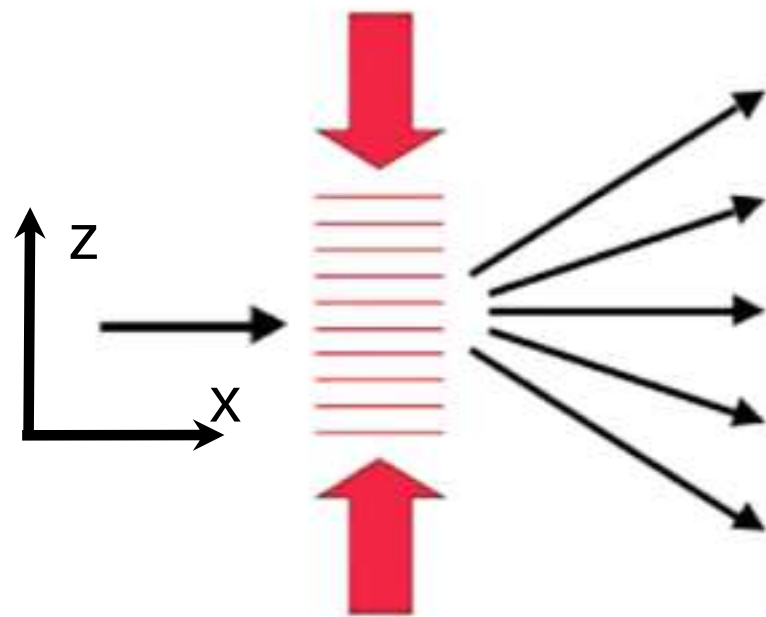
$$= \sum_{n=-\infty}^{n=+\infty} (i)^n J_n \left(\frac{U_0 T}{2\hbar} \right) |p_0 - 2n\hbar k\rangle$$



sodium atoms
Pritchard, 1985



C_{60} molecules
Zeilinger, 2001



weight:

$$\left| J_n\left(\frac{U_0 T}{2\hbar}\right) \right|^2$$

Stationary problem: total energy is conserved

For small momenta along z , $p_0 \leq \hbar k$ and $n \sim 1$

The kinetic energy along z changes by $\Delta E_z = \left| \frac{p_z^2}{2m} - \frac{(p_z \pm 2\hbar k)^2}{2m} \right| \sim E_R$

It must be compensated by a change along x : $\Delta E_z = -\Delta E_x$

Finite transit time enables momentum changes along x of $\sim \hbar / w$

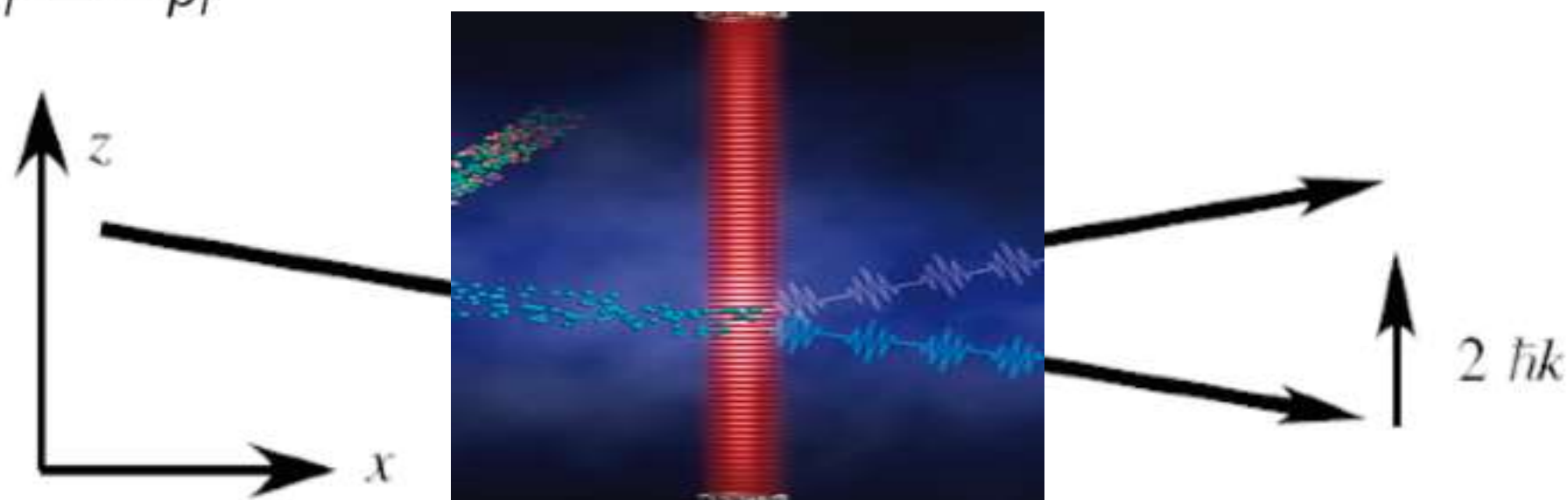
Change in kinetic energy $\Delta E_x = \frac{(p_x \pm \hbar / w)^2}{2m} - \frac{p_x^2}{2m} \sim \frac{p_x \hbar}{mw} = \frac{\hbar}{T}$

→ As long as $\frac{\hbar}{T} \geq E_R$, the total energy can be conserved and exchanged between x and z

Thick grating : Bragg diffraction

Energy and momentum are conserved for particular initial momenta $p_0 = (2n - 1)\hbar k$ along z :

$$\Rightarrow p_f = -p_i$$

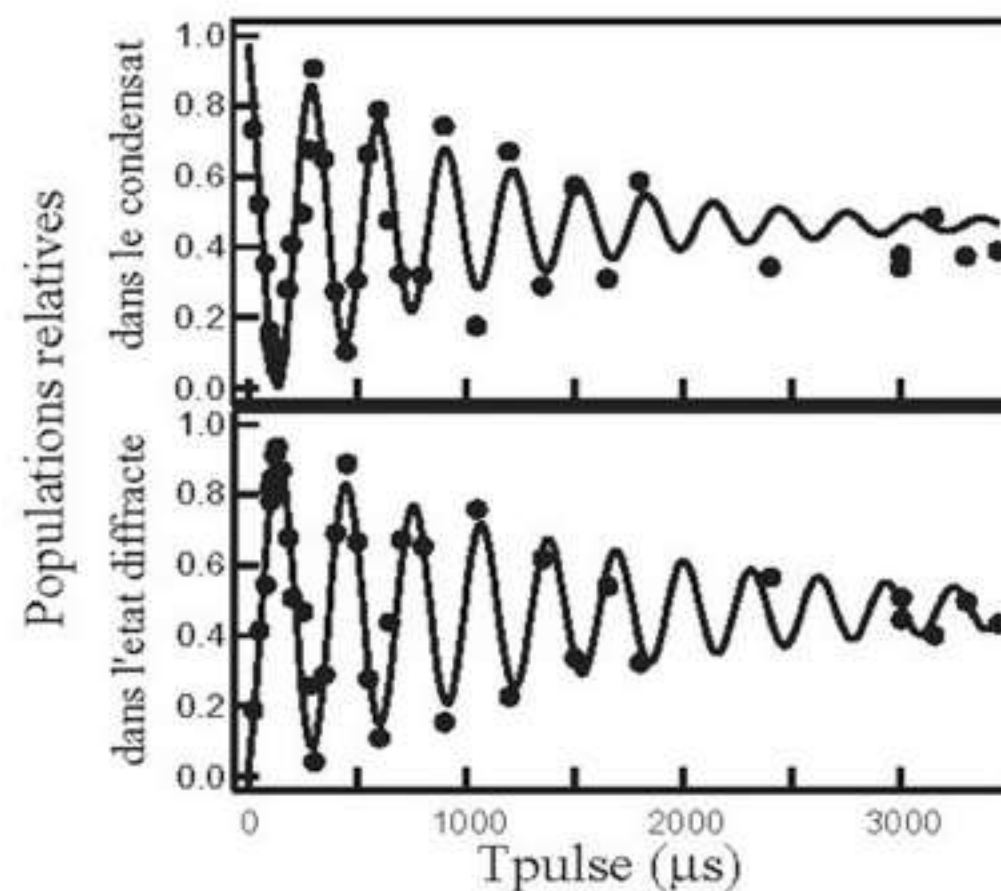
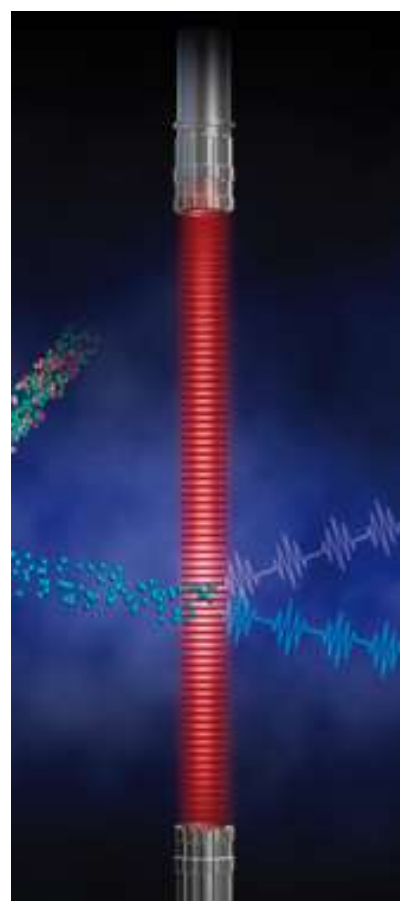


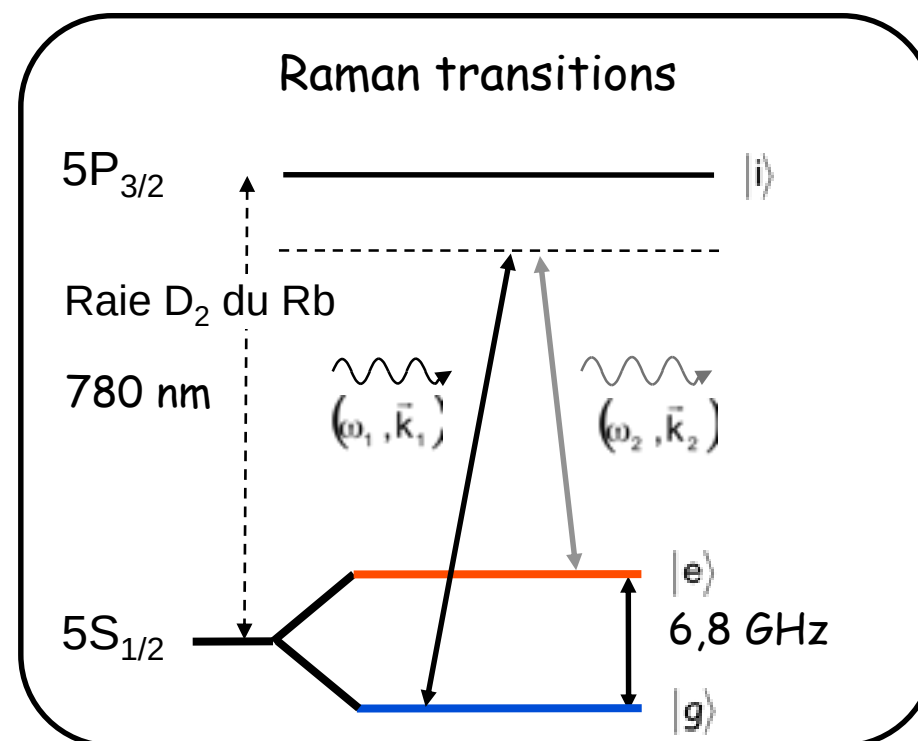
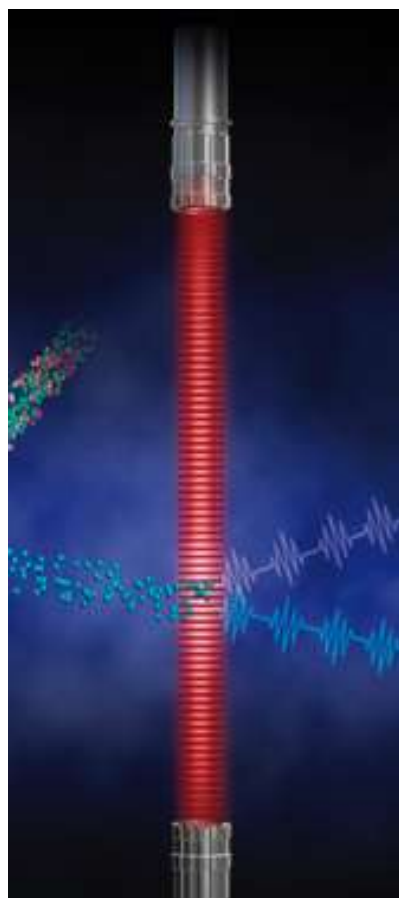
zeroth order: **Rabi oscillations** between $|p_z = -\hbar k\rangle$ and $|p_z = \hbar k\rangle \Rightarrow$ beamsplitter with adjustable weights!

- ✓ Absorption of photon from on laser and stimulated emission to the retroreflected beam

⇒ Rabi oscillation between 2 (or more) state

⇒ Different momenta since photons carry momentum

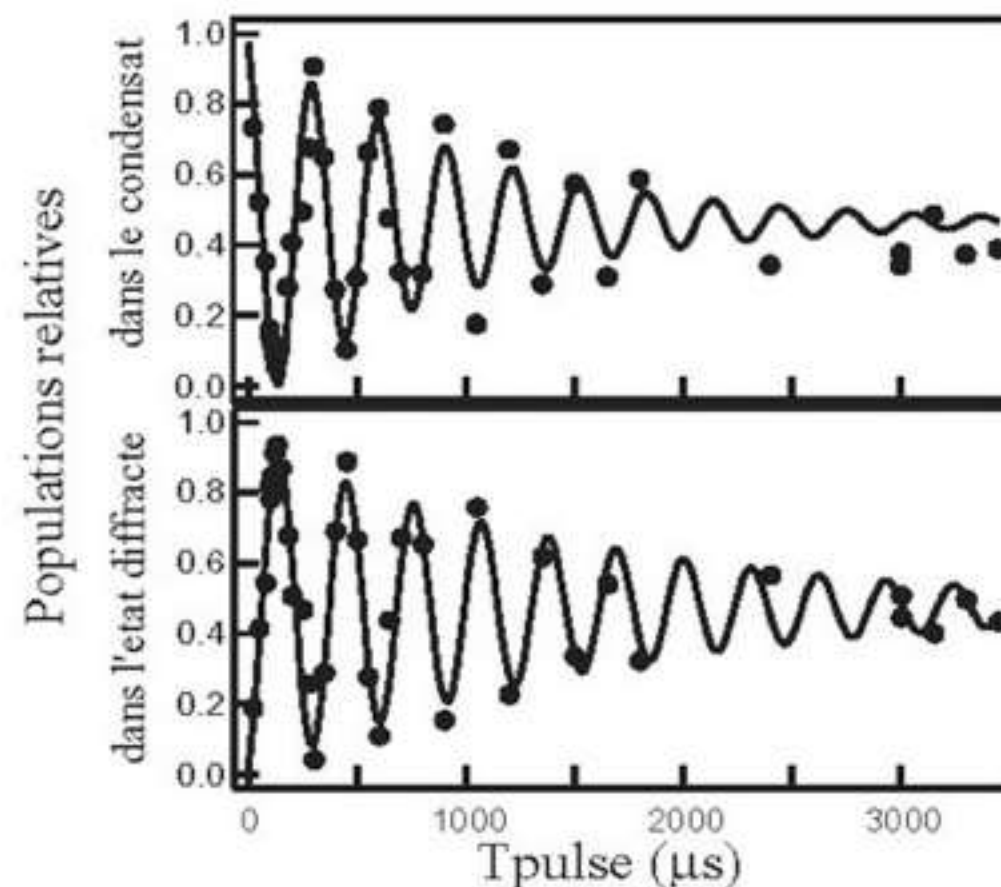
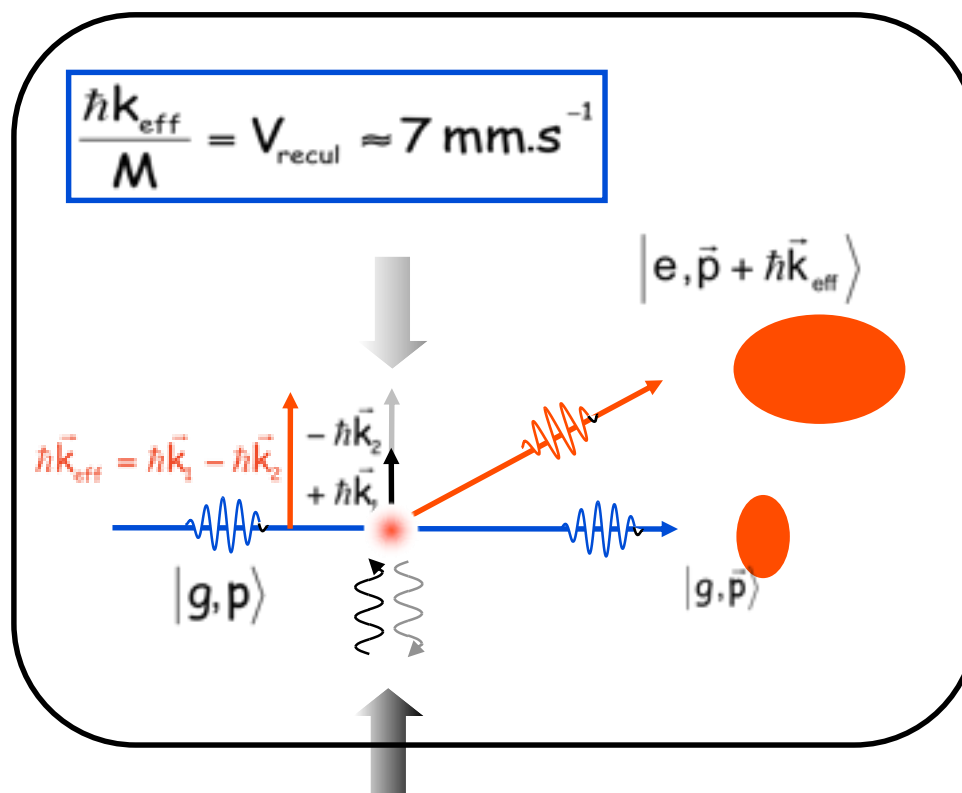




✓ Absorption of photon from on laser and stimulated emission to the retroreflected beam

⇒ **Rabi oscillation between 2 (or more) state**

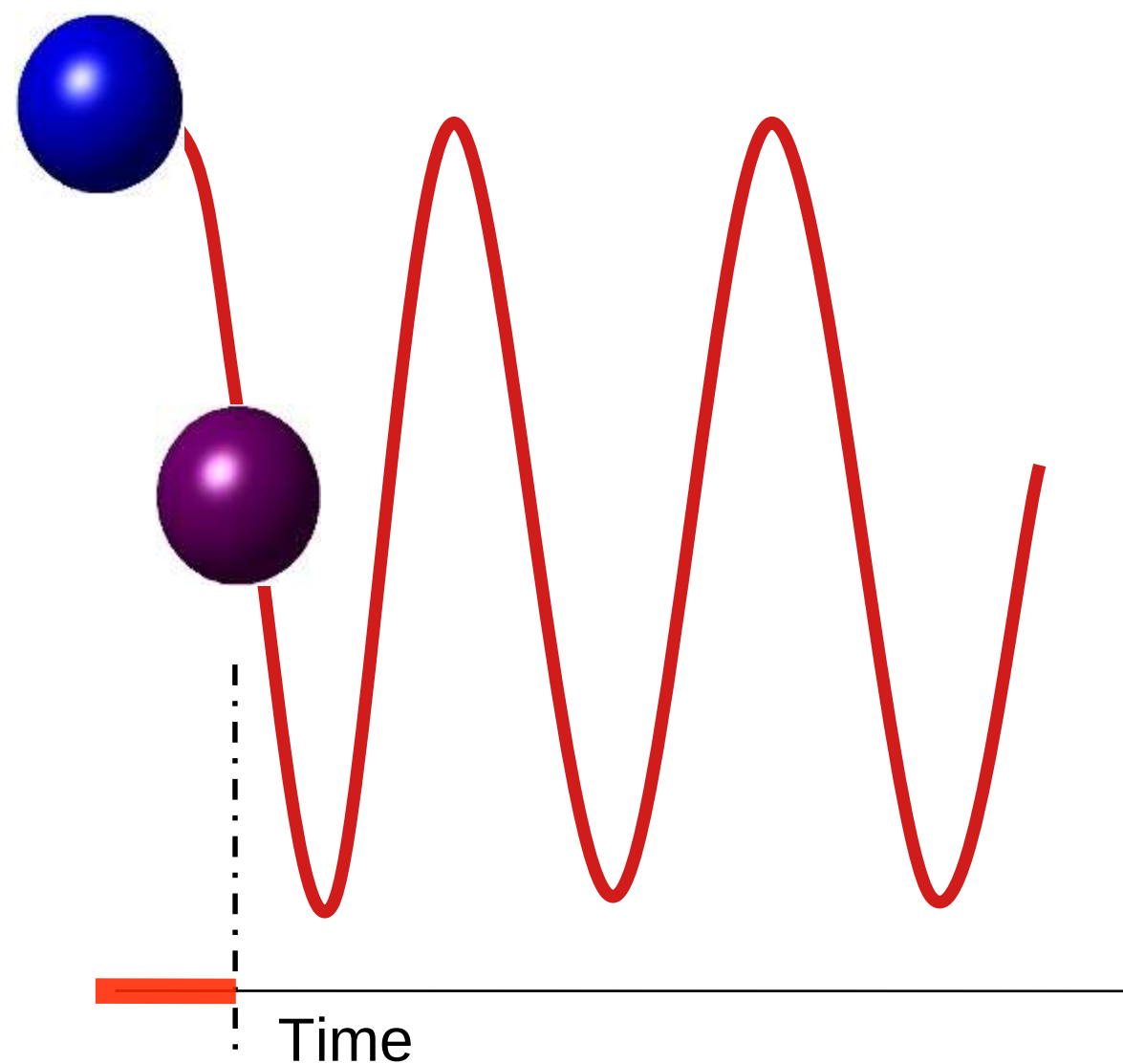
⇒ **Different momenta since photons carry momentum**



✓ Controlling the interaction time controls the result of the oscillation

⇒ Half way between red and blue

⇒ $\lambda/2$ pulse



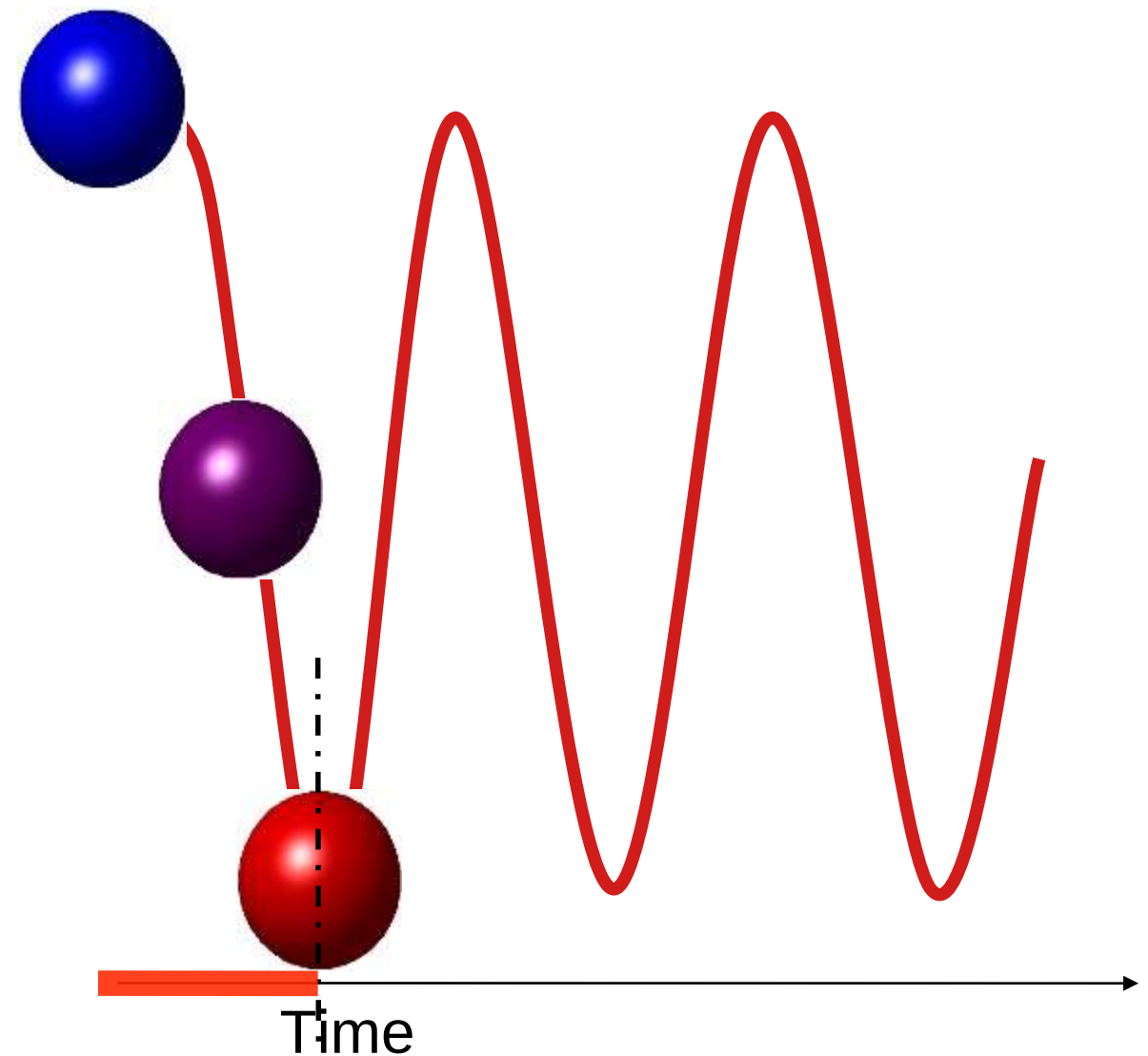
✓ Controlling the interaction time controls the result of the oscillation

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⇒ $\pi/2$ pulse

⇒ Another half : all the way from red to blue

⇒ π pulse



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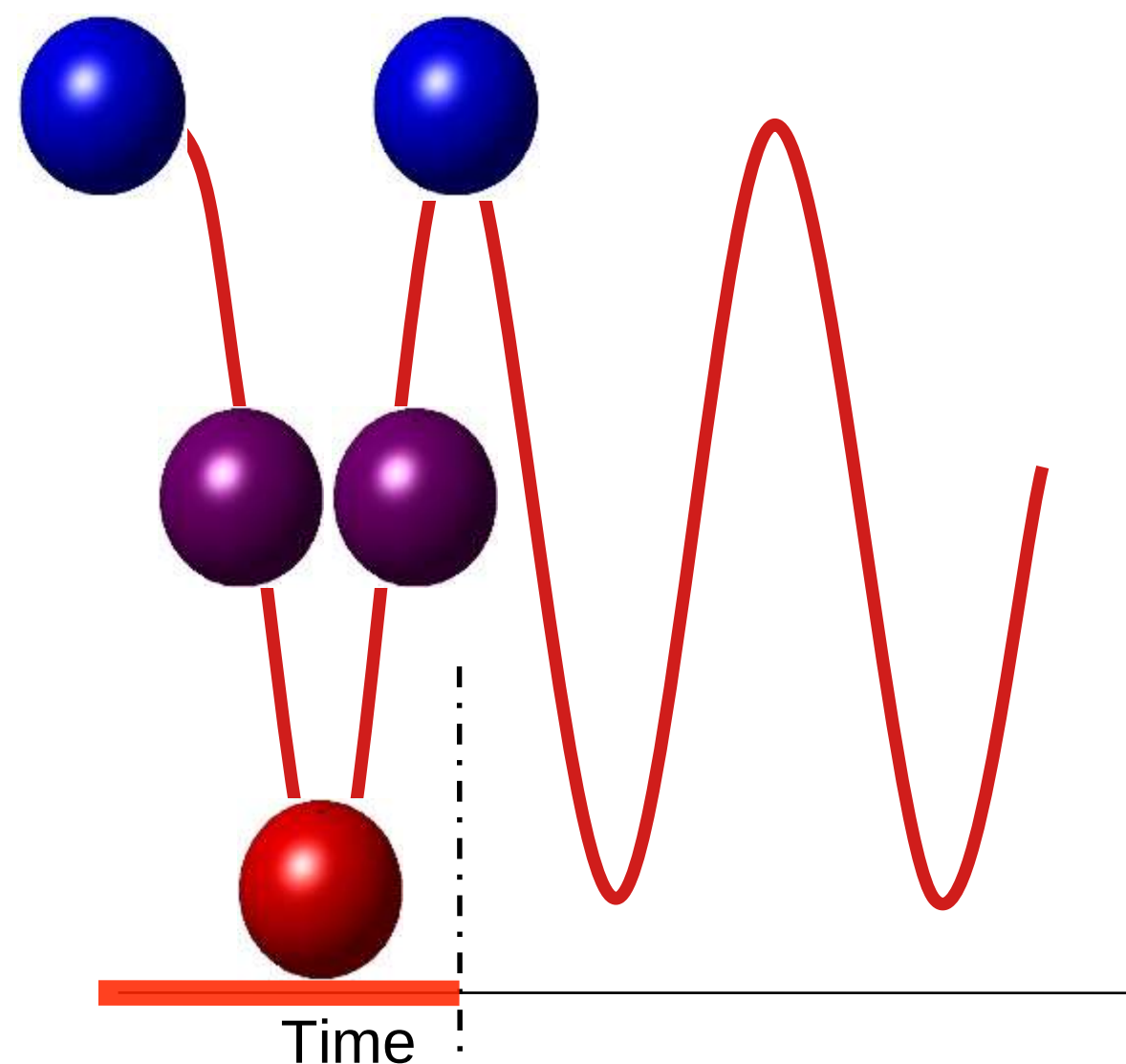
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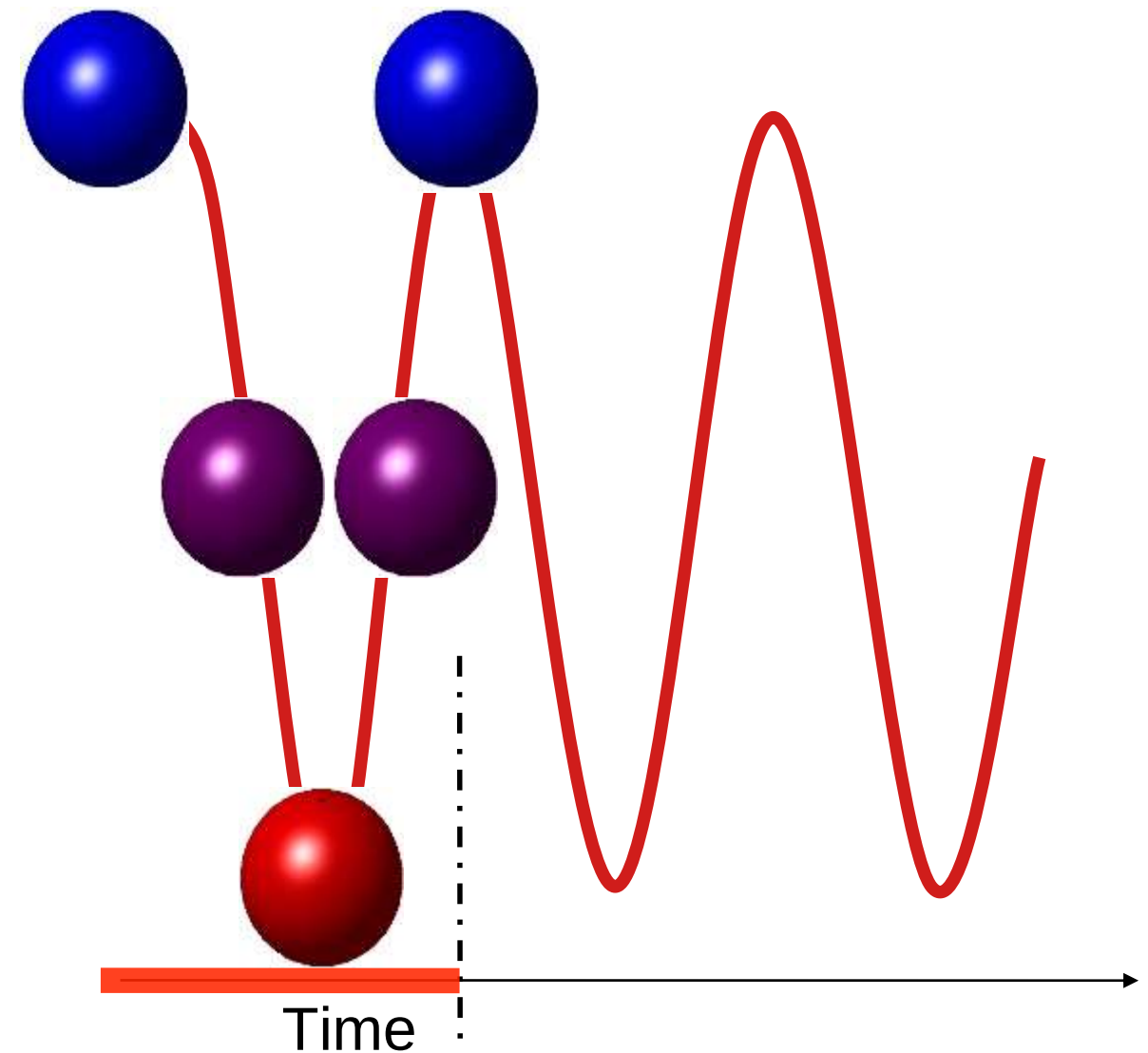
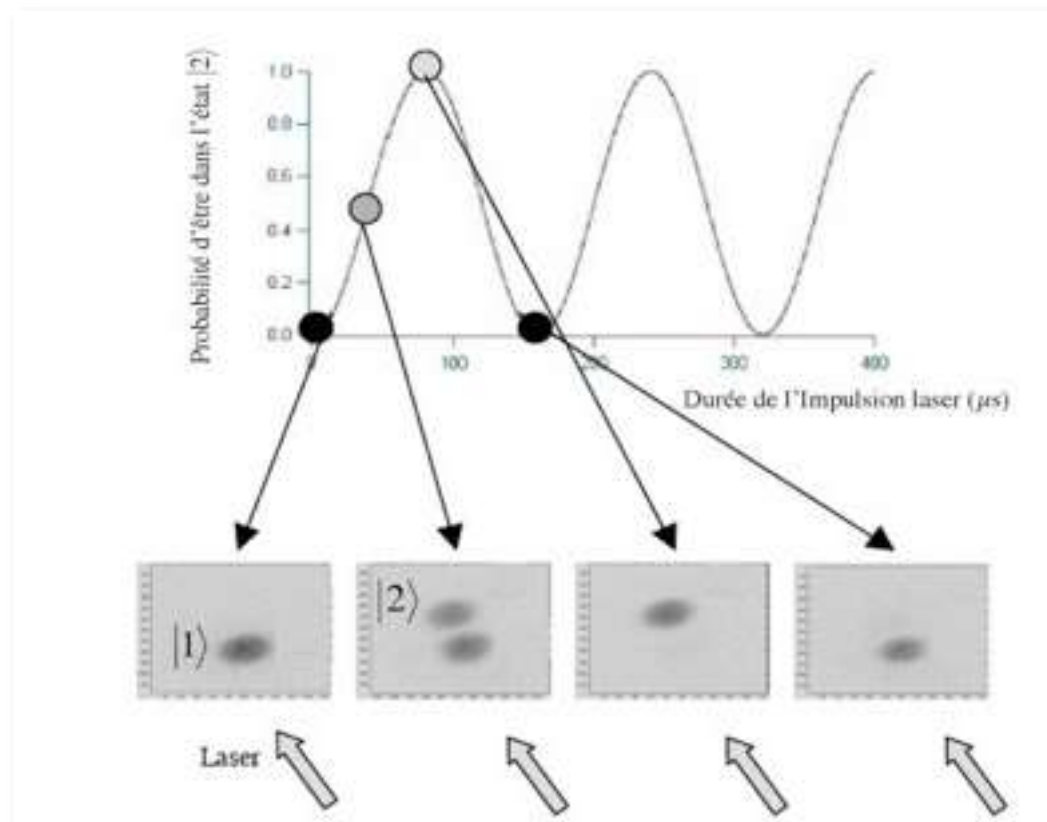
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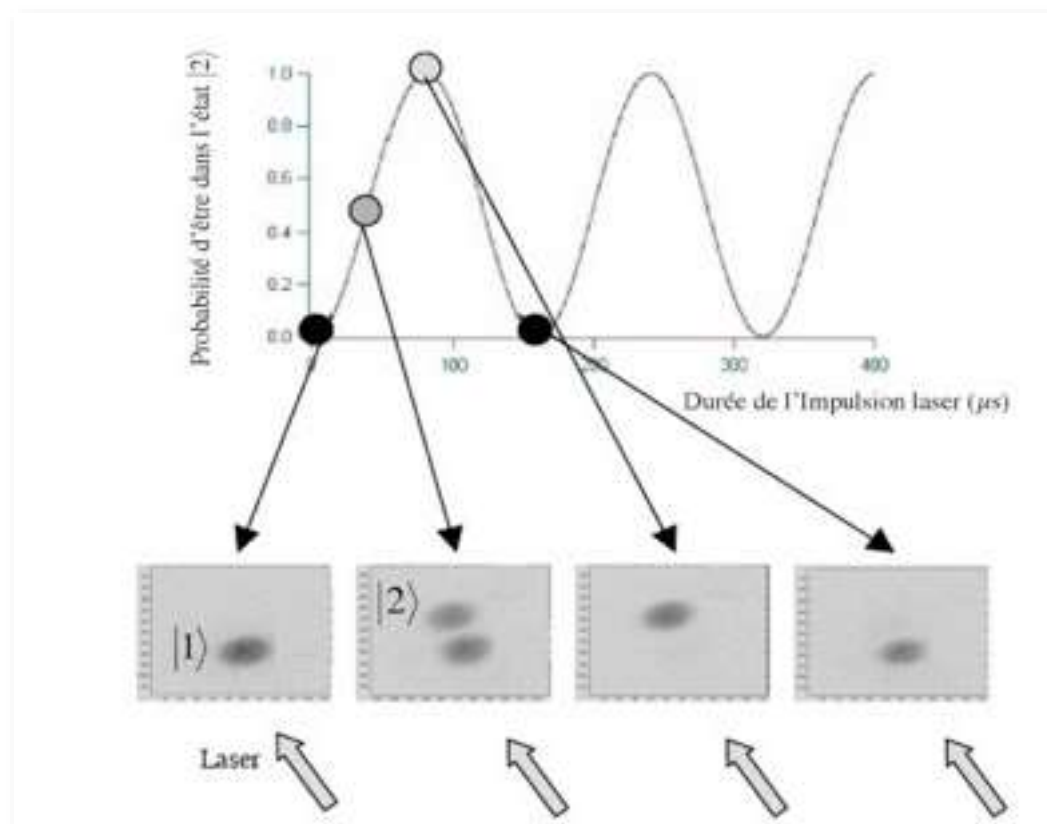
$$V_{\text{at-las}} = \frac{1}{2} \underbrace{|2\rangle\langle 1|}_{\text{Internal}} e^{iKz} \dots$$

external

$$V_{\text{at-las}} |1, p\rangle = \frac{1}{2} |2\rangle\langle 1| \otimes e^{iKz} |p\rangle$$

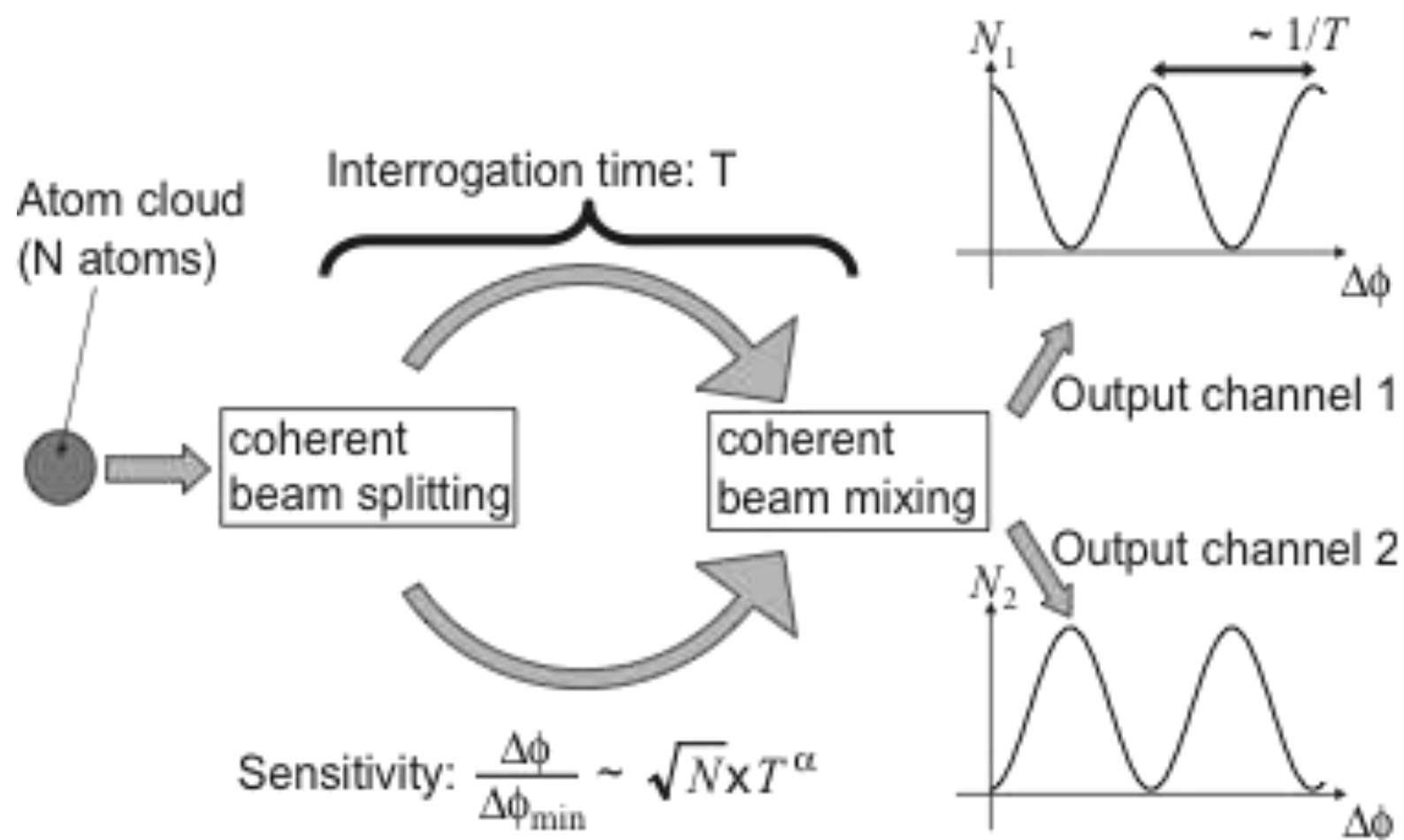
$$= |2\rangle \otimes |p + \frac{1}{2}K\rangle$$

$$|2, p + \frac{1}{2}K\rangle$$



$$|\psi(t)\rangle = C_1(t) |1, p\rangle + C_2(t) |2, p + \frac{1}{2}K\rangle$$

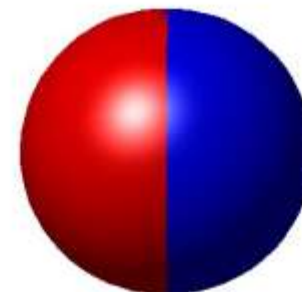
- ✓ An atom interferometer will use a series (at least two) coherent splitting processes to “create” multiple paths that will interfere.



- ✓ The first $\pi/2$ pulse - beam splitter
 - ⇒ Creates the coherent superposition



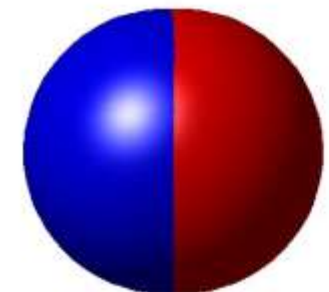
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 - ⇒ Creates the coherent superposition
- ✓ The two parts of the atom separate
 - ⇒ Splitting between the two parts



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 - ⇒ Creates the coherent superposition
- ✓ The two parts of the atom separate
 - ⇒ Splitting between the two parts
- ✓ Apply the π pulse - mirror
 - ⇒ Changes blue to red
 - ⇒ Velocity from 0 to recoil
 - ⇒ Changes red to blue
 - ⇒ Velocity from recoil to 0



- ✓ The first $\pi/2$ pulse - beam splitter
 - ⇒ Creates the coherent superposition
- ✓ The two parts of the atom separate
 - ⇒ Splitting between the two parts
- ✓ Apply the π pulse - mirror
 - ⇒ Changes blue to red
 - ⇒ Velocity from 0 to recoil
 - ⇒ Changes red to blue
 - ⇒ Velocity from recoil to 0
- ✓ Apply last $\pi/2$ pulse when the two parts overlap again

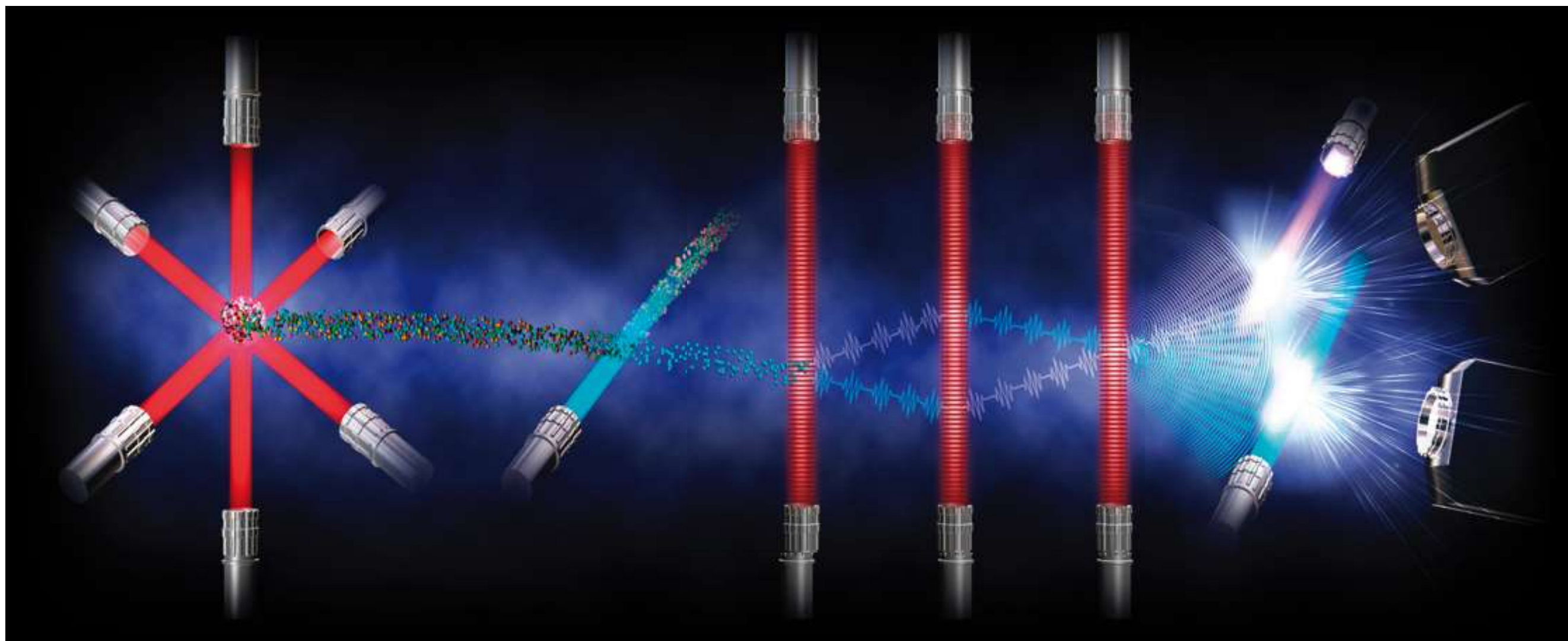


Cold atom source

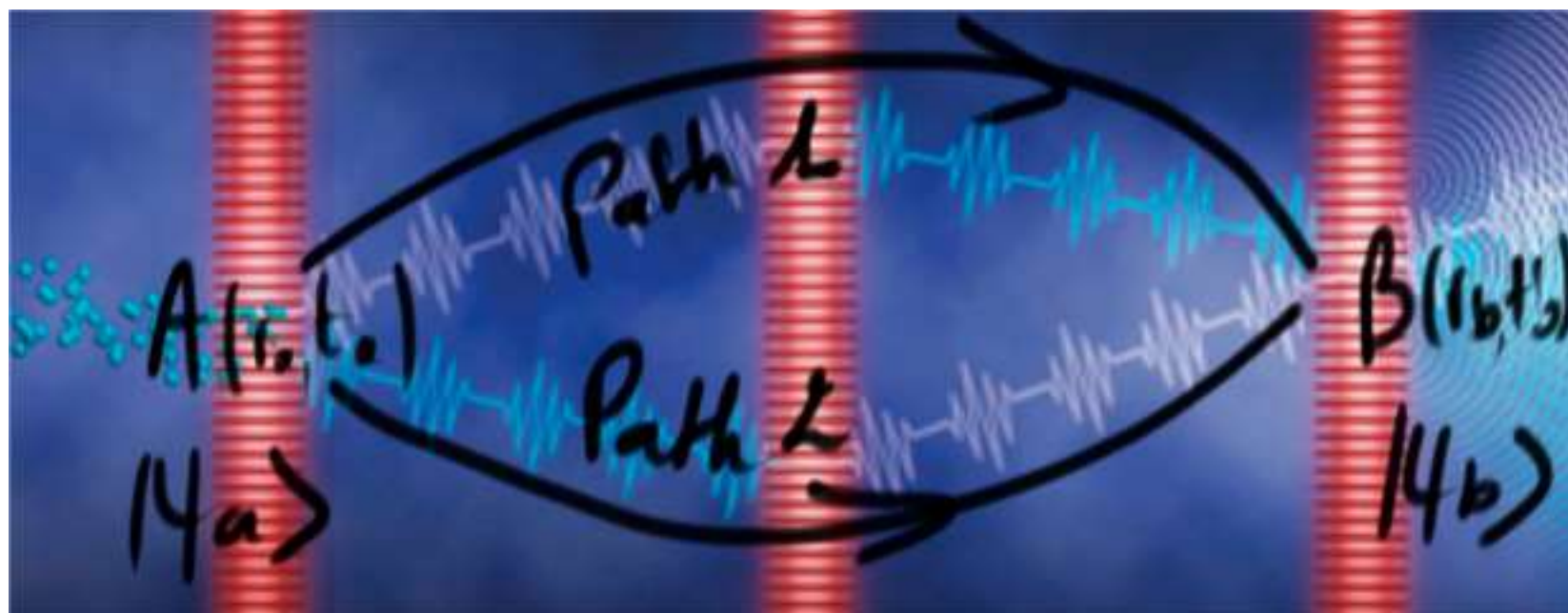
State prep.

3-pulse interferometer

Detection



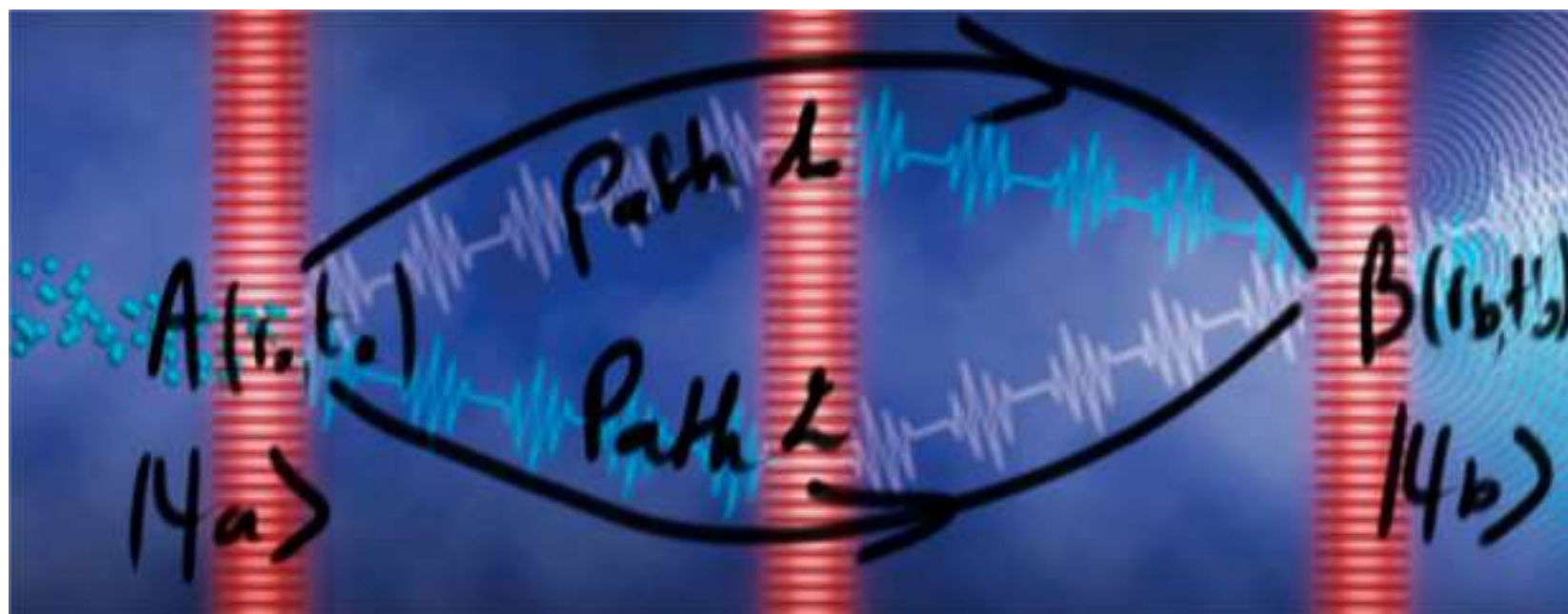
HOW TO CALCULATE THE PHASE SHIFTS



$$\text{PATH 1: } |\sigma_I|^2 = \langle 4a | 4a \rangle_I / 2$$

$$\text{PATH 2: } |\sigma_{II}|^2 = \langle 4a | 4a \rangle_{II} / 2$$

$$\begin{aligned} \text{2 PATHS: } |\sigma|^2 &= |\sigma_I + \sigma_{II}|^2 \\ &= |\sigma_I|^2 + |\sigma_{II}|^2 + \\ &\quad 2 \operatorname{Re} \sigma_I \sigma_{II}^* \end{aligned}$$



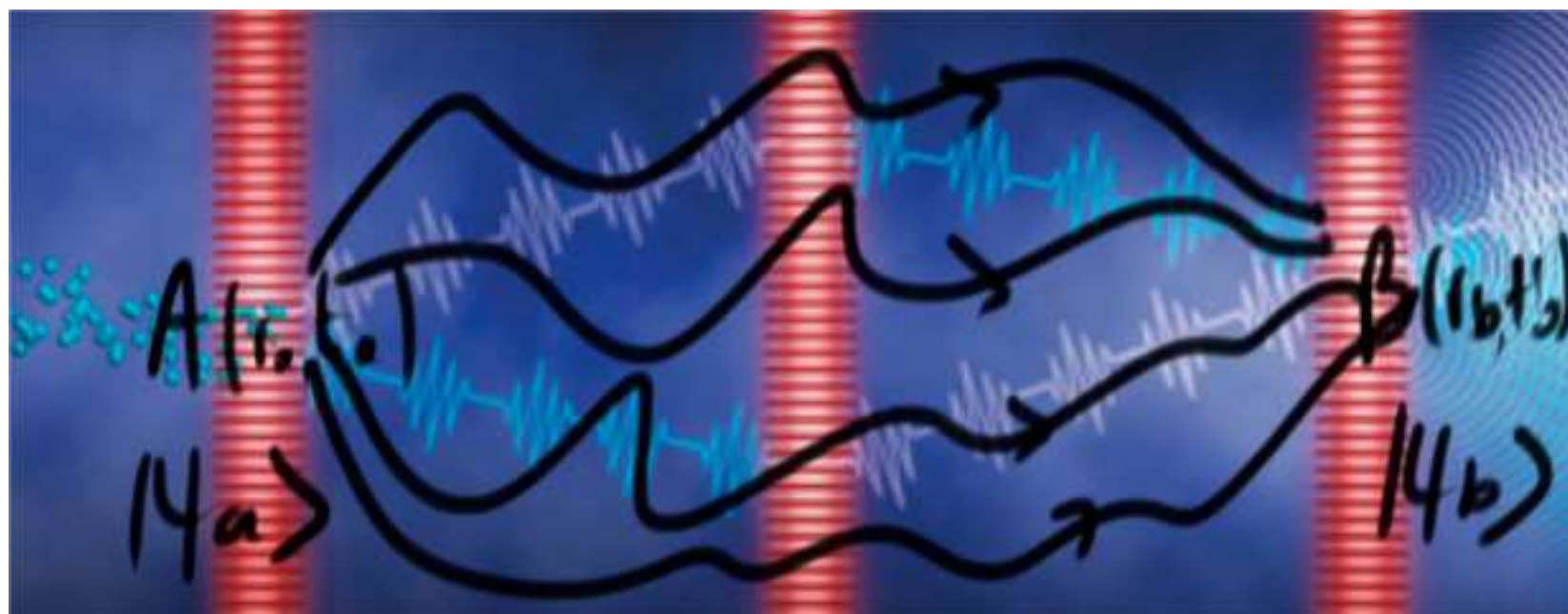
$$\text{PATH 1: } |\mathcal{K}_I|^2 = \langle \psi_a | \mathcal{K}_I | \psi_b \rangle^2$$

$$\text{PATH 2: } |\mathcal{K}_{II}|^2 = \langle \psi_a | \mathcal{K}_{II} | \psi_b \rangle^2$$

$$\begin{aligned} \text{2 PATHS: } |\mathcal{K}|^2 &= |\mathcal{K}_I + \mathcal{K}_{II}|^2 \\ &= |\mathcal{K}_I|^2 + |\mathcal{K}_{II}|^2 + \\ &\quad 2 \operatorname{Re} \mathcal{K}_I \mathcal{K}_{II}^* \end{aligned}$$

The probability amplitude for a particle to travel from $A(\mathbf{r}_a, t_a)$ to $B(\mathbf{r}_b, t_b)$ is the **Feynmann propagator**

$$K(\mathbf{r}_b, t_b; \mathbf{r}_a, t_a) = \sum_{\Gamma} e^{iS_{\Gamma}/\hbar}$$

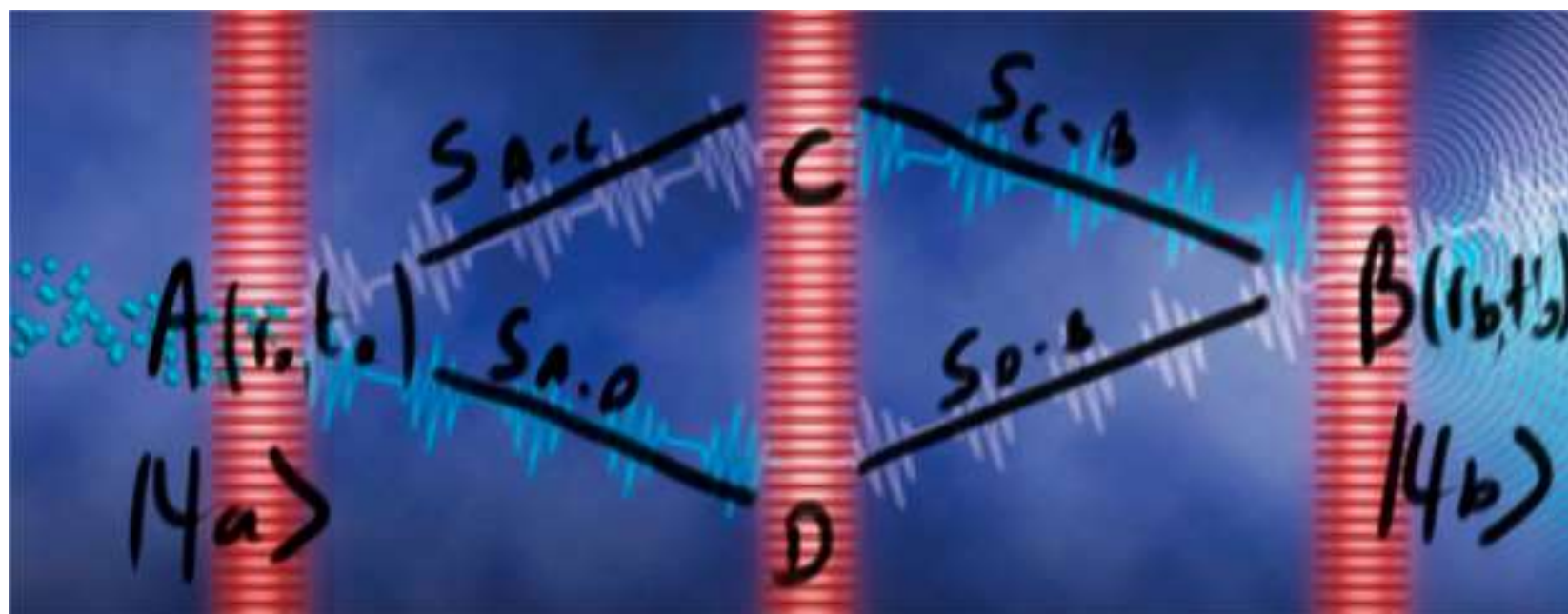


The sum is over all paths from A to B . The wave function at B is

$$\psi(\mathbf{r}_b, t_b) = \int K(\mathbf{r}_b, t_b; \mathbf{r}_a, t_a) \psi(\mathbf{r}_a, t_a) d\mathbf{r}_a$$

and the action S_Γ is deduced from the Lagrangian

$$S_\Gamma = \int_{t_a}^{t_b} \mathcal{L}(\mathbf{r}(t), \dot{\mathbf{r}}(t), t) dt$$



For a Lagrangian at most quadratic in $\mathbf{r}$ and $\dot{\mathbf{r}}$, K is deduced from the **classical action**: $K \propto e^{iS_{cl}/\hbar}$

Why?

Summation over all paths $\sum_{\Gamma} e^{iS_{\Gamma}/\hbar}$: only **stationary phase** contributes significantly

$\Rightarrow$ keep paths Γ minimizing the action, i.e. **classical trajectories**

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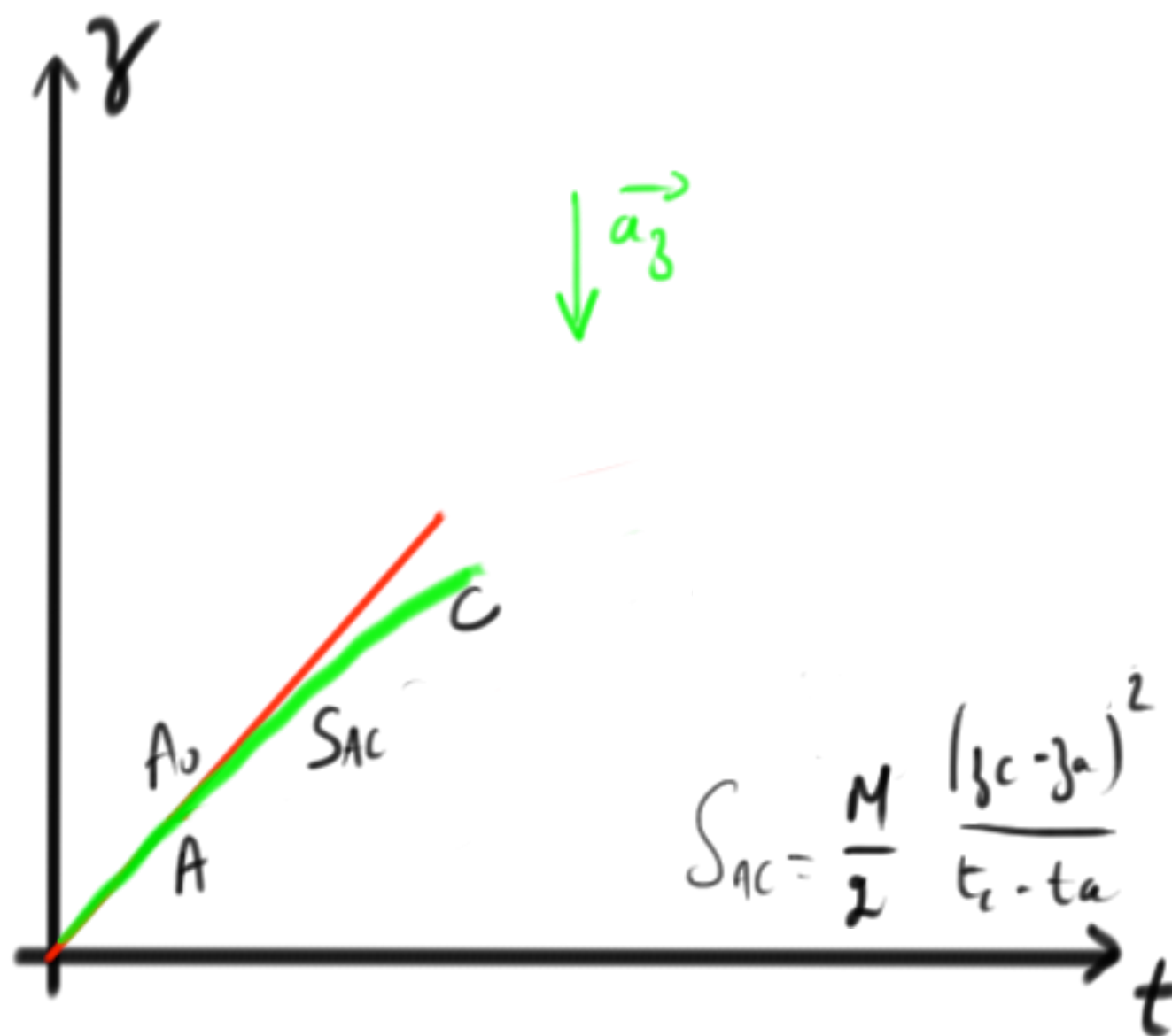
Why?

over all paths $\sum_{\Gamma} e^{iS_{\Gamma}/\hbar}$: only **stationary phase** contributes significantly
 $\Rightarrow$ keep paths Γ minimizing the action, *i.e.* **classical trajectories**

- free particle: $\mathcal{L} = M\dot{\mathbf{r}}^2/2$
- particle in gravitational field: $\mathcal{L} = M\dot{\mathbf{r}}^2/2 - Mgz$
- particle in an harmonic trap: $\mathcal{L} = M\dot{\mathbf{r}}^2/2 - M\omega_0^2 r^2/2$
- particle in a rotating frame:

$$\mathcal{L} = M\dot{\mathbf{r}}^2/2 + M\dot{\mathbf{r}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}) + M(\boldsymbol{\Omega} \times \mathbf{r})^2/2$$

PHASE SHIFT FROM MATTER WAVE PROPAGATION



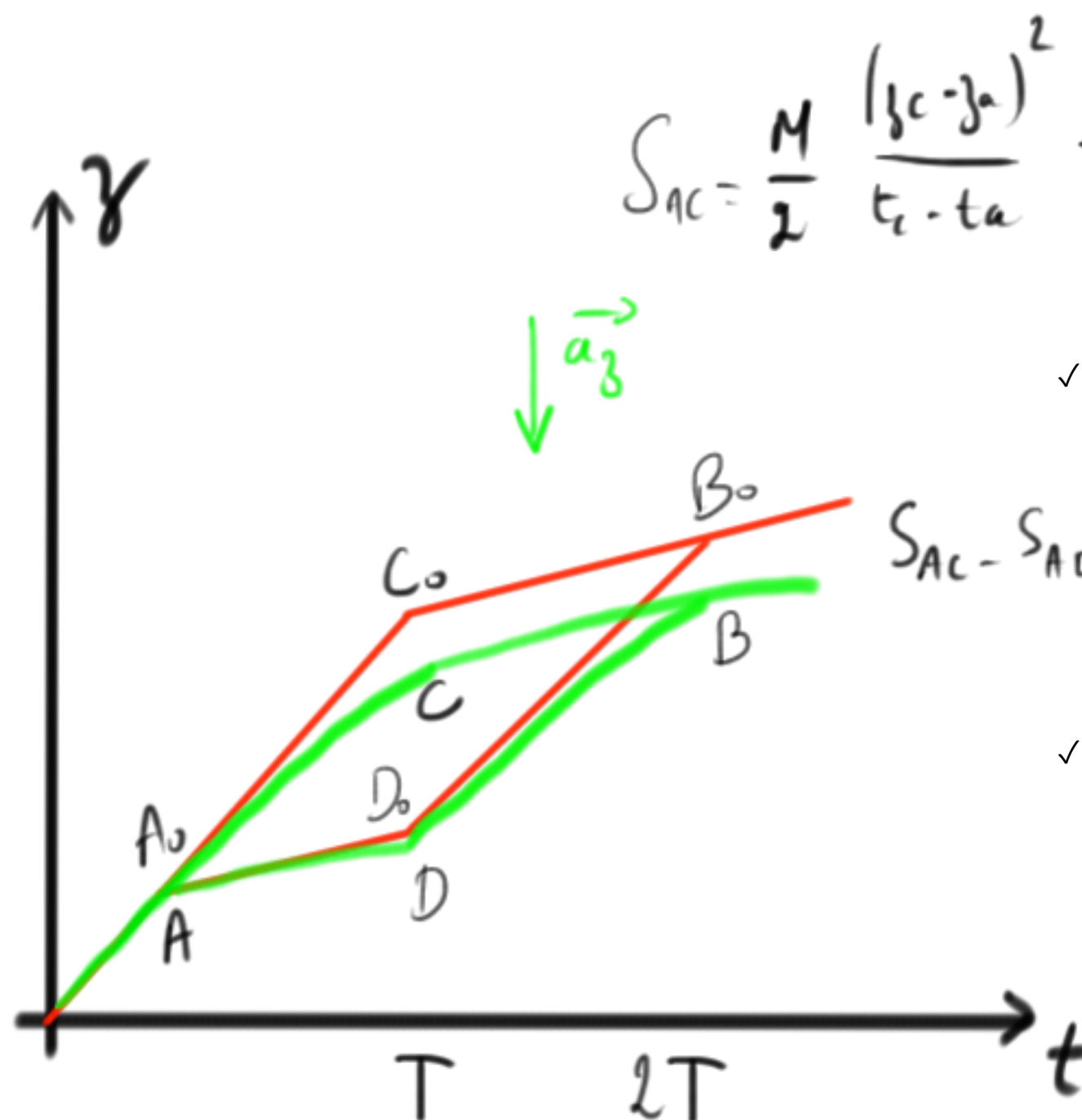
$$\mathcal{L}(z, \dot{z}) = \frac{1}{2} M \dot{z}^2 - M g z$$

$$\begin{cases} z_c = z_A + v_A(t_c - t_a) - \frac{1}{2} a_z (t_c - t_a)^2 \\ v_c = v_A - a_z (t_c - t_a) \end{cases}$$

$$S_{A-C} = S(z_c, t_c; z_a, t_a)$$

$$S_{AC} = \frac{M}{2} \frac{(z_c - z_a)^2}{t_c - t_a} - \frac{M g}{2} (z_c + z_a)(t_c - t_a) - \frac{M g^2}{24} (t_c - t_a)^3$$

PHASE SHIFT FROM MATTER WAVE PROPAGATION



$$S_{AC} = \frac{M}{2} \frac{(\gamma_c - \gamma_a)^2}{t_c - t_a} - \frac{M}{2} (\gamma_c + \gamma_a)(t_c - t_a) - \frac{M}{24} (\gamma_c^2 - \gamma_a^2)(t_c - t_a)^3$$

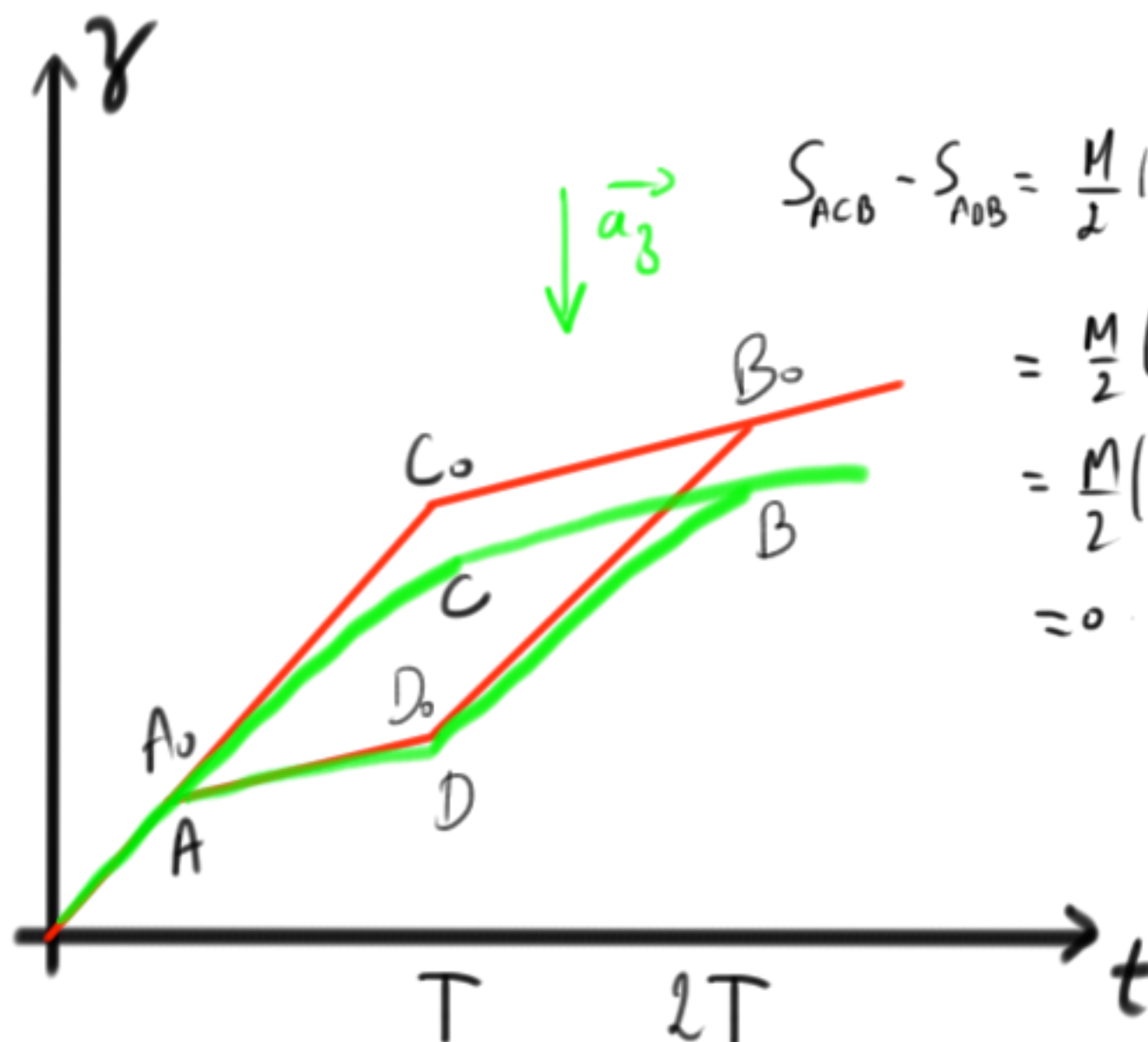
✓ Calculate $S_{AC}-S_{AD}$ and $S_{CB}-S_{DB}$

$$S_{AC} - S_{AD} = \frac{M}{2T} (\gamma_c - \gamma_d) [\gamma_c + \gamma_d - 2\gamma_a - gT^2]$$

✓ Sum $S_{AC}-S_{AD}$ and $S_{CB}-S_{DB}$

PHASE SHIFT FROM MATTER WAVE PROPAGATION

✓ Sum $S_{AC}-S_{AD}$ and $S_{CB}-S_{DB}$



$$\begin{aligned}
 S_{ACB} - S_{ADB} &= \frac{M}{2} (\gamma_C - \gamma_D) \left[\gamma_C + \gamma_D - \gamma_A - \gamma_B - g^T^2 \right] \\
 &= \frac{M}{2} (\gamma_C - \gamma_D) \left[(\gamma_{C0} - \frac{1}{2} g^T^2) + (\gamma_{D0} - \frac{1}{2} g^T^2) - \gamma_{A0} - (\gamma_{B0} - \frac{1}{2} g^T^2) \right] \\
 &= \frac{M}{2} (\gamma_C - \gamma_D) \left[(\gamma_{C0} + \gamma_{D0}) - (\gamma_{A0} + \gamma_{B0}) \right] \\
 &= 0
 \end{aligned}$$

No phase shift from matter wave propagation

Three contributions to interferometer phase shift:

$$\Delta\phi_{\text{total}} = \Delta\phi_{\text{prop}} + \Delta\phi_{\text{laser}} + \Delta\phi_{\text{sep}}$$

Propagation
shift:

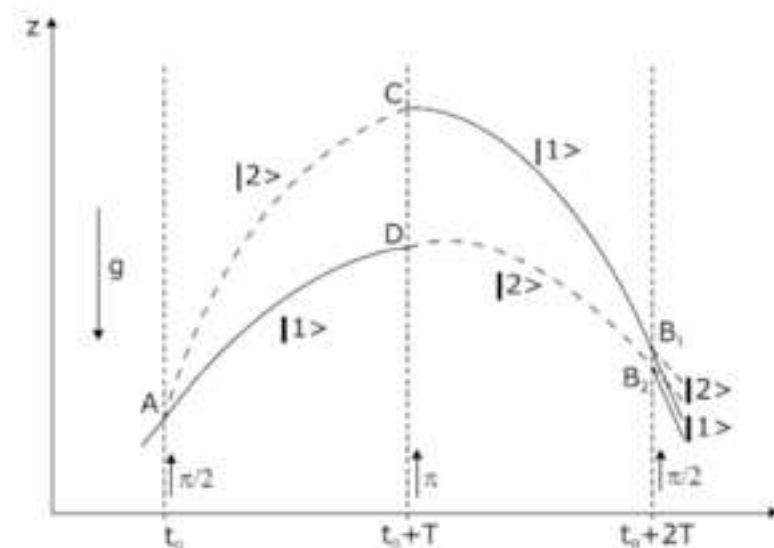
$$\frac{S_{\text{cl},B} - S_{\text{cl},A}}{\hbar}$$

Laser fields
(Raman
interaction):

$$k(z_c - z_b + z_d - z_a) + \phi_I - 2\phi_{II} + \phi_{III}$$

Wavepacket
separation at
detection:

$$\vec{p} \cdot \Delta\vec{r} / \hbar$$



See Bongs, et al., quant-ph/0204102
(April 2002) also App. Phys. B, 2006.

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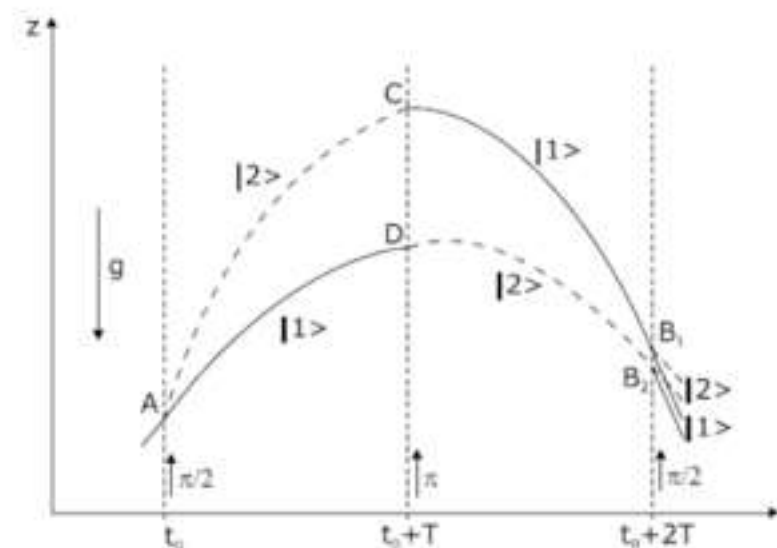
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See Bongs, et al., quant-ph/0204102
(April 2002) also App. Phys. B, 2006.

- ✓ Each time the atom changes state, the laser imprints its phase on the atom



«Stationary» Laser Phase e^{ikx}

$$|\psi(t)\rangle = c_1(t)|1, p_0\rangle + c_2(t)|2, p_0\rangle$$

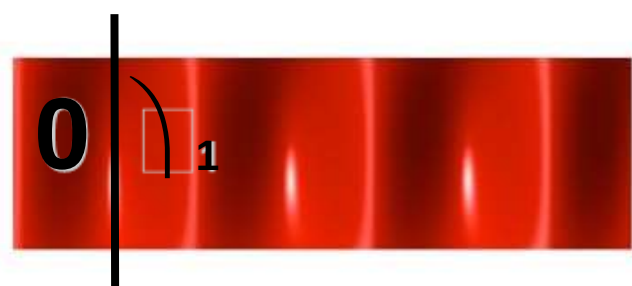
before pulse

$$c_1(t_-) \text{ and } c_2(t_-)$$

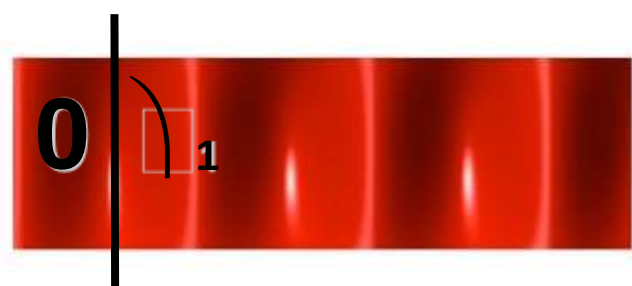
after pulse

$$c_1(t_+) = \cos\left(\frac{\Omega T}{2}\right) c_1(t_-) - i \sin\left(\frac{\Omega T}{2}\right) e^{i(\Delta t + \varphi)} c_2(t_-) \dots$$

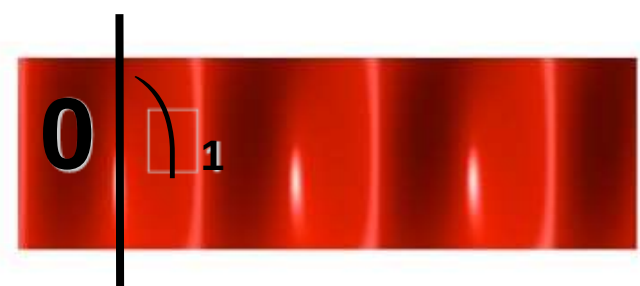
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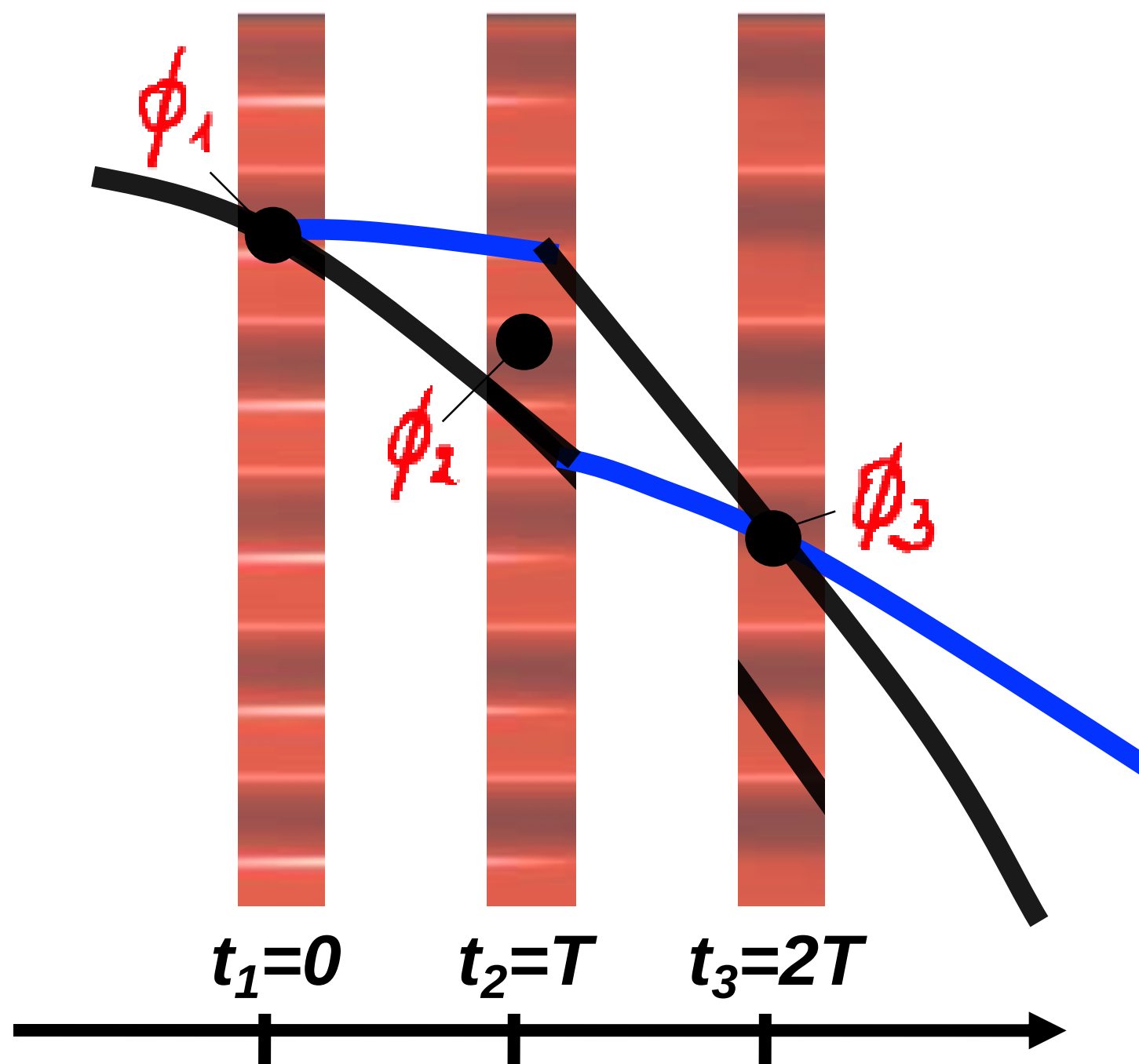


- ✓ Each time the atom changes state, the laser imprints its phase on the atom



Final phase difference

$$(\phi_1 - \phi_{2r}) - (\phi_{2l} - \phi_3)$$



□ The fact that each splitting process “imprints” the laser phase makes the laser act as a “ruler”

□ We read the atom position with the laser

□ The phase depends on the center of mass position of the atomic WP :

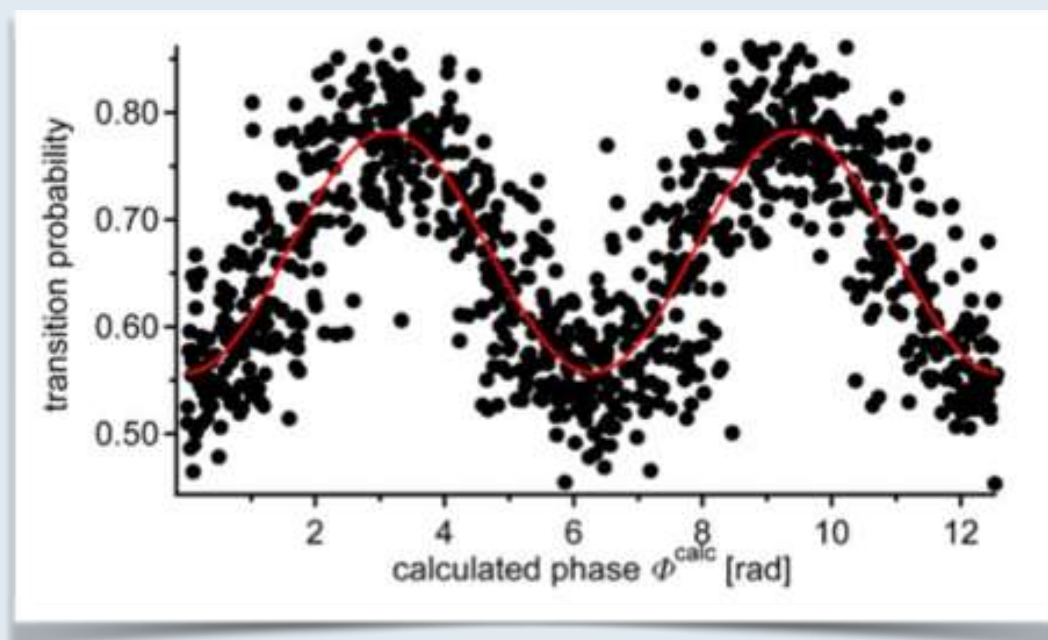
$$\Phi(t_i) = kx(t_i) + \Phi_0$$

$$\Delta\Phi = \phi_1 - 2\phi_2 + \phi_3$$

$$\Delta a_{\text{min}} = \frac{\Delta v}{\sqrt{N}} = \frac{1}{\Delta T^2 \sqrt{N}}$$

- The atoms “reads” its position and we get it with atom interferometry
 - ⇒ velocity measurement proportional to T
 - ⇒ Acceleration measurement proportional to T^2

- Interference fringes : $N_{at} \sim \cos(2\pi a T^2 / \lambda + \Phi)$



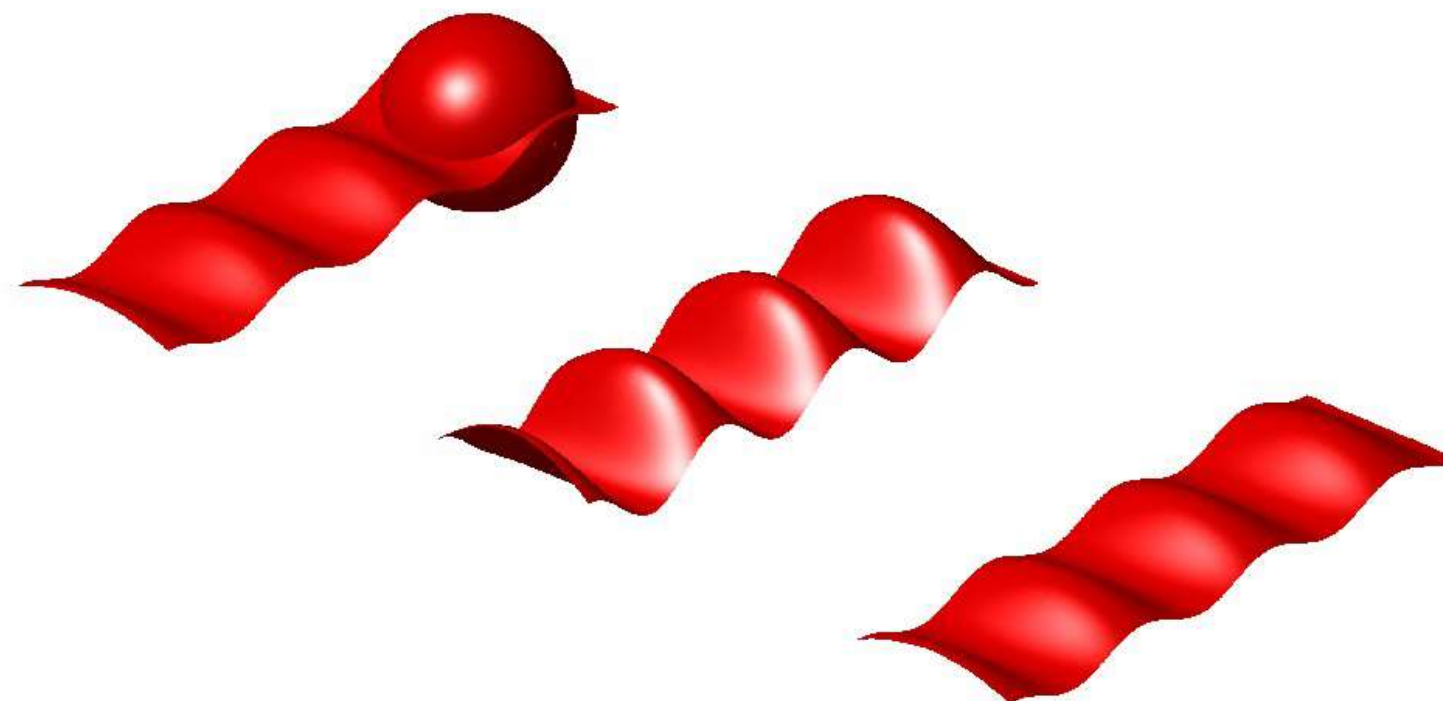
$$\Delta\phi \sim \frac{2\pi}{\lambda_{laser}} g T^2$$

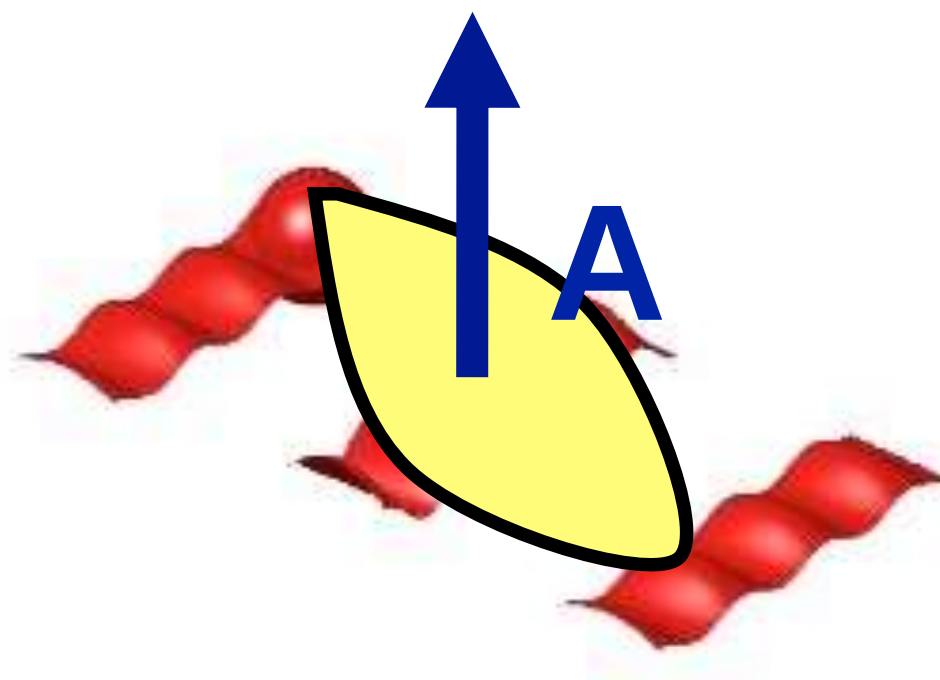
⇒ If we have a S/N ratio of 1000, the sensitivity of the accelerometer is ($T = 1s$) :

$$\Delta g_{min} \sim \frac{\lambda_{laser}}{6000} \frac{1}{T^2} \sim \frac{6 \cdot 10^{-7}}{6 \cdot 10^3} \sim 10^{-10} \text{ m.s}^{-2}$$



- ✓ **3 separated diffraction zones**
- ✓ **Coriolis acceleration comes from rotating lase**

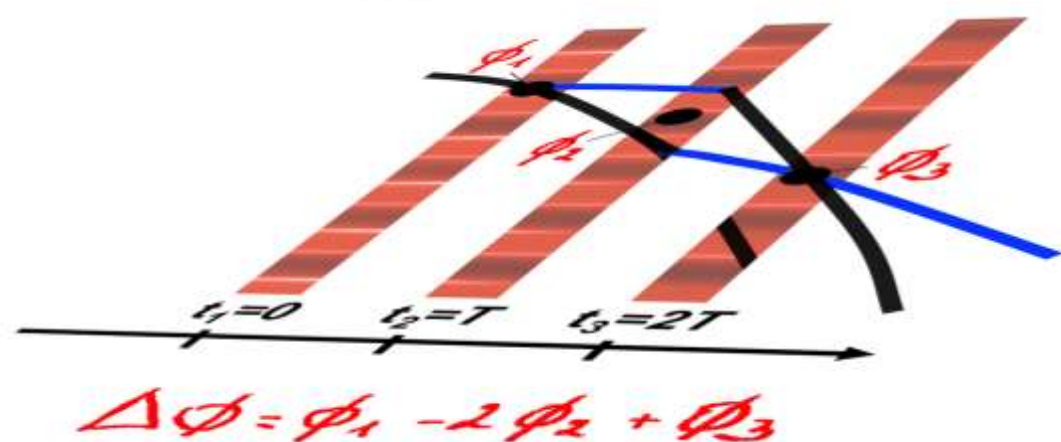




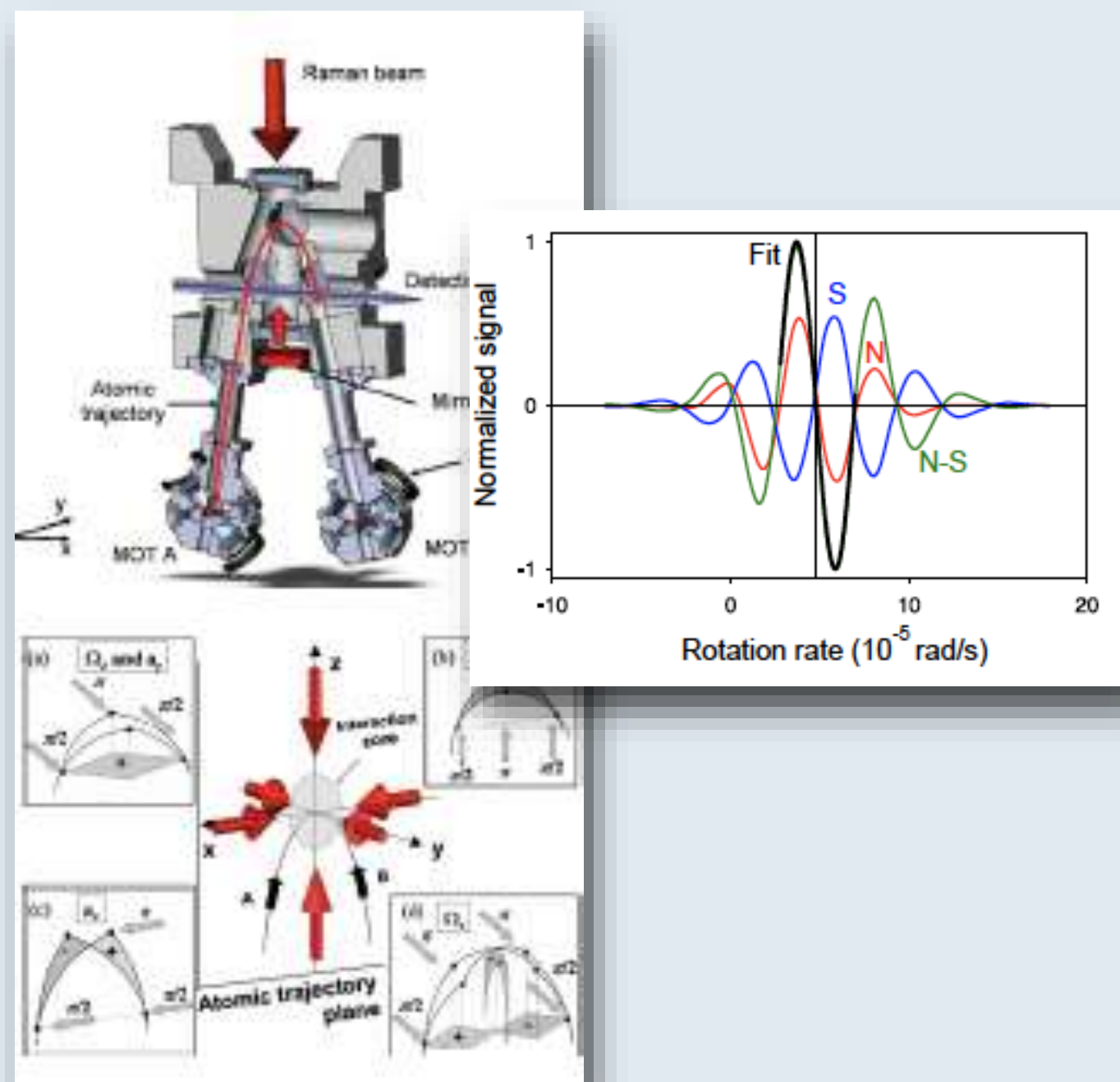
$\Delta R = \frac{d\phi_{\text{AO}}}{d\phi_{\text{ph}}} = \frac{c \lambda_{\text{ph}}}{V \lambda_{\text{AO}}} \approx \frac{N c t}{h \omega_0} \approx \sqrt{N} 10^{11}$

1 GeV/nucléon

1 eV



$$\Delta\Phi_{\text{rot}} = 2\pi \frac{2m_{\text{at}}}{h} \mathbf{A} \times \mathbf{\Omega} = L^2 \frac{V_t}{V_L} \mathbf{\Omega}$$



AI gyroscope, demonstrated laboratory performance:
 2×10^{-6} deg/hr $^{1/2}$ ARW; $< 10^{-4}$ deg/hr bias stability

LECTURE 2 : MEASURING ACCELERATION AND ROTATIONS - METHOD, PRECISION, ACCURACY

✓ First pulse is the 1st beam splitter

⇒ Coherent superposition

⇒ Different speed

⇒ The two components split

✓ second pulse : mirror

⇒ redirect wavepackets

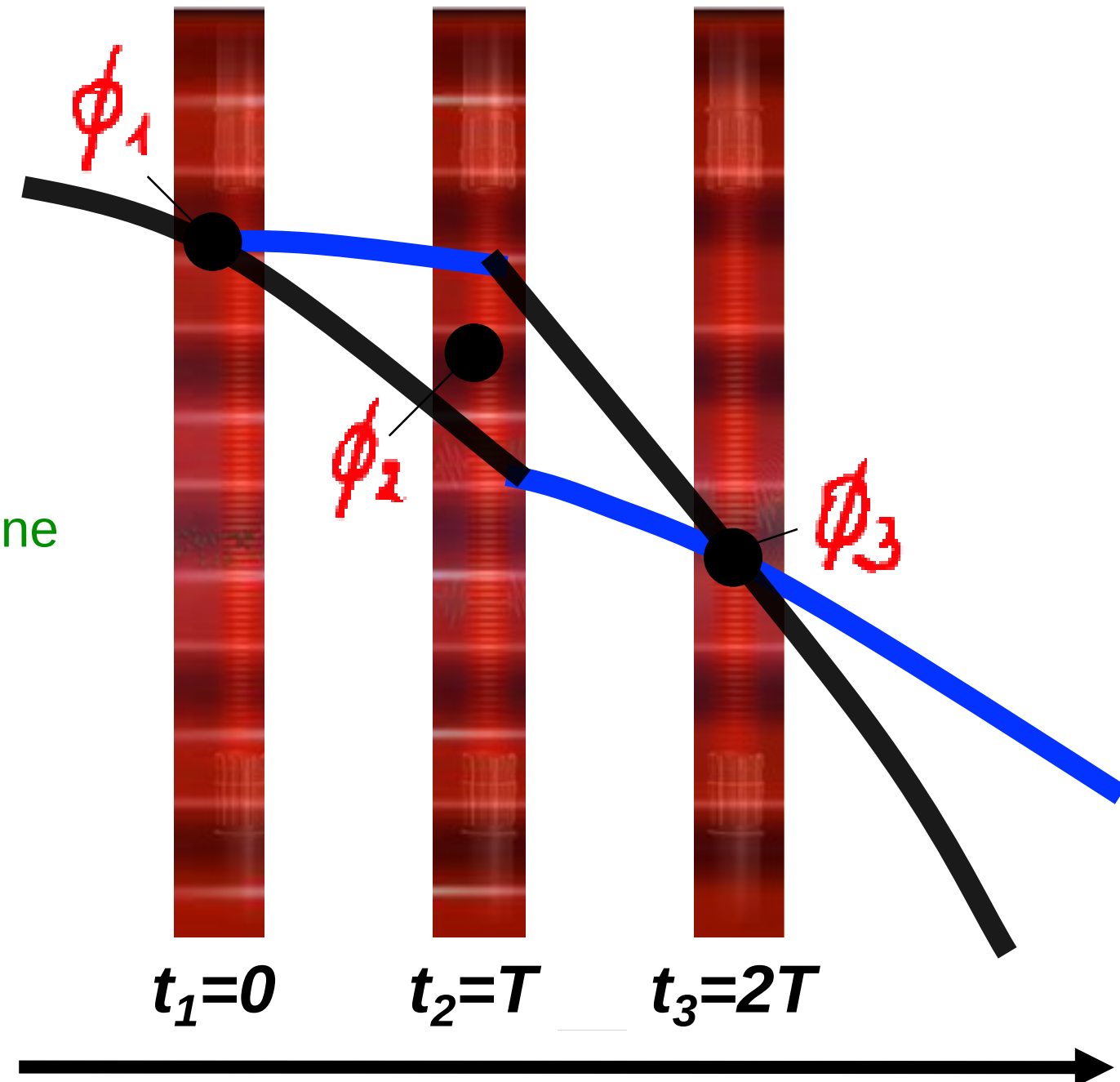
✓ Last $\pi/2$ pulse when wavepackets recombine

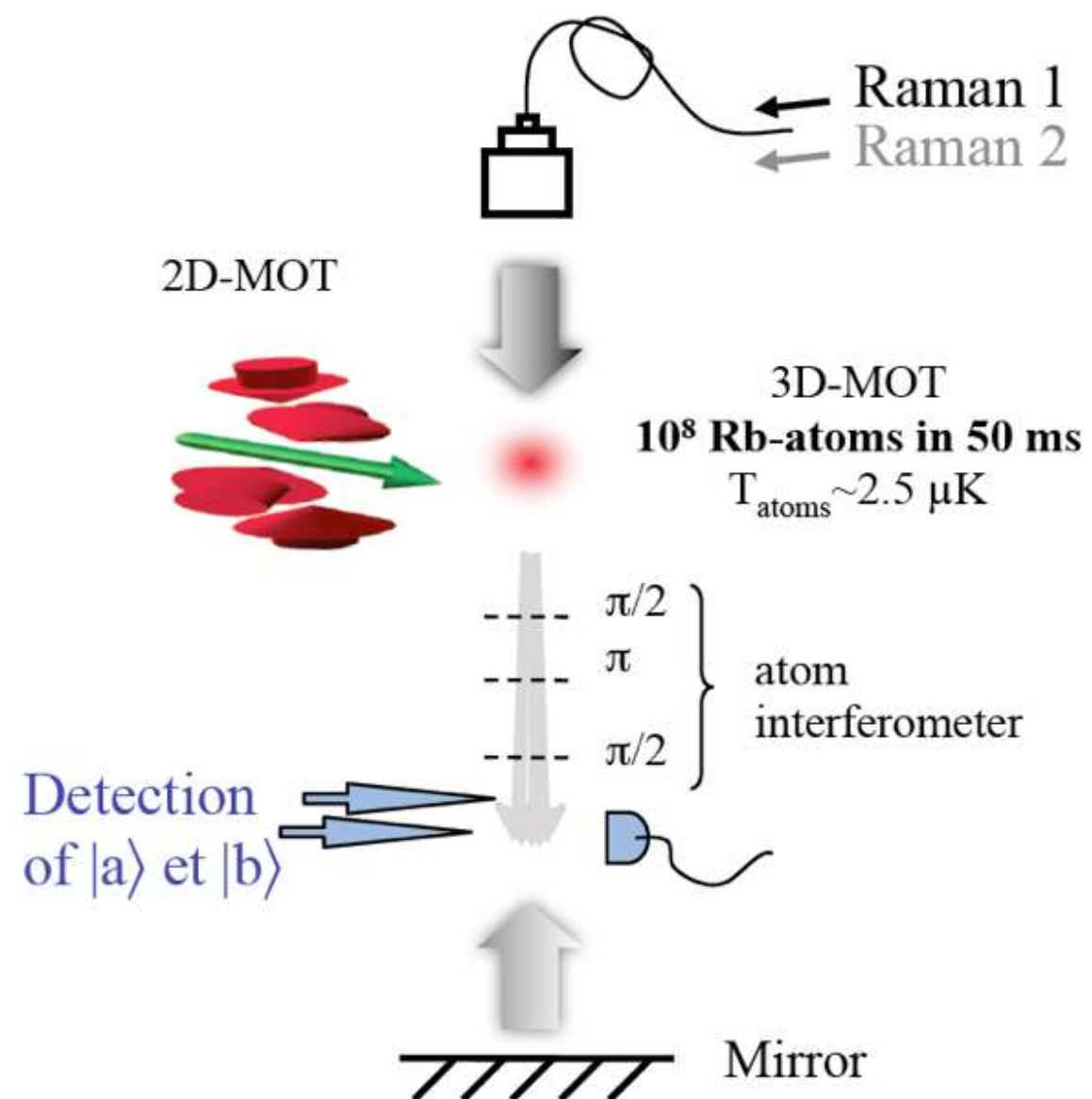
⇒ output depends on phase difference

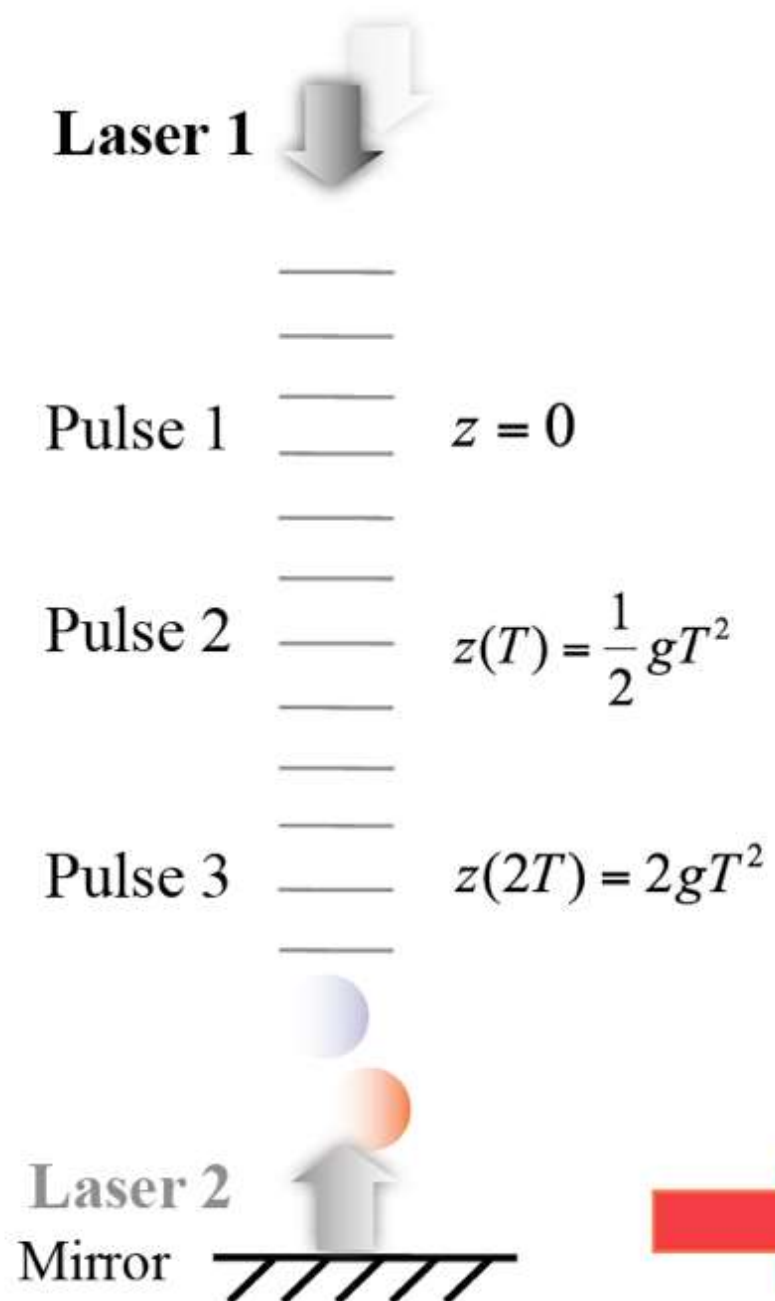
□ The phase depends on the center of mass position of the atomic WP :

$$\Phi(t_i) = kx(t_i) + \Phi_0$$

$$\Delta\Phi = \phi_1 - 2\phi_2 + \phi_3$$







Vertical interferometer
 $\Rightarrow$ Free fall along the equiphase planes

Sensitivity to gravity acceleration :

$$\Delta\Phi_{\text{int}} = -\vec{k}_{\text{eff}} \vec{g} T^2 + \delta\Phi_{\text{noise}} + \delta\Phi_{\text{sys}}$$

Two overlapped Raman beams retroreflected
 $\Rightarrow$ Equiphases defined by the mirror position
 $\propto (z_1 - 2z_2 + z_3)$

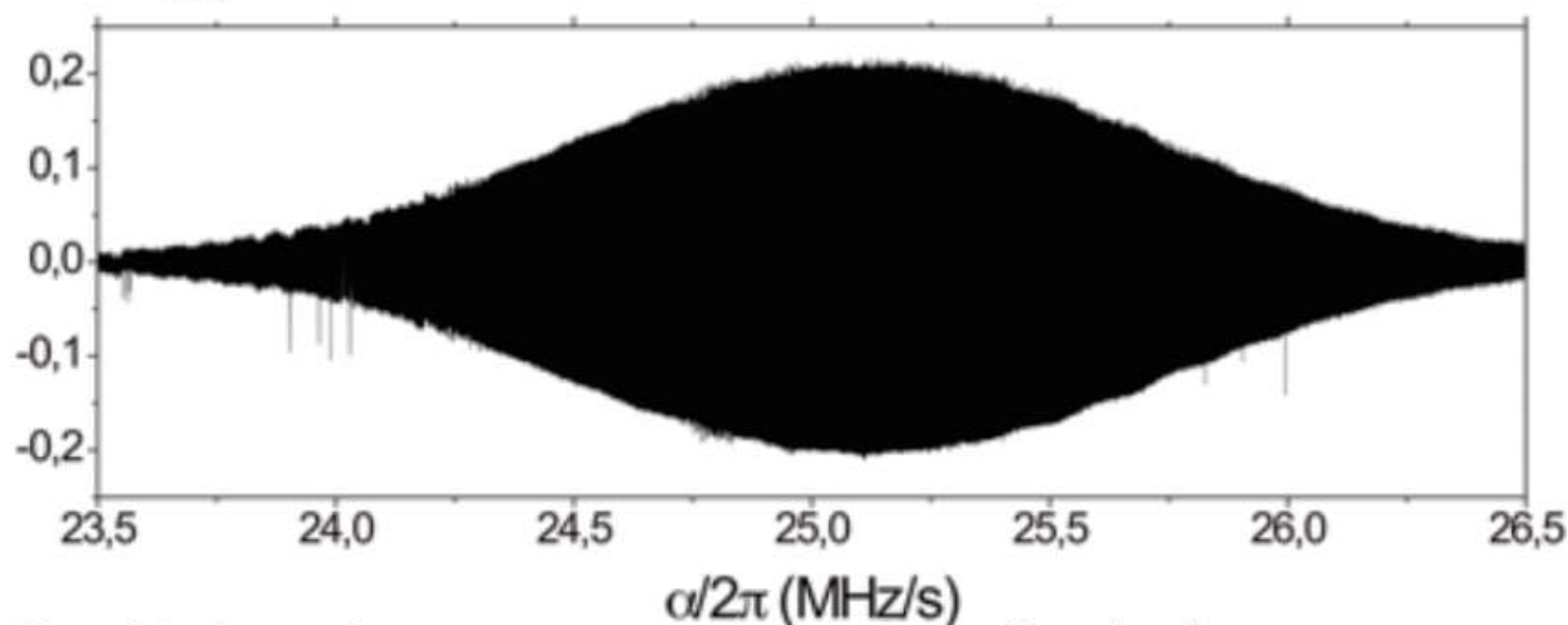
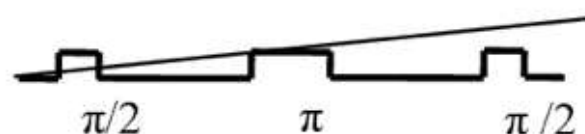
measure of the relative displacement
atoms/mirror with a very precise ruler

- Free fall $\rightarrow$ Doppler shift of the resonance condition of the Raman transition

$$\omega_1 - \omega_2 = \omega_e - \omega_f + \vec{k}_{eff} \vec{v}(t) + \frac{hk_{eff}^2}{2m}$$

- Ramping of the frequency difference to **stay on resonance** : $\omega = \omega_0 + \alpha t$

$$\Delta\Phi = \vec{k}_{eff} \vec{g} T^2 - \alpha T^2$$

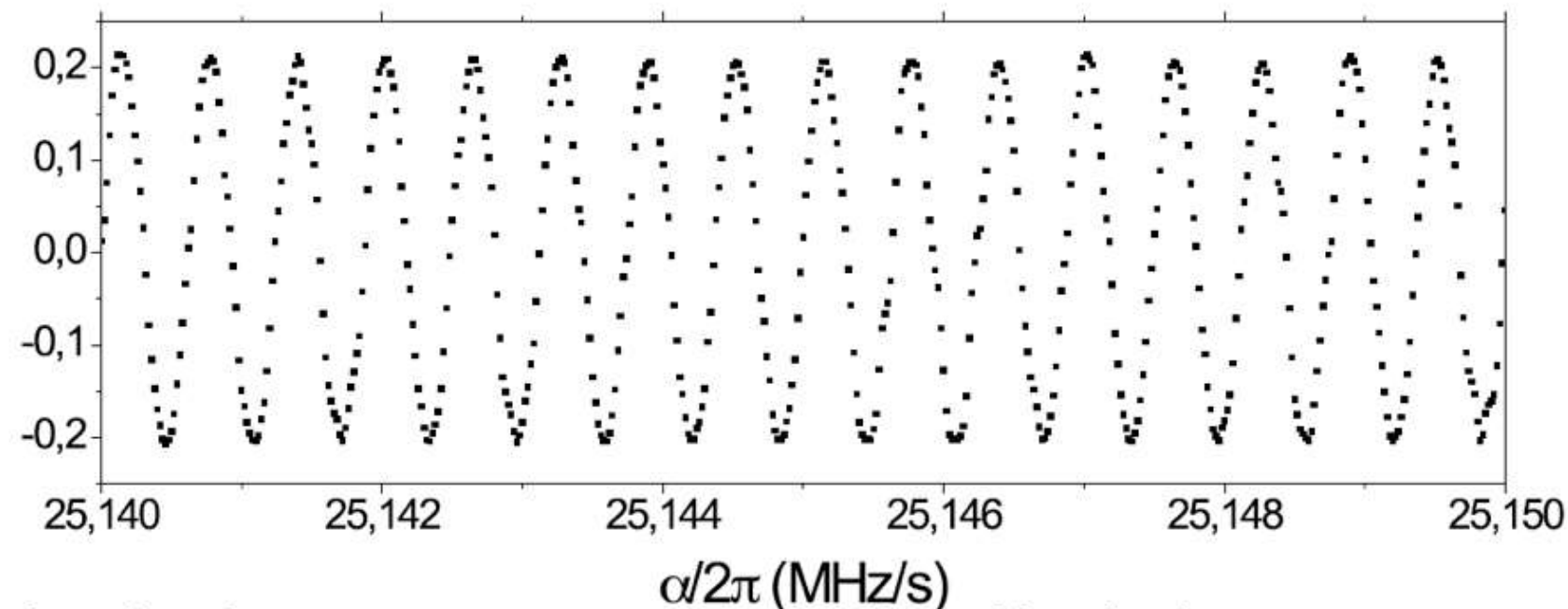
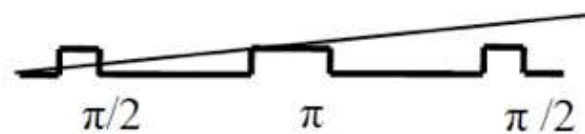


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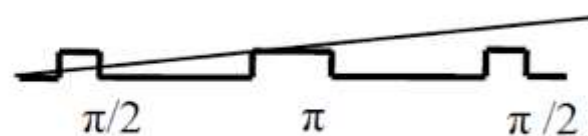


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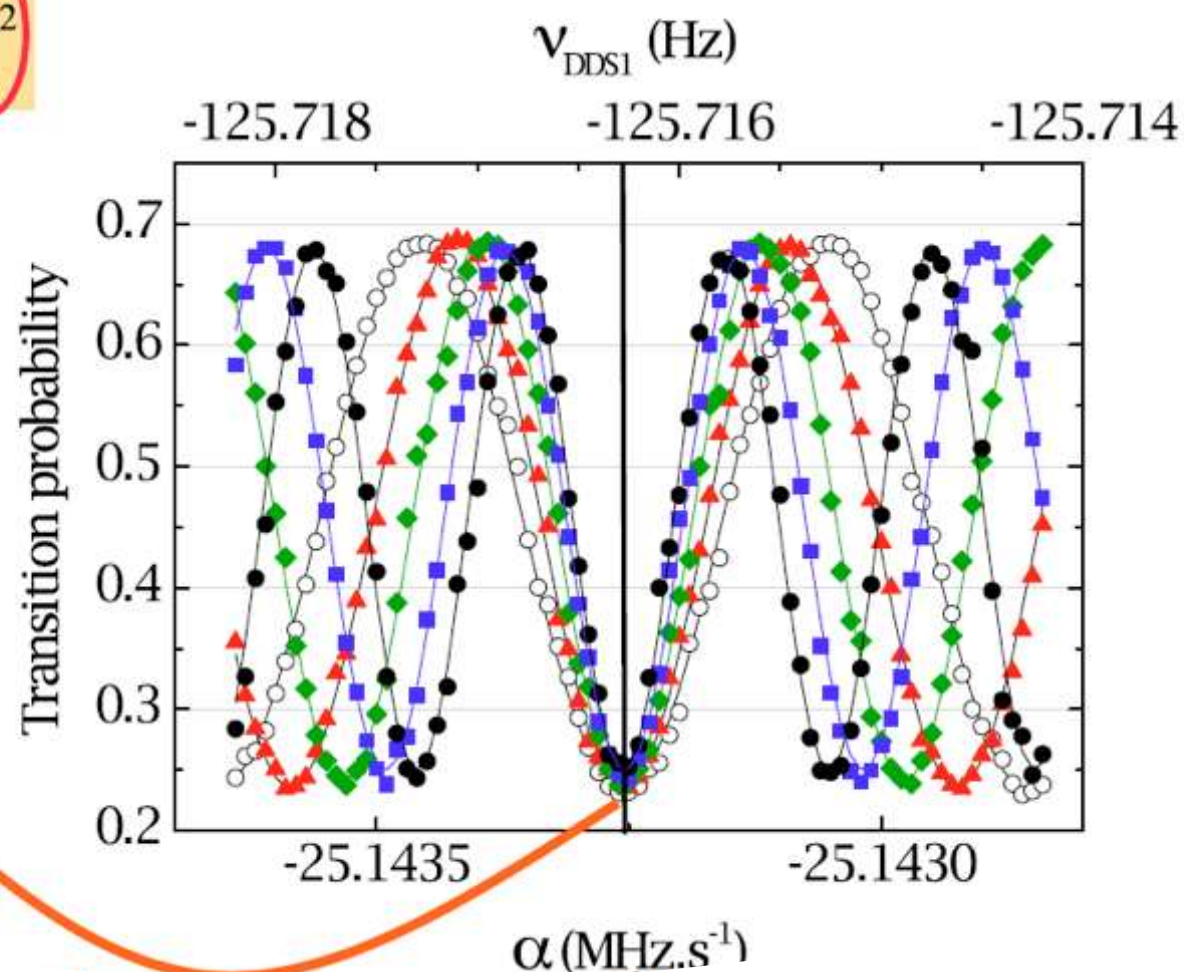
- Ramping of the frequency difference to **stay on resonance** : $\omega = \omega_0 + \alpha t$

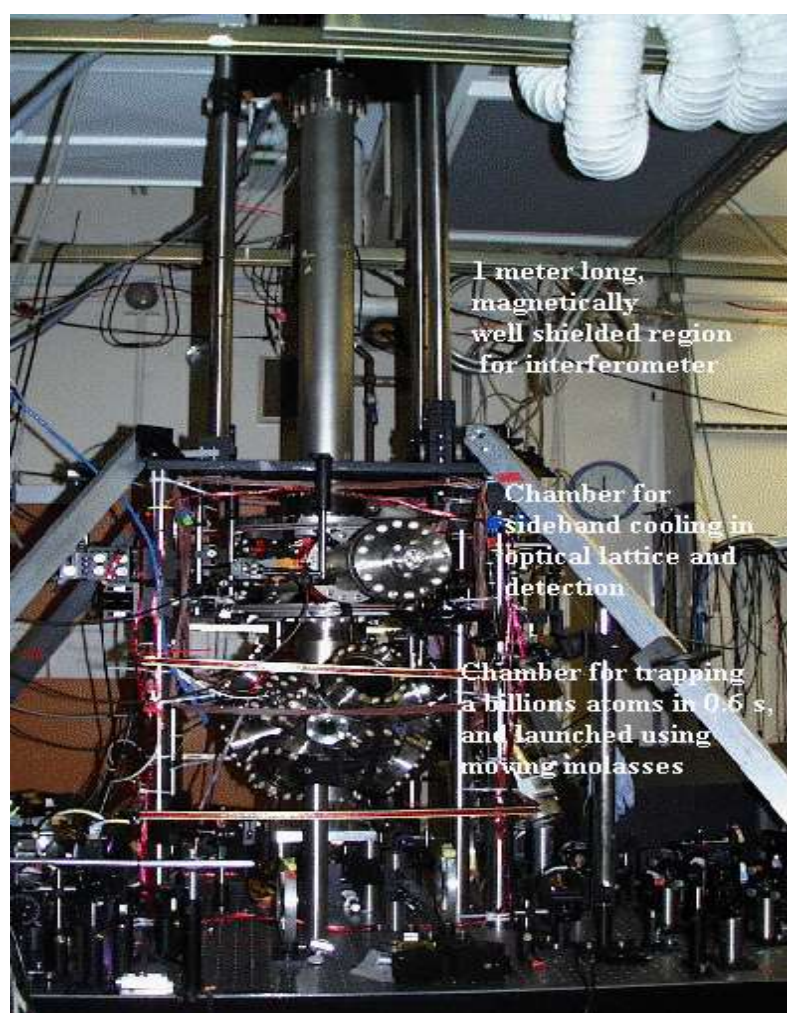
$$\Delta\Phi = \vec{k}_{eff} \vec{g} T^2 - \alpha T^2$$



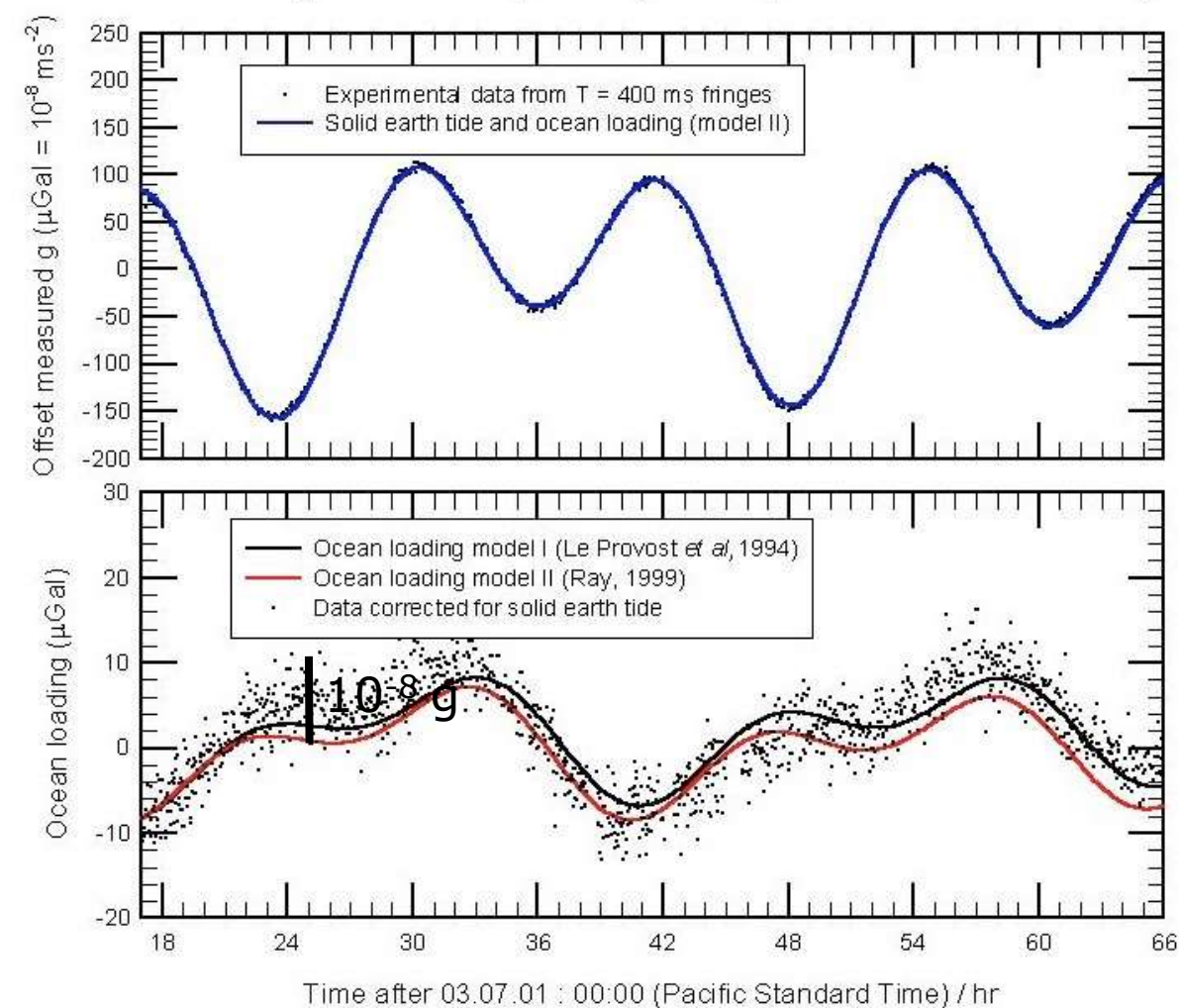
- Dark fringe :
independent of T

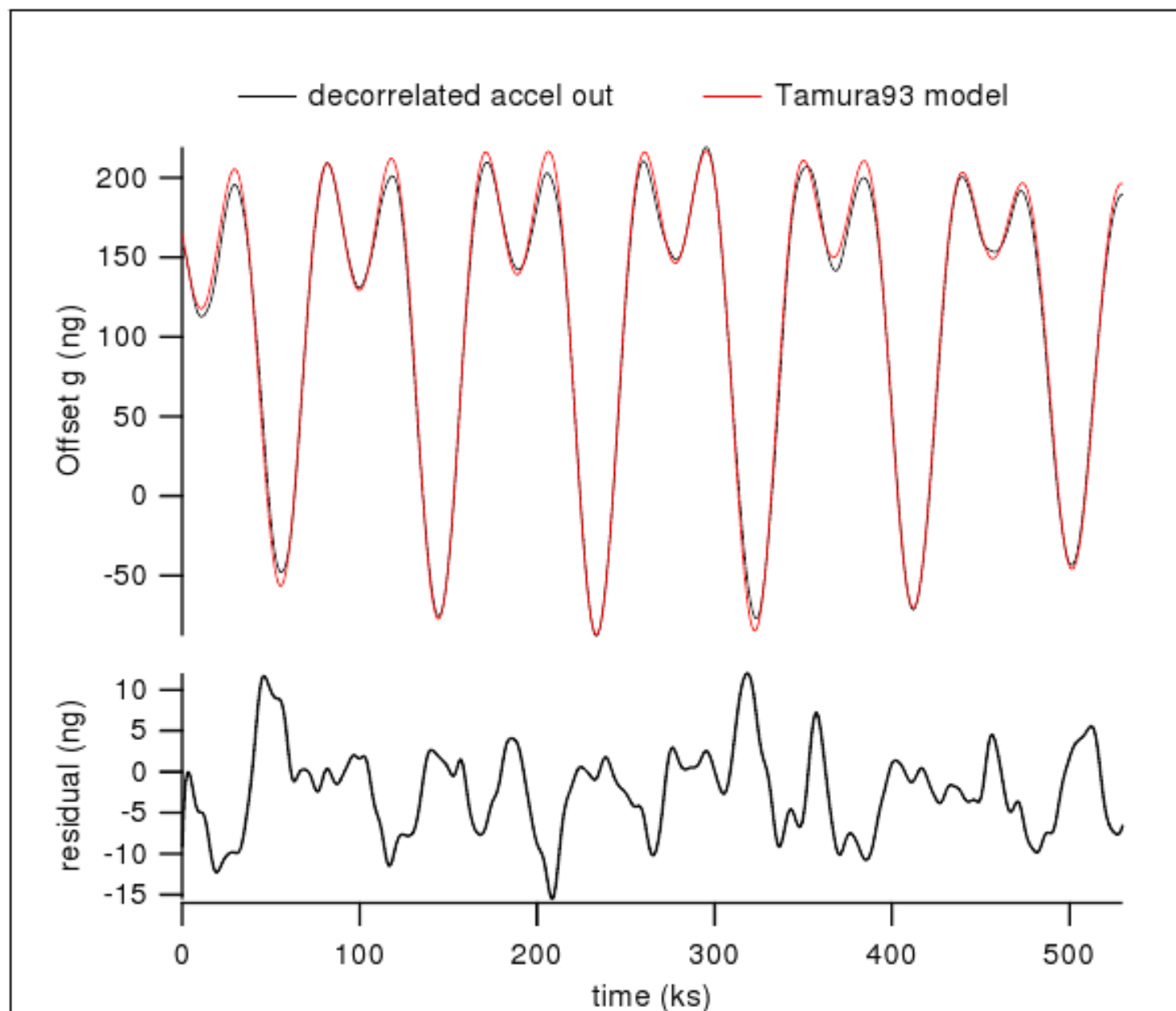
$$g = \frac{\alpha_0}{k_{eff}}$$

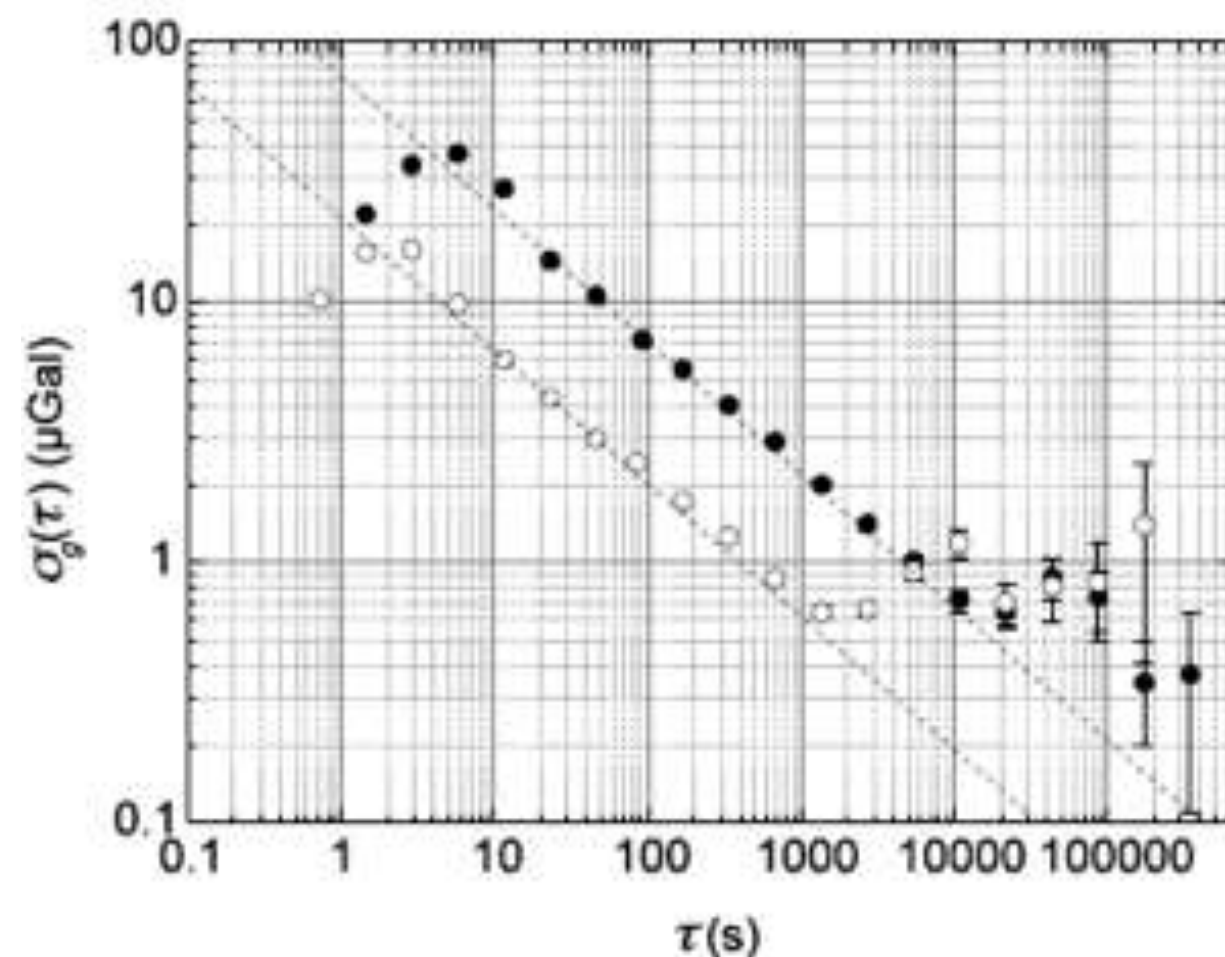
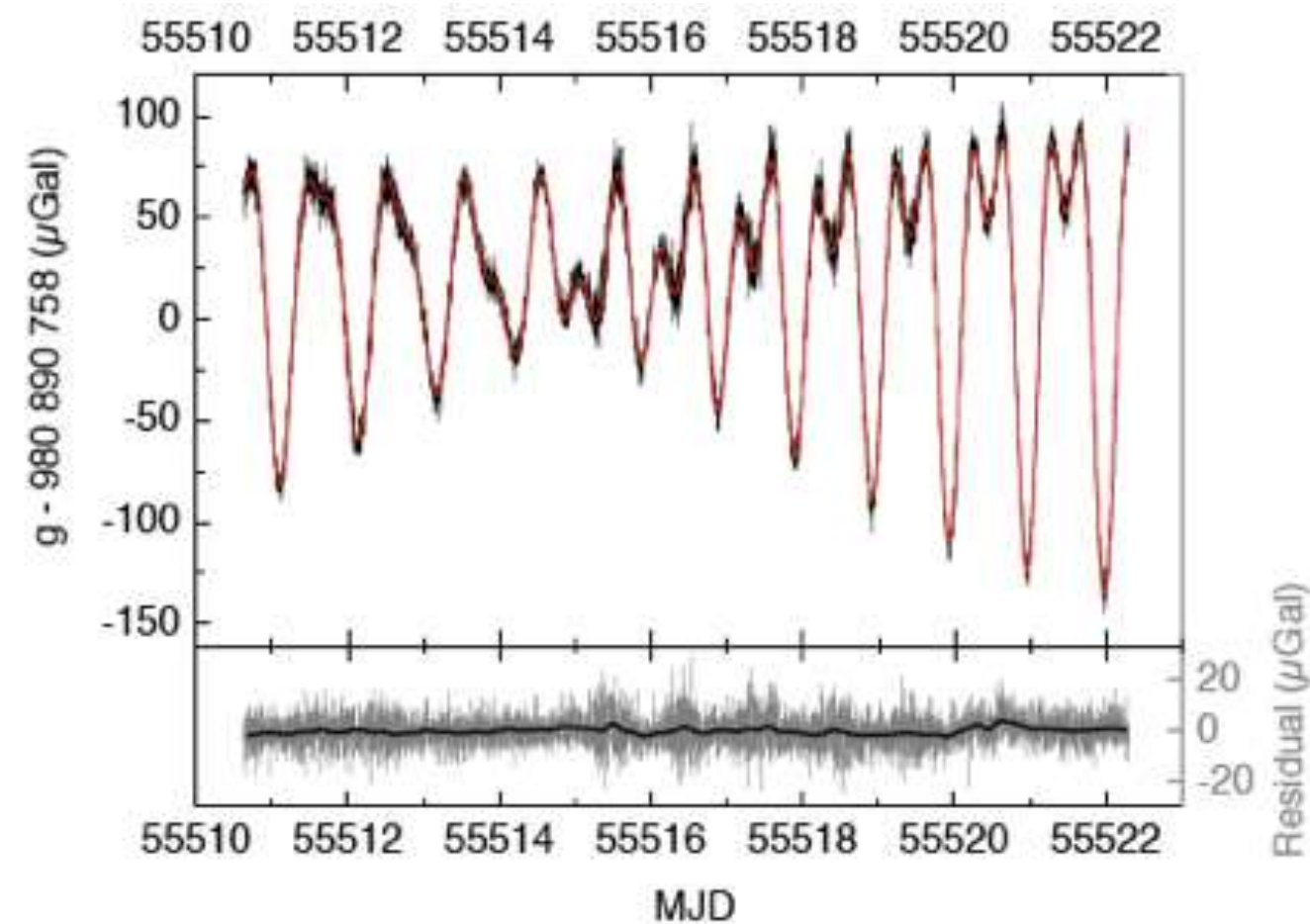




Monitoring of local gravity using $T = 400$ ms fringes







✓ Excellent agreement measurement/tide model

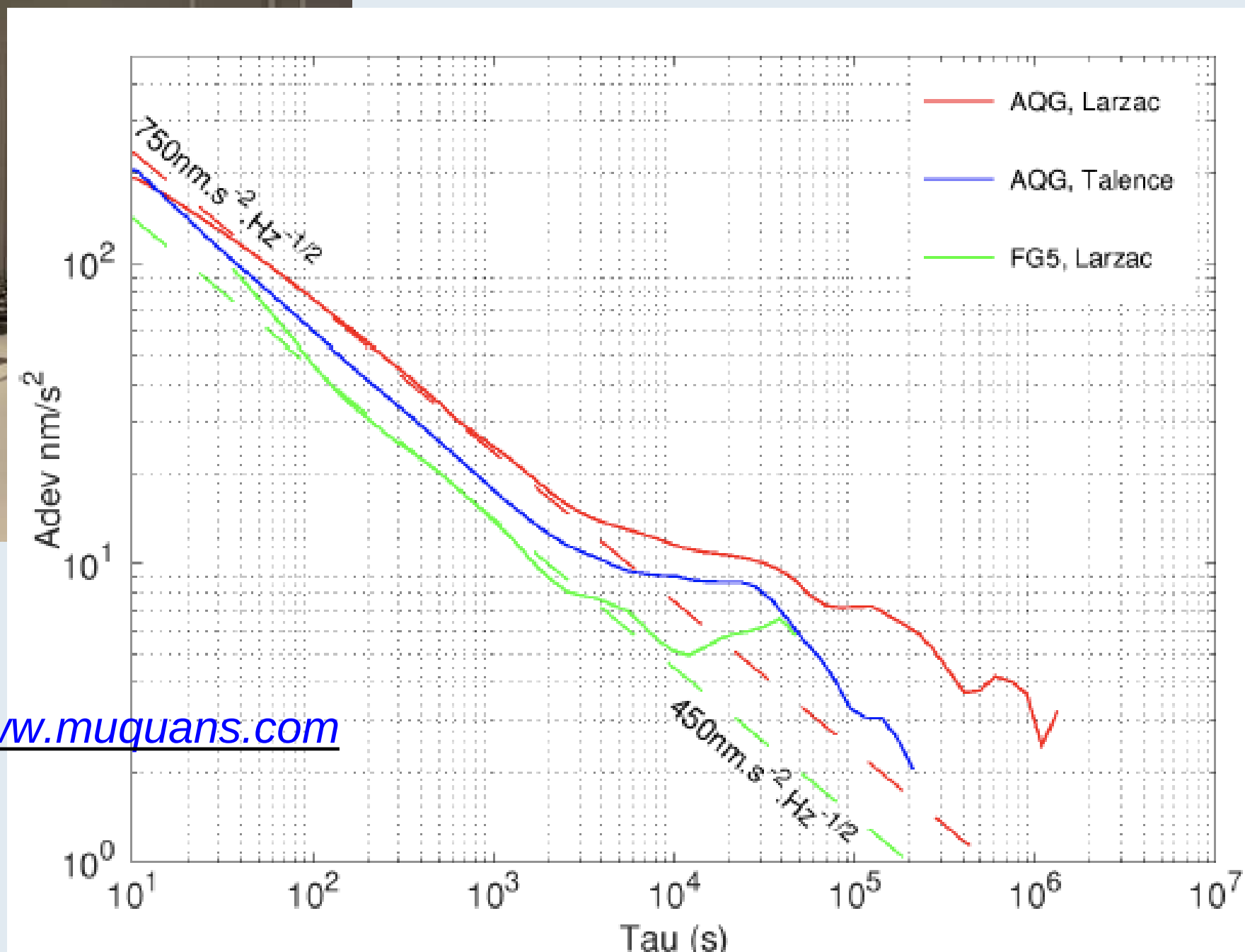
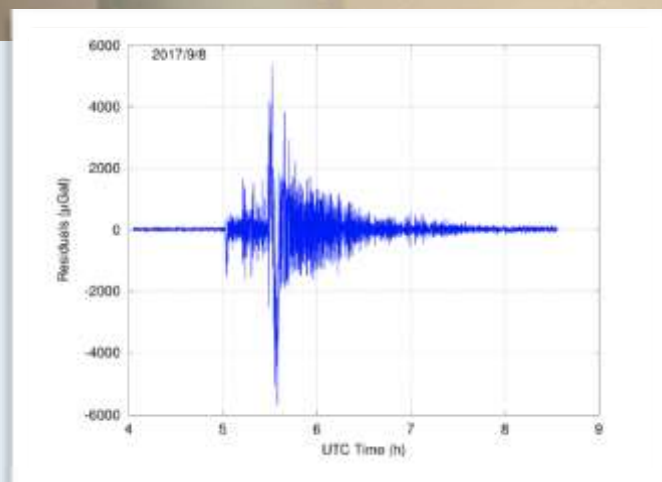
✓ Tides : $\pm 10^{-7} \text{ g}$

✓ ($1 \mu\text{Gal} = 10^{-8} \text{ m.s}^{-2}$, ou $\sim 10^{-9} \text{ g}$)

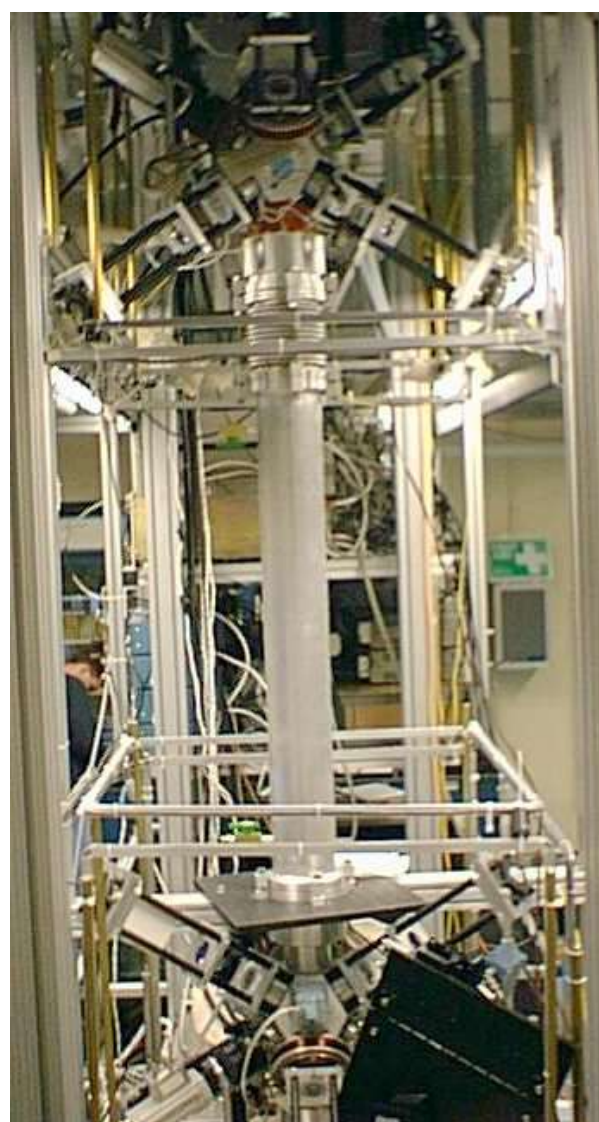
✓ Accuracy at $5 \cdot 10^{-9}$ level

✓ Long term stability : $< 7 \cdot 10^{-10} \text{ g}$

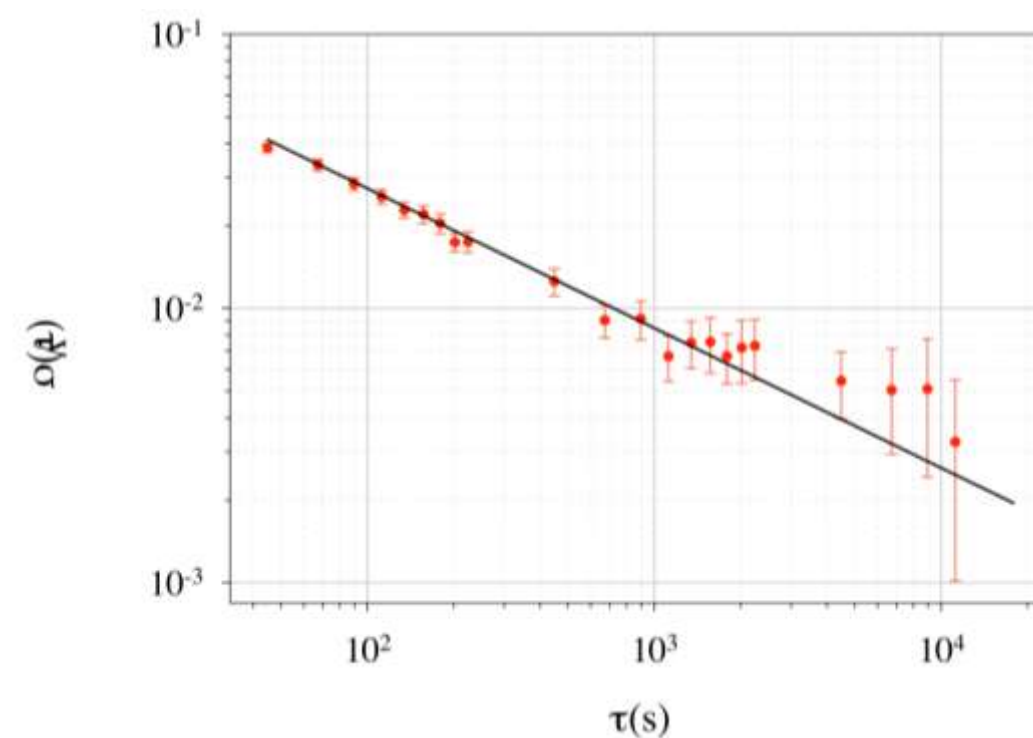
✓ International gravity comparison in BIPM 2009 : S. Merlet, et al., Metrologia 47, L9-L11 (2010); A. Louchet-Chauvet et al, IEEE (2011), arXiv:1008.2884.



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1.4 m

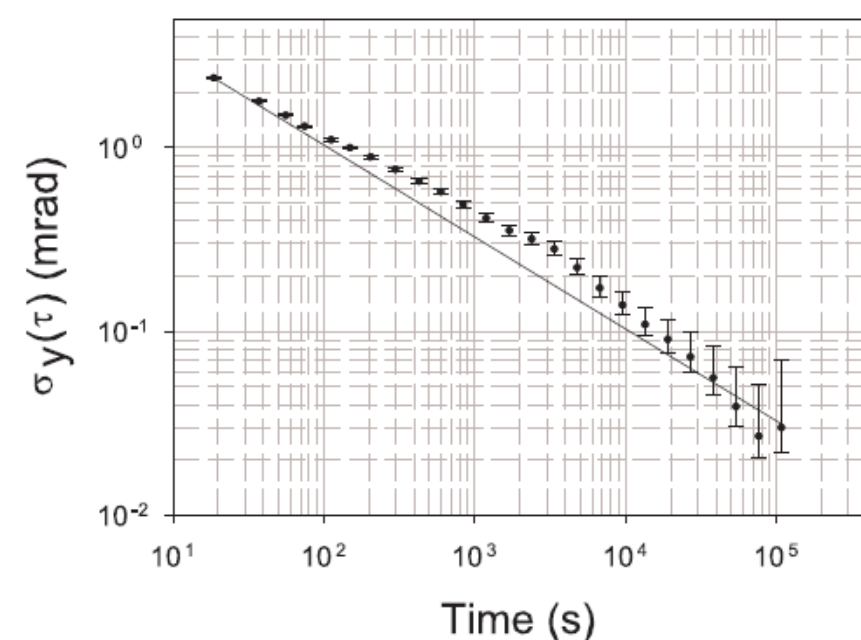
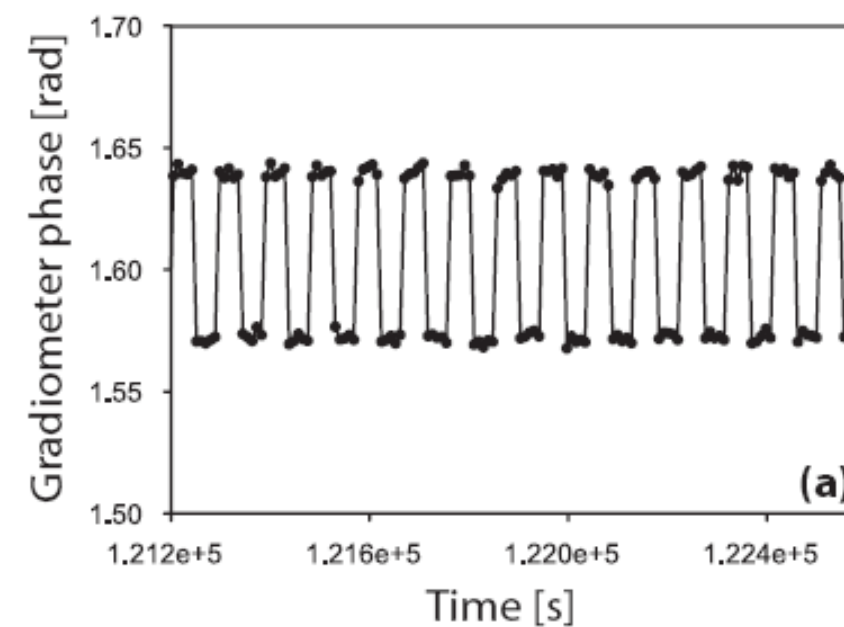


Demonstrated differential acceleration sensitivity:

$$4 \times 10^{-9} \text{ g/Hz}^{1/2}$$

($2.8 \times 10^{-9} \text{ g/Hz}^{1/2}$ per accelerometer)

Distinguish gravity induced accelerations from those due to platform motion with differential acceleration measurements.

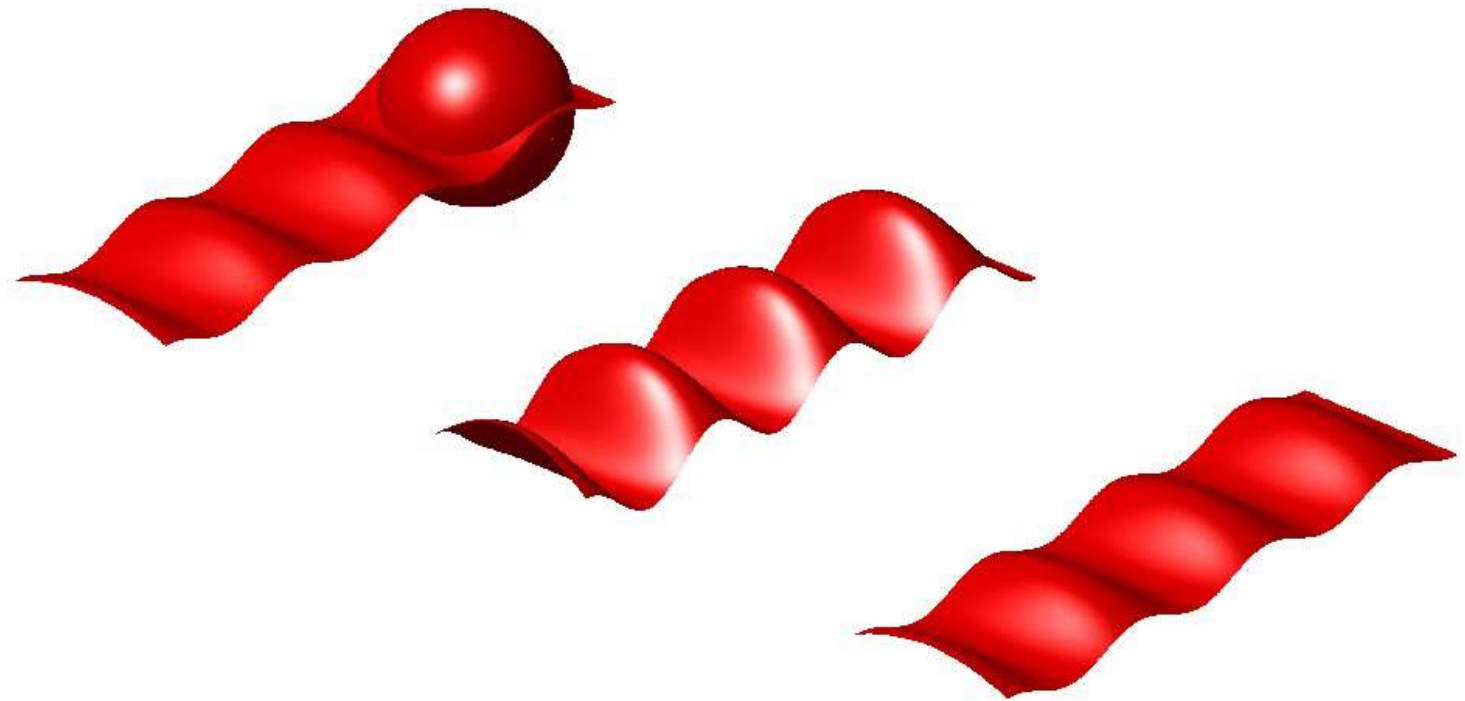


Demonstrated accelerometer resolution: $\sim 10^{-11}$ g.

- ✓ 3 separated diffraction zones
- ✓ Coriolis acceleration comes from rotating laser

$$\Delta\Phi_{\text{rot}} = 2\pi \frac{2 m_{\text{at}}}{h} A \times \Omega = L^2 \frac{v_t}{v_L} \Omega$$

- ✓ How can we discriminate between acceleration and rotation ?
- ✓ Use two atom interferometers in opposite directions



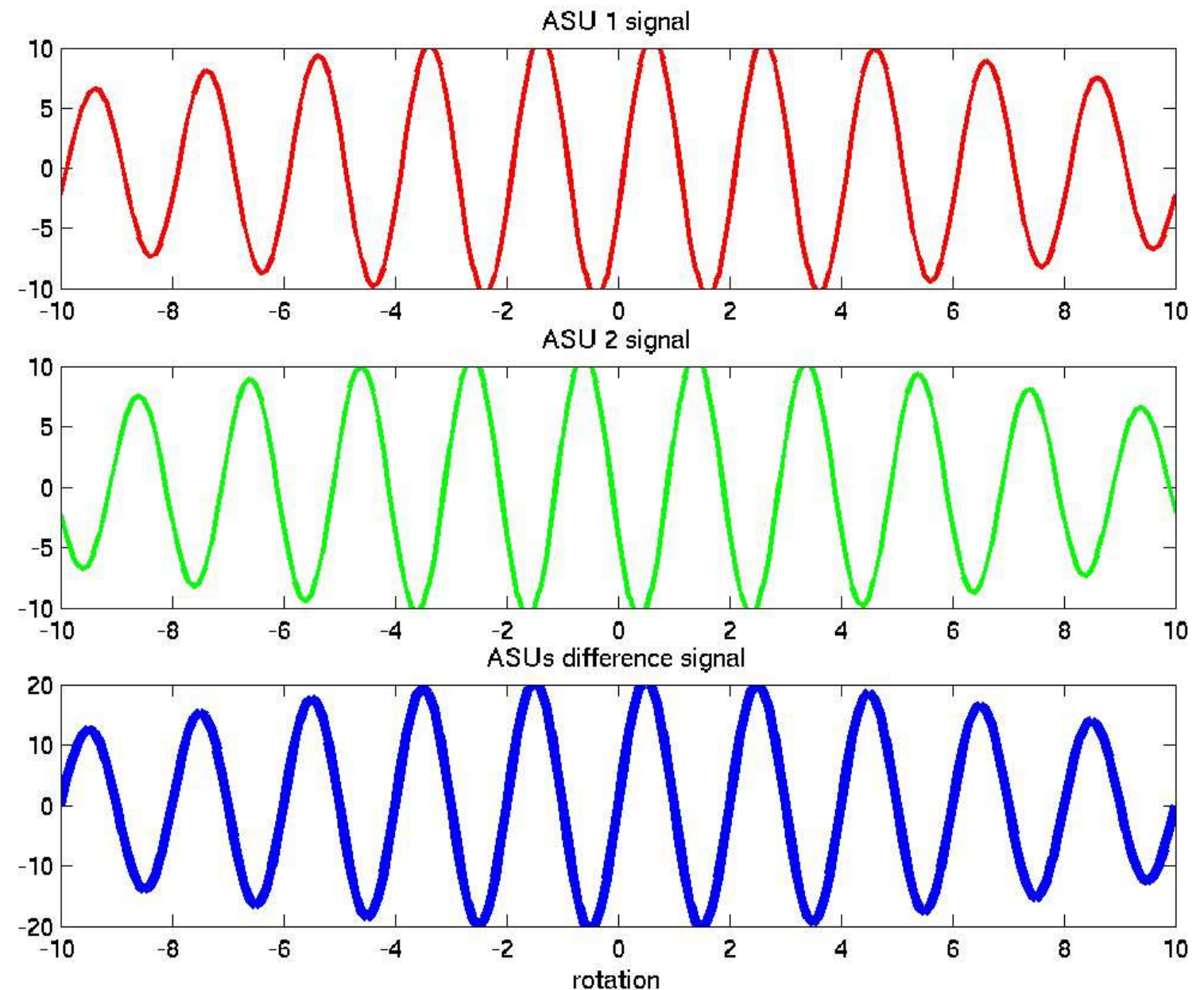
- ✓ 3 separated diffraction zones
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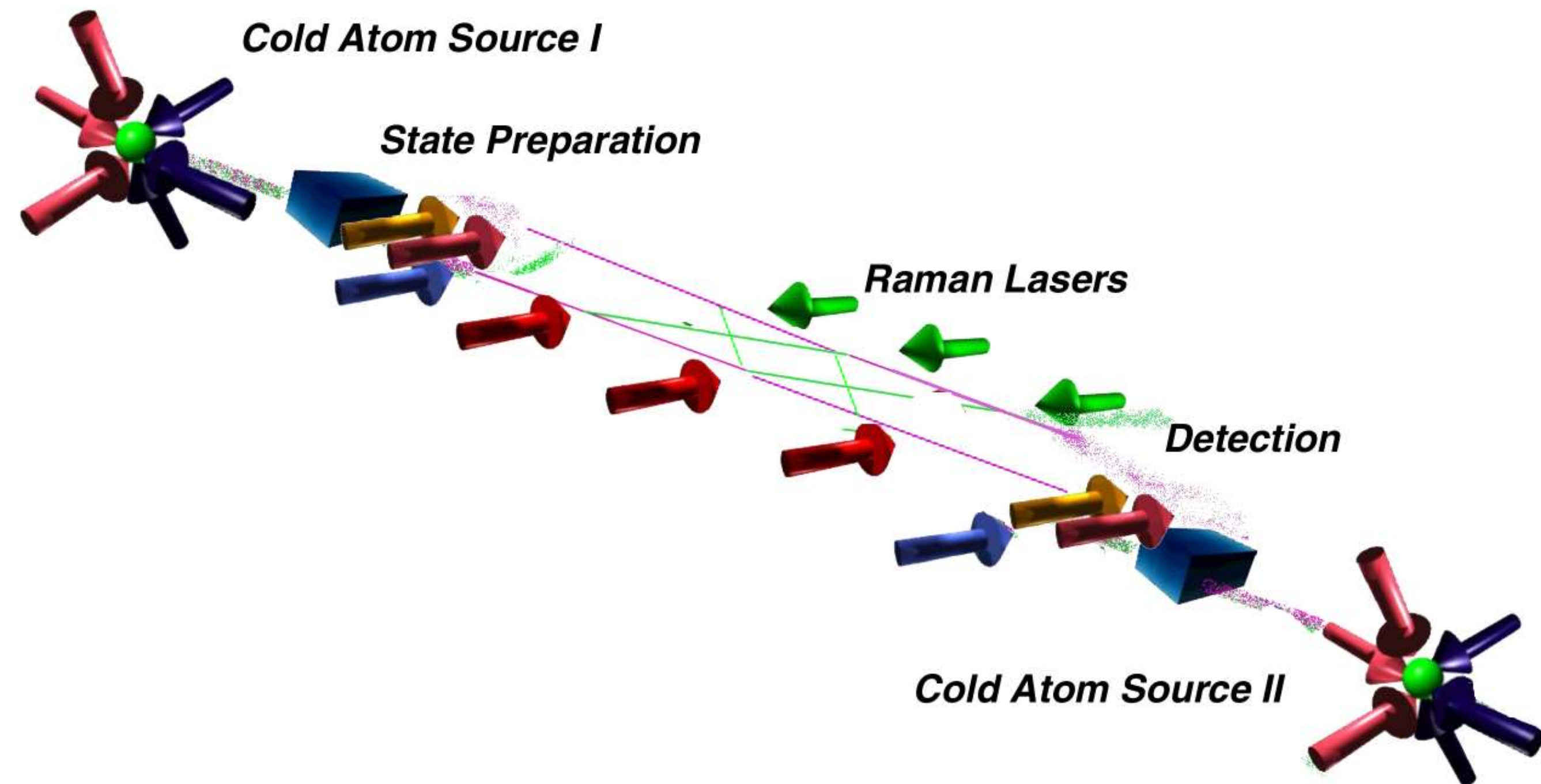
$$\Delta\Phi_{\text{rot}} = 2\pi \frac{2 m_{\text{at}}}{h} A \times \Omega = L^2 \frac{v_t}{v_L} \Omega$$

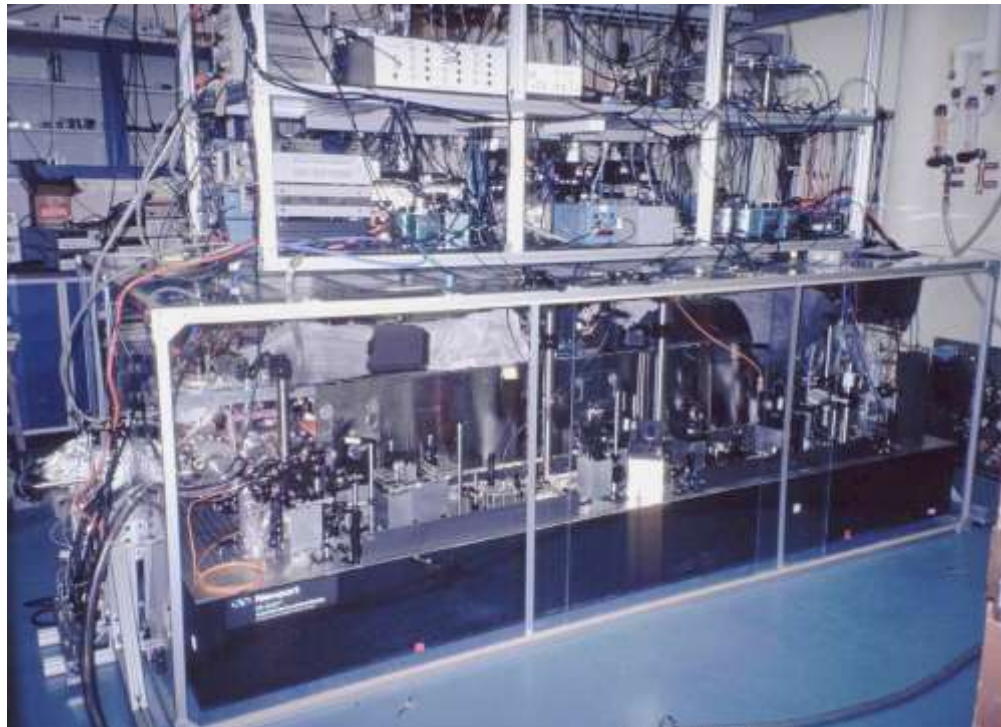
- ✓ How can we discriminate between acceleration and rotation ?

$$\Delta\Phi_{\text{acc}} = k T_{\text{drift}}^2 a = \frac{L^2}{v_L^2} a$$

- ✓ Use two atom interferometers in opposite directions







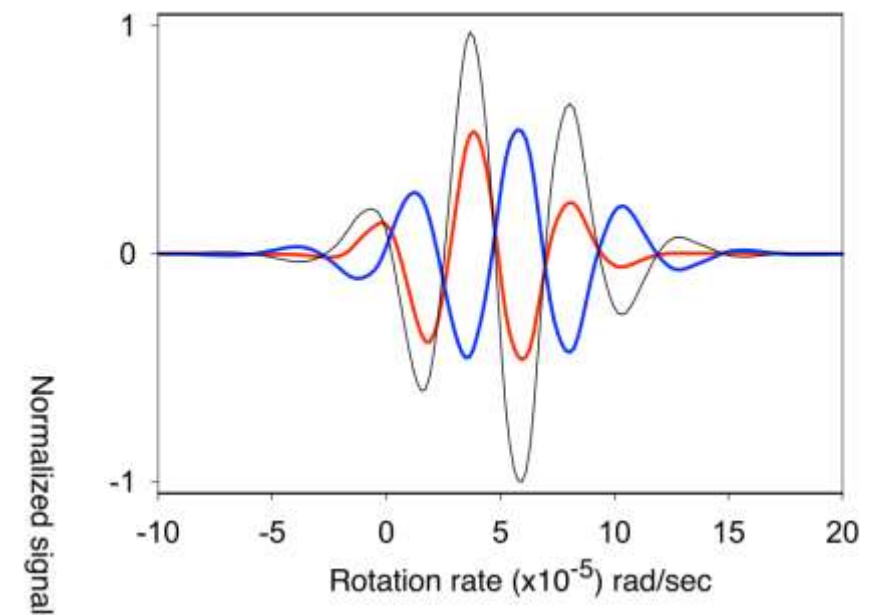
AI gyroscope, demonstrated
laboratory performance:

2×10^{-6} deg/hr^{1/2} ARW

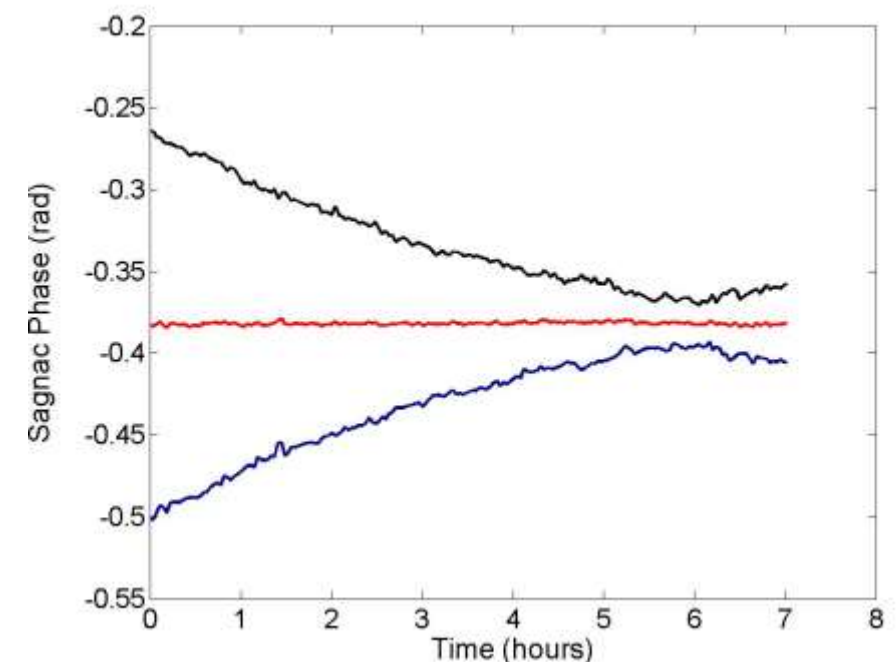
$< 10^{-4}$ deg/hr bias stability

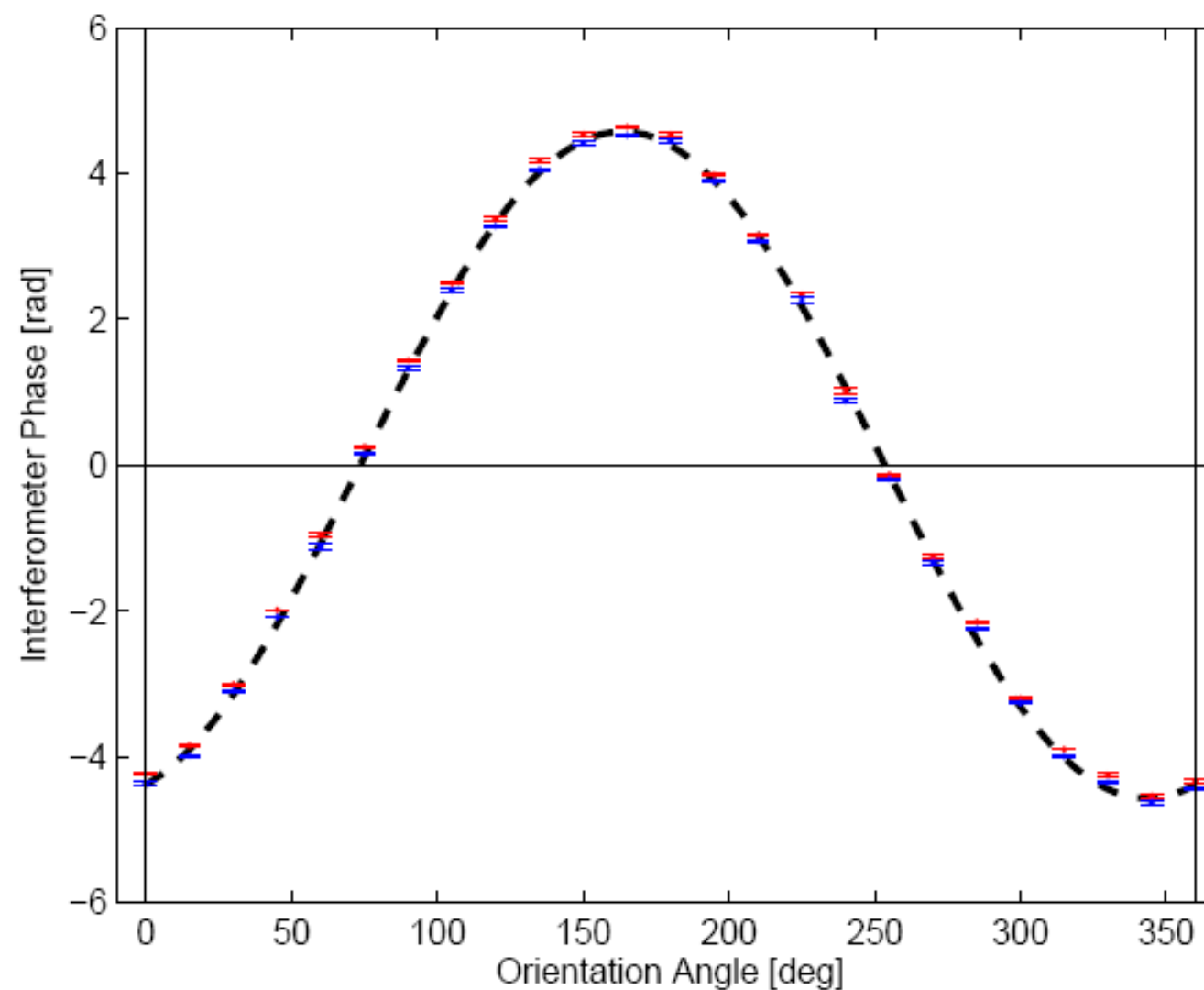
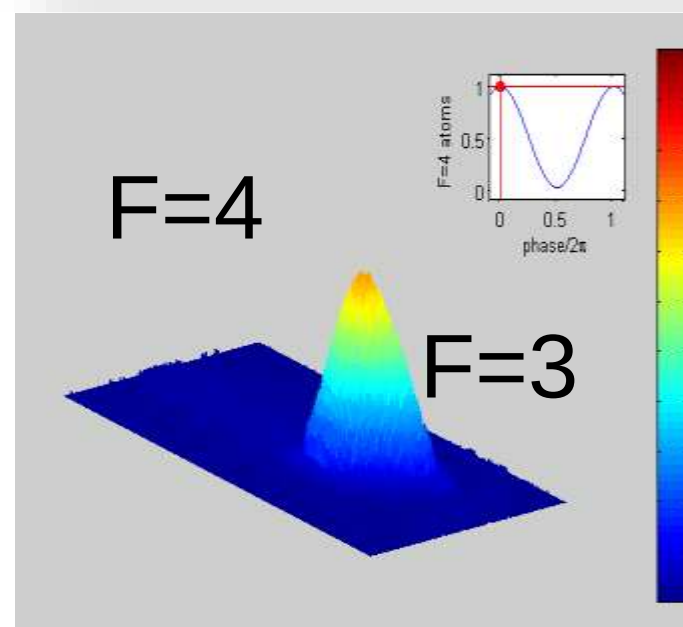
Compact, fieldable (navigation) and dedicated very high-sensitivity (Earth rotation dynamics, tests of GR) geometries possible.

Rotation signal



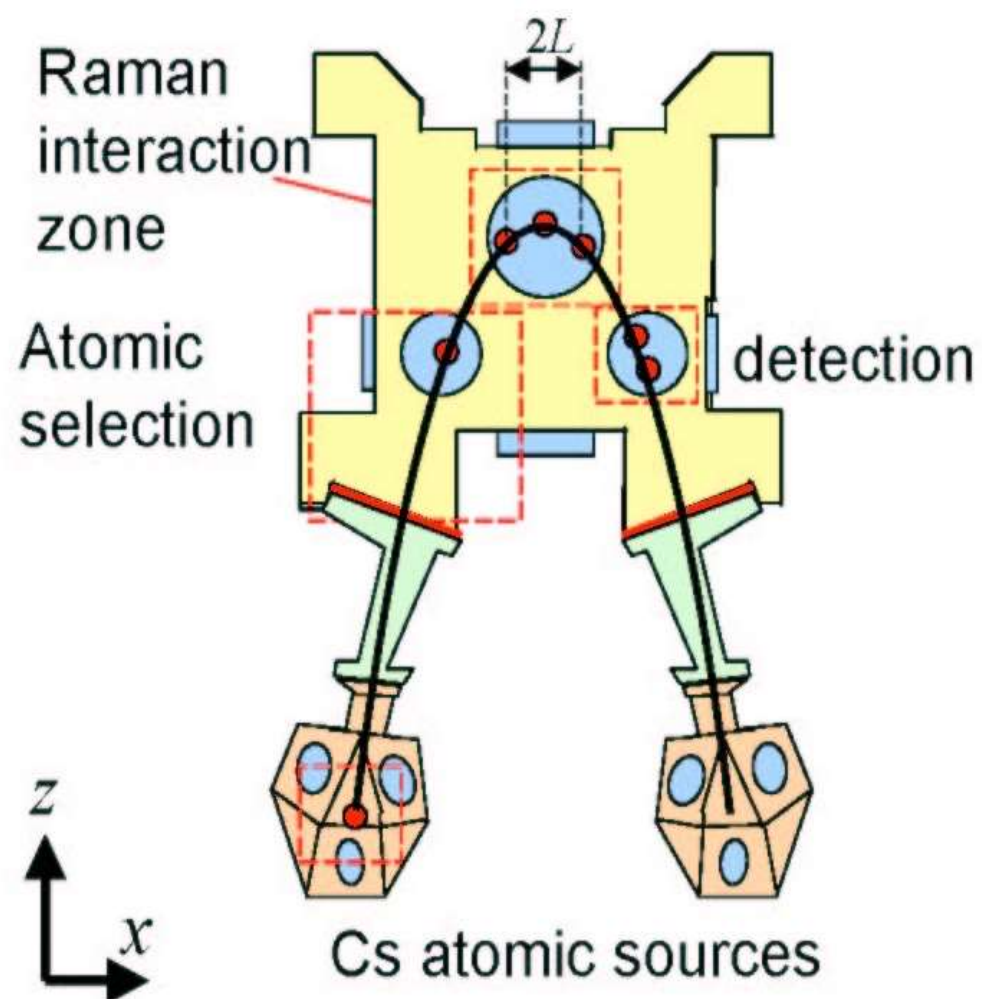
Bias stability





Gyroscope output vs. orientation.

$200 \mu\text{deg/hr}^{1/2}$

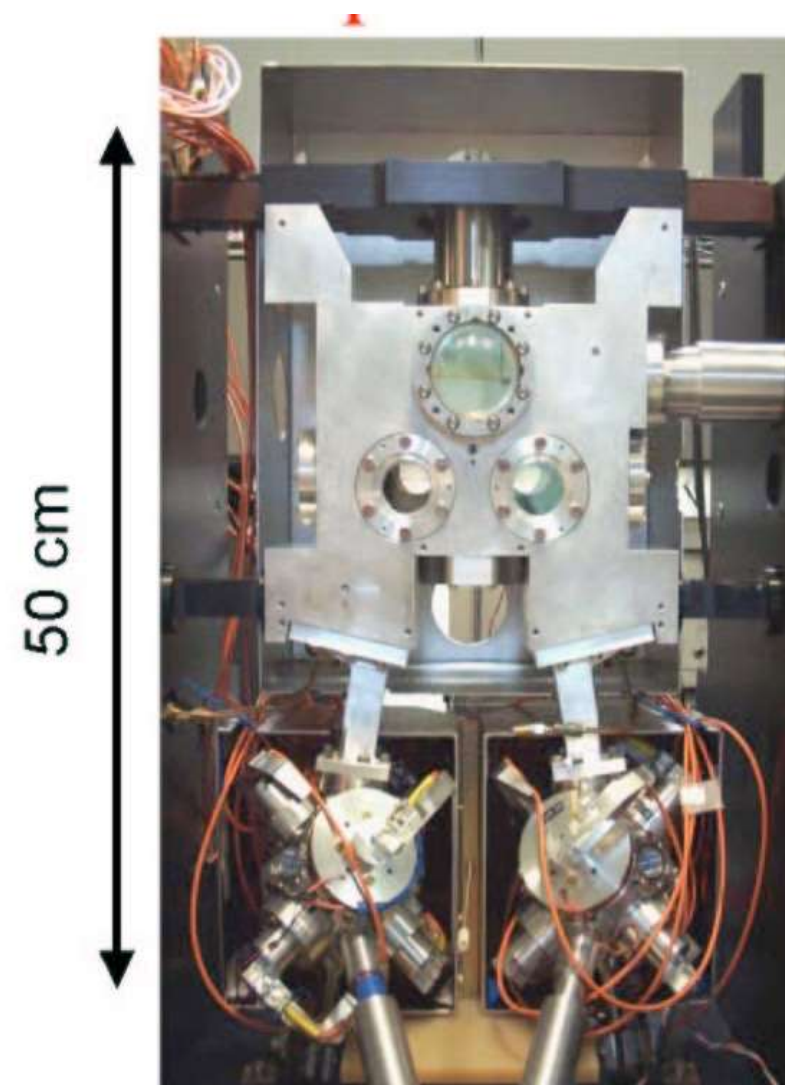


Launching velocity :

$$V = 2,6 \text{ m.s}^{-1}$$

$$\theta = 8^\circ$$

Expected sensitivity of $30 \text{ nrad.s}^{-1}.\text{Hz}^{-1/2}$



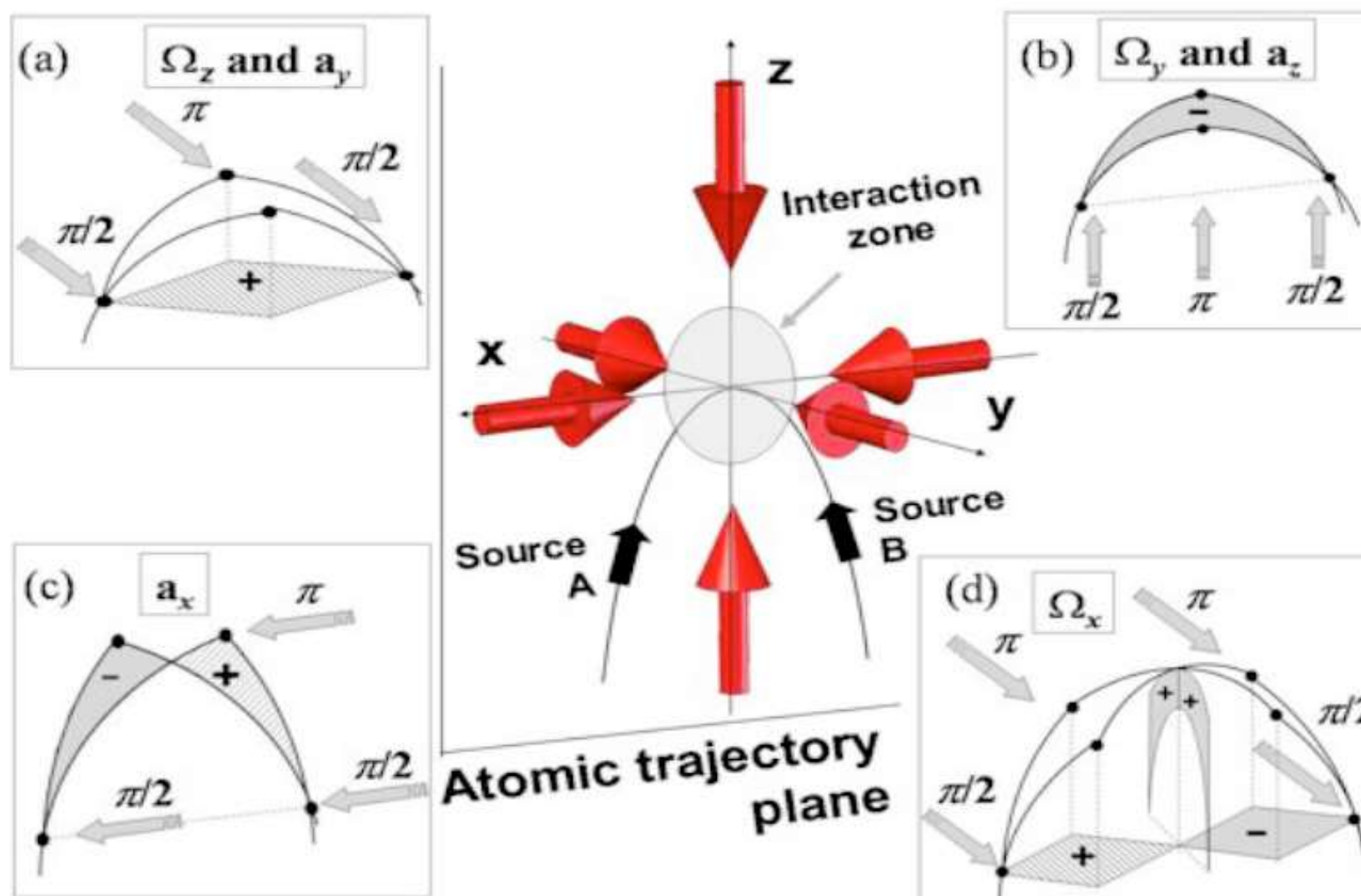
Interrogation zone :

$$2L = 30 \text{ mm}$$

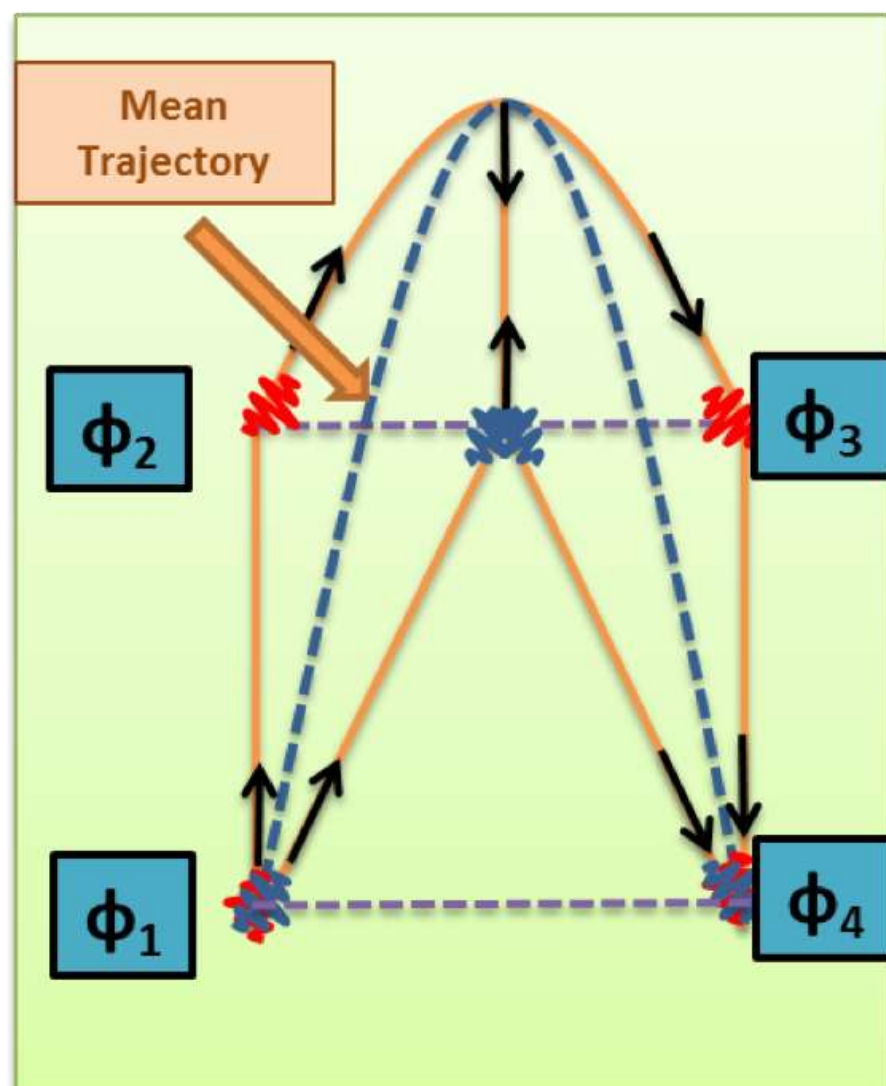
$$2T = 100 \text{ ms}$$

$$V_x = 30 \text{ cm.s}^{-1}$$

$$\text{area} = 6 \text{ mm}^2$$



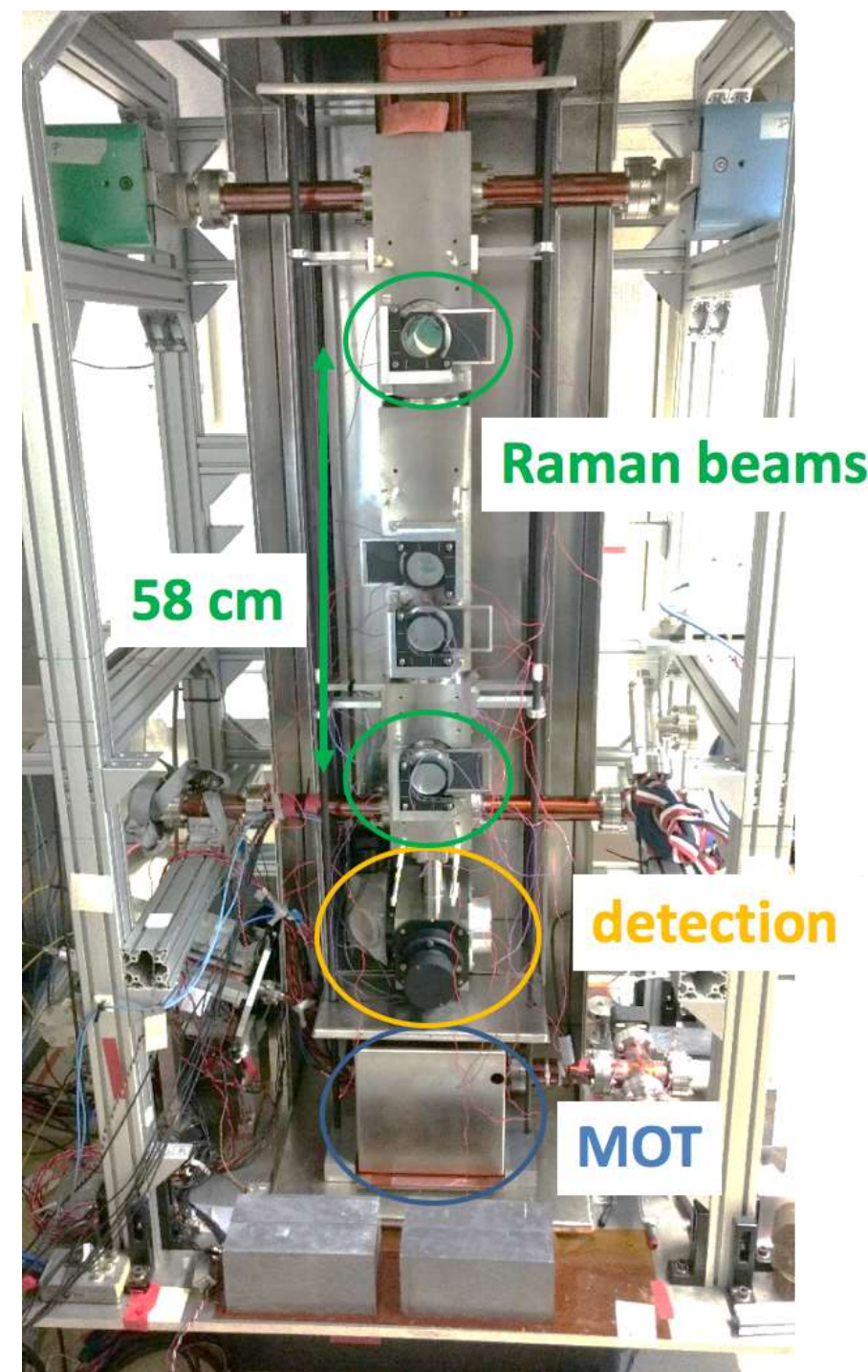
- ✓ Ability to measure the **six axes of inertial** in the same apparatus (rotations and accelerations) : B. Canuel et al., PRL 97, 010402 (2006)
- ✓ Rotation measurement sensitivity limited by standard QPN competitive with best gyro technologies : A. Gauguier, et al., Phys. Rev A 80 (2009), 063604

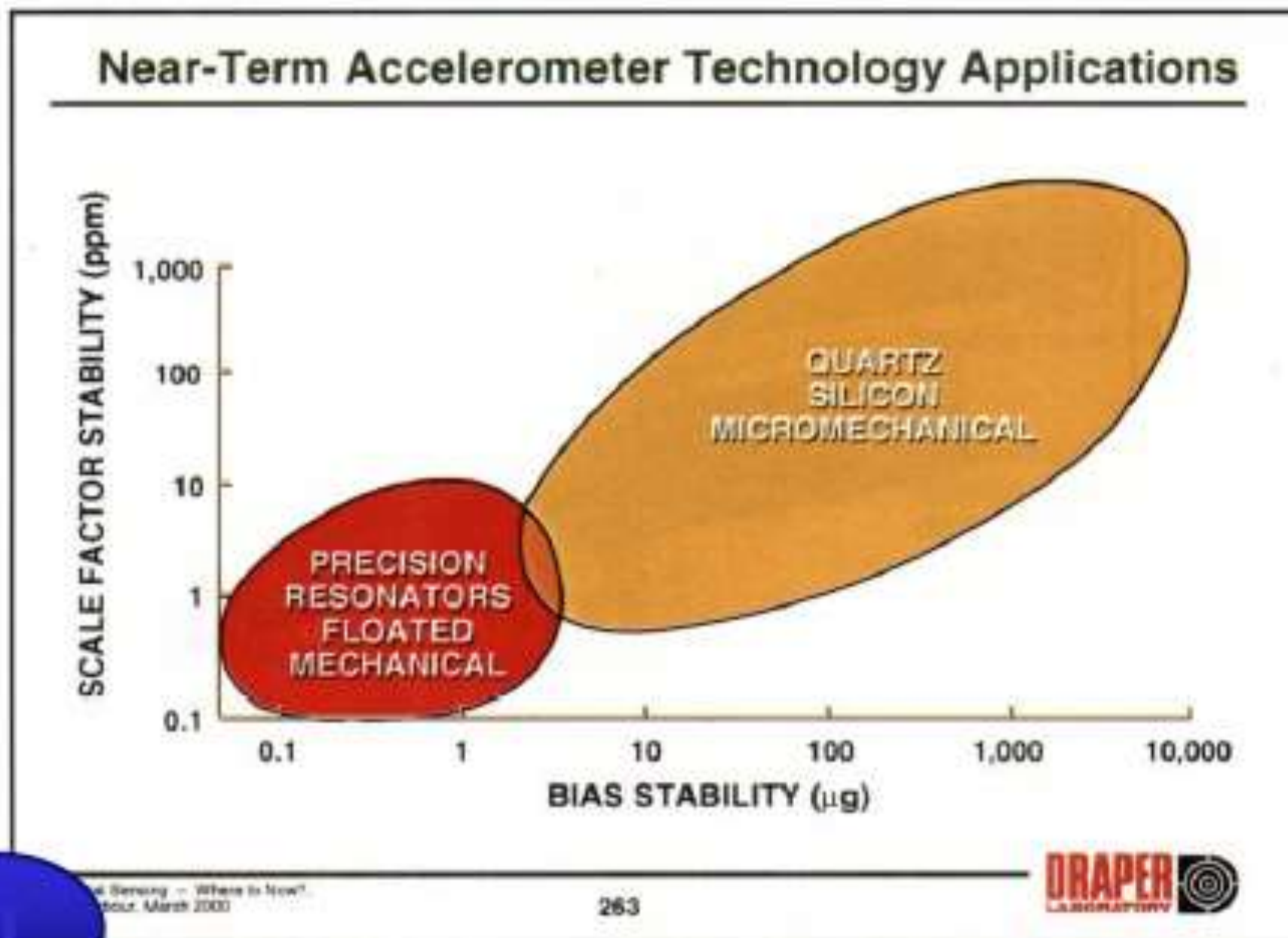


« Butterfly » configuration

$$\Phi_{\Omega} = \frac{1}{2} \vec{k}_{\text{eff}} \cdot (\vec{g} \times \vec{\Omega}) T^3$$

- 4×10^7 Cesium atoms
@ $1.2 \mu\text{K}$ launched
vertically at 5 m/s
- Relative alignment
of the beams $< 2 \mu\text{rad}$
- passive isolation
platform ($> 0.4 \text{ Hz}$)





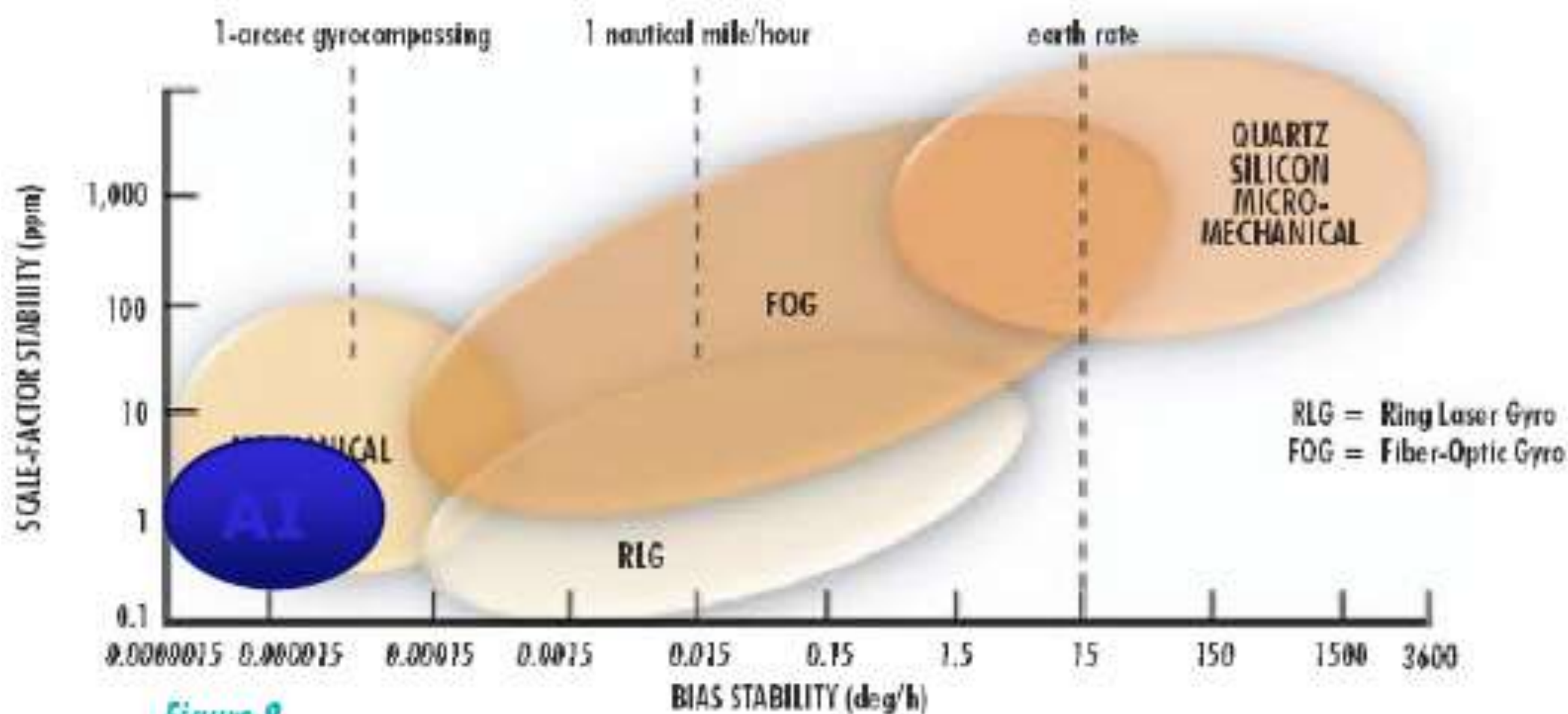
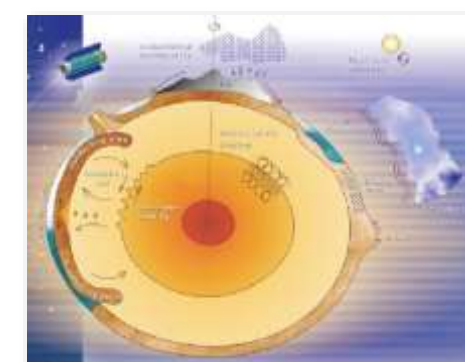
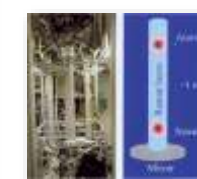
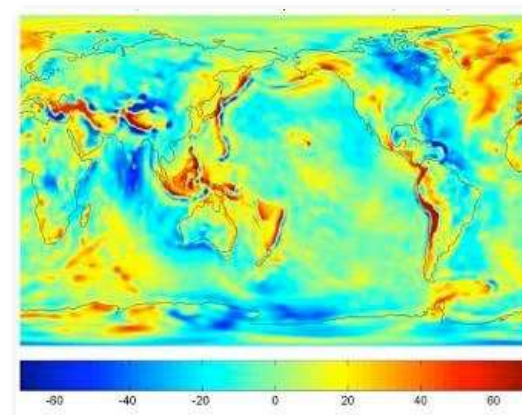
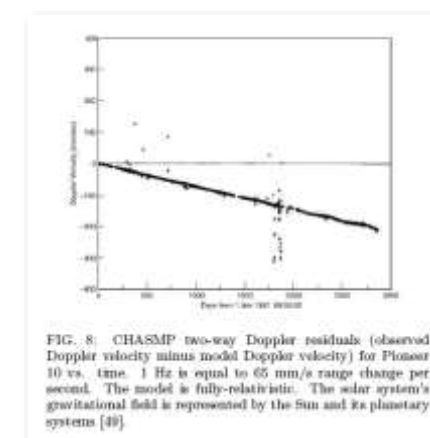
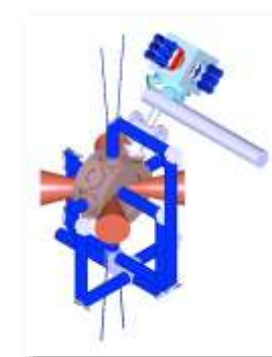
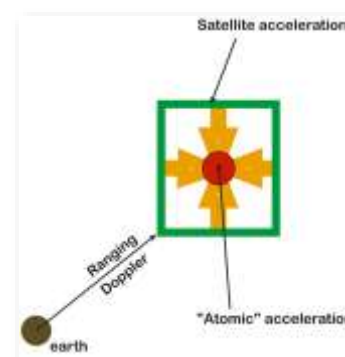
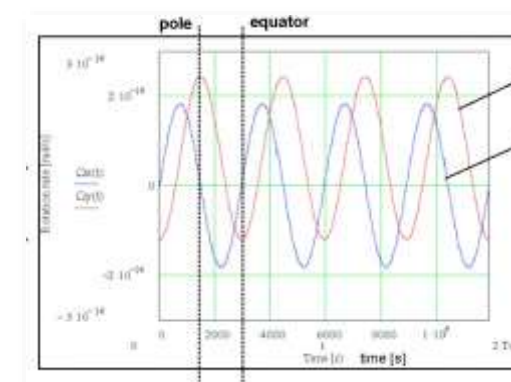
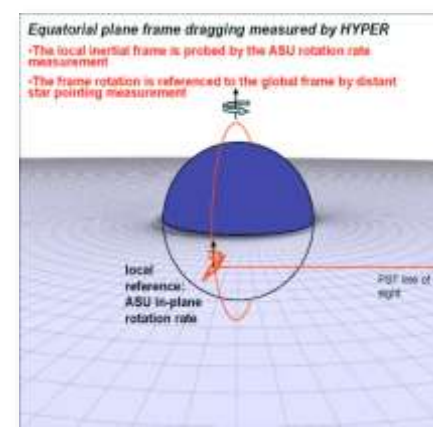


Figure 8

Source: Proc. IEEE/Workshop on
Autonomous Underwater Vehicles

- ✓ Gravimetry
 - ✓ Geodesy/Earthquake prediction
 - ✓ Oil/mineral/resource management
 - ✓ Gravity anomaly detection
- ✓ Low cost, compact, navigation grade IMU
 - ✓ Autonomous vehicle navigation
 - ✓ Gravity compensated IMU (grav grad/gyro)
 - ✓ GPS-free high accuracy navigation
- ✓ Gravitational physics
 - ✓ Equivalence Principle
 - ✓ Gravity-wave detection
 - ✓ Post-Newtonian gravity, tests of GR
 - ✓ Tests of the inverse square law
 - ✓ Dark matter/energy signatures?
- ✓ Beyond Standard model
 - ✓ Charge neutrality
 - ✓ h/m , tests of QED



The sensitivity function is a natural tool to characterize the influence of the noise of the environment on the the interferometric phase.

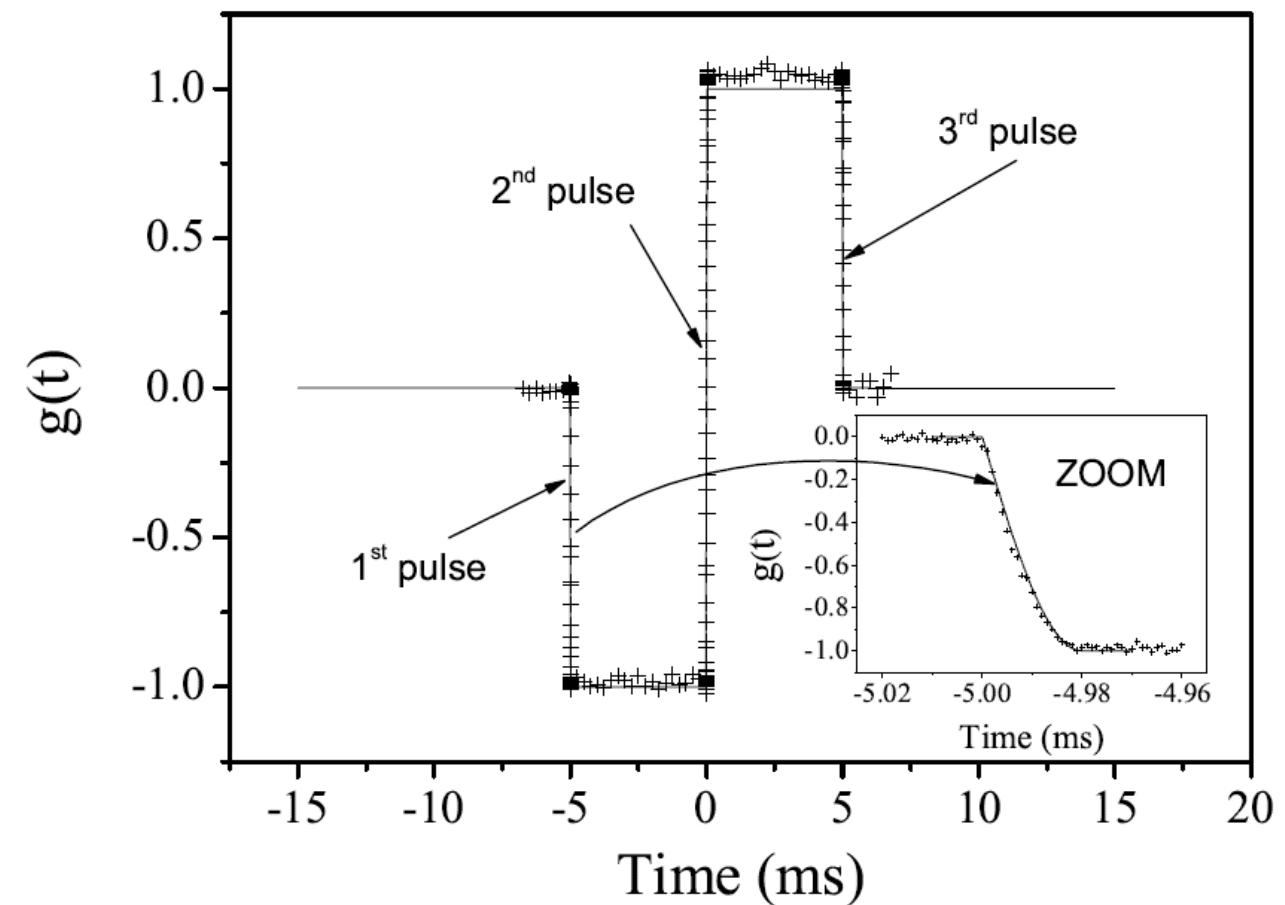
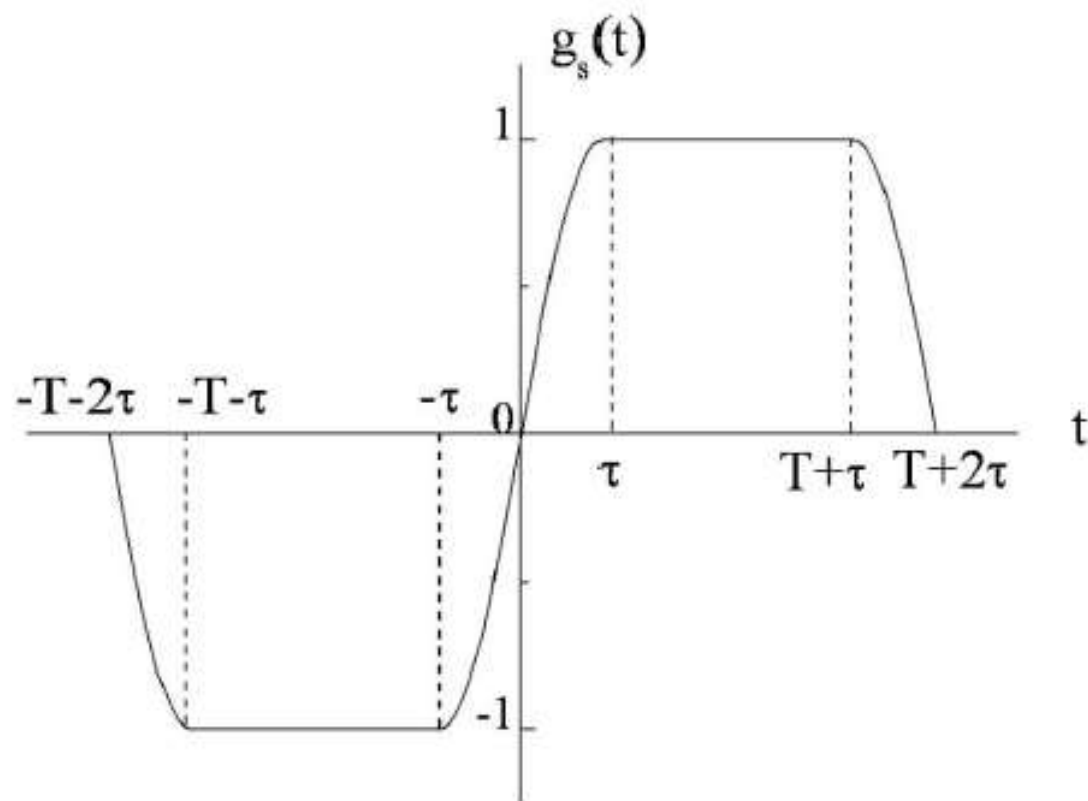
Let's assume a phase jump $\delta\phi$ occurs on the laser phase at time t during the interferometer sequence.

→ change in the transition probability $\delta P(\delta\phi, t)$

→ Sensitivity function $g(t) = 2 \lim_{\delta\phi \rightarrow 0} \frac{\delta P(\delta\phi, t)}{\delta\phi}$

$$g(t) = \begin{cases} \sin(\Omega_R t) & 0 < t < \tau_R \\ 1 & \tau_R < t < T + \tau_R \\ -\sin(\Omega_R (T - t)) & T + \tau_R < t < T + 2\tau_R \end{cases}$$

$$g(t) = \begin{cases} -2\sin(\Omega_R (T - t)) & T + \tau_R < t < T + 2\tau_R \end{cases}$$



$$g(t) = \begin{cases} \sin(\Omega_R t) & 0 < t < \tau_R \\ 1 & \tau_R < t < T + \tau_R \\ -\sin(\Omega_R (T - t)) & T + \tau_R < t < T + 2\tau_R \end{cases}$$

$$g(t) = \begin{cases} -\sin(\Omega_R (T - t)) & T + \tau_R < t < T + 2\tau_R \end{cases}$$

How to recover the interferometric sensitivity

- Integrate the sensitivity function

$$\delta\Phi = \int_{-\infty}^{+\infty} g_s(t) d\phi(t) = \int_{-\infty}^{+\infty} g_s(t) \frac{d\phi(t)}{dt} dt$$

- Compute the phase evolution during atomic free fall in the presence of gravity (for a gravimeter) :

$$\phi(t) = k \left(\frac{gt^2}{2} + v_0 t \right) + \phi^0(t)$$

$g_s(t) = g_s(-t)$
 0
 0 (supposed fixed)

How to recover the interferometric sensitivity

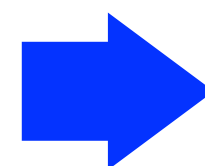
- Integrate the sensitivity function

$$\delta\Phi = \int_{-\infty}^{+\infty} g_s(t) d\phi(t) = \int_{-\infty}^{+\infty} g_s(t) \frac{d\phi(t)}{dt} dt$$

- Compute the phase evolution during atomic free fall in the presence of gravity (for a gravimeter) :

$$\phi(t) = k \left(\frac{gt^2}{2} + v_0 t \right) + \phi^0(t)$$

g_s(t) = g_s(t) 0 (supposed fixed)



$$\delta\Phi = \int_{-\infty}^{+\infty} g_s(t) k g t dt$$

$$\delta\Phi = k g (T + 2\tau) \left(T + \frac{4\tau}{\pi} \right)$$

Sensitivity to laser phase noise

→ Model of the phase noise

$$\phi(t) = A_0 \cos(\omega_0 t + \psi) \quad \frac{d\phi(t)}{dt} = -A_0 \omega_0 \sin(\omega_0 t + \psi)$$

→ Go to Fourier space

$$G(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} g_s(t) dt = \int_{-\infty}^{+\infty} -i \sin(\omega t) g_s(t) dt$$

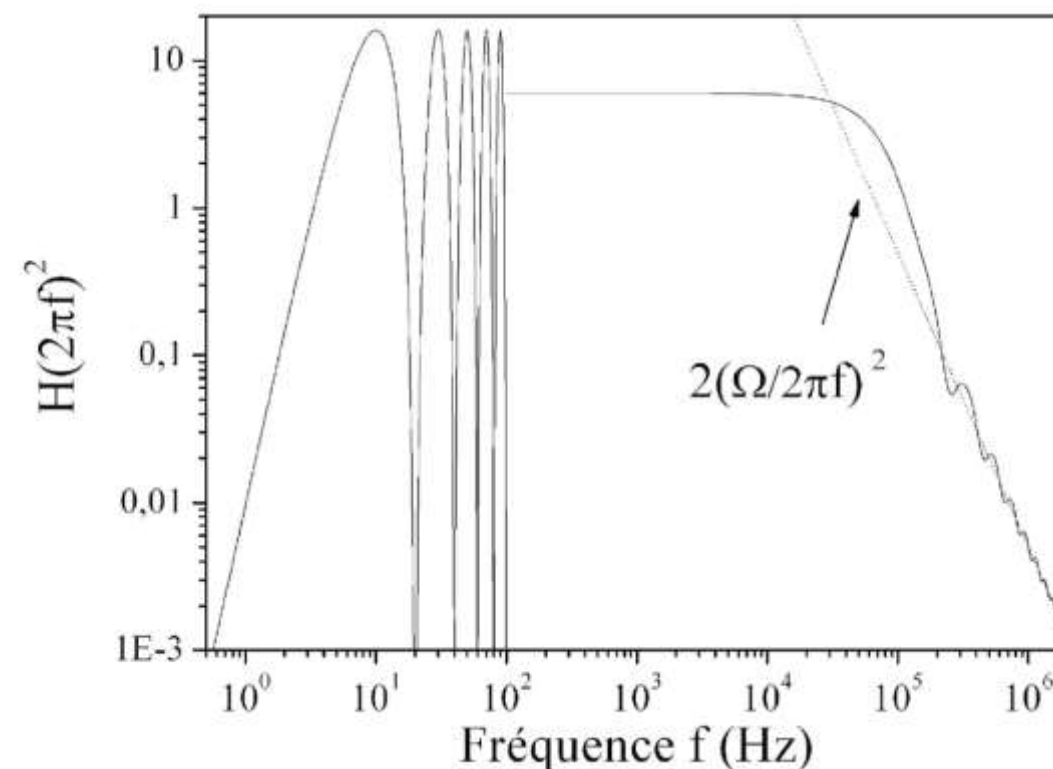
→ Get the phase sensitivity for a phase fluctuation at frequency ω_0

$$\delta\Phi = -iA_0\omega_0 G(\omega_0) \cos(\psi) = -A_0\omega_0 \cos(\psi) |G(\omega_0)|$$

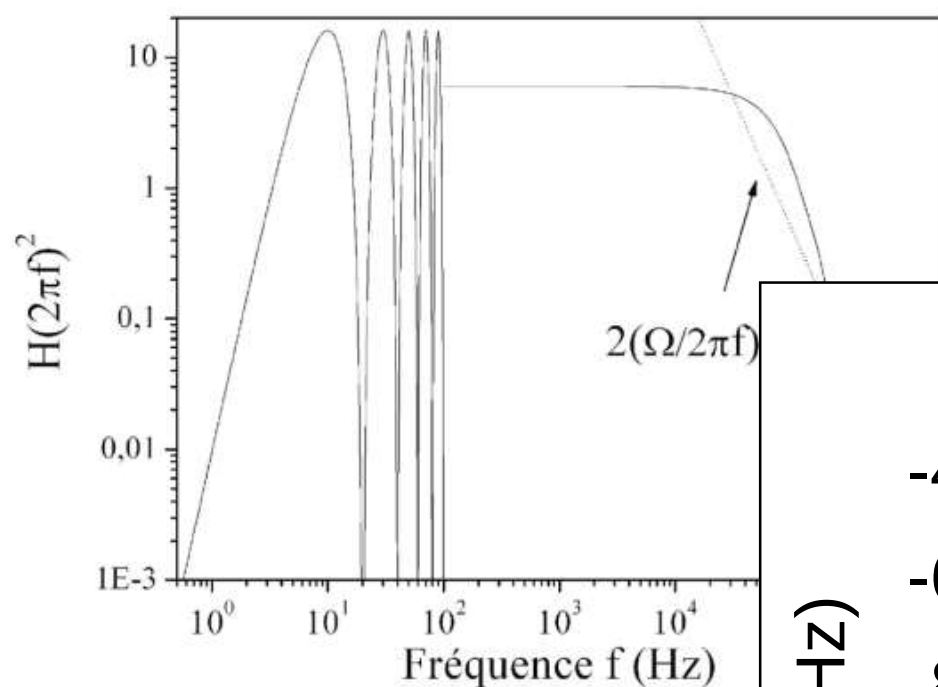
$$G(\omega) = \frac{4i\Omega_R}{\omega^2 - \Omega_R^2} \sin\left(\frac{\omega(T + 2\tau)}{2}\right) \left(\cos\left(\frac{\omega(T + 2\tau)}{2}\right) + \frac{\Omega_R}{\omega} \sin\left(\frac{\omega T}{2}\right) \right)$$

Interferometer transfer function (acts as a filter)

→ Introduce the transfer function $H(\omega) = |\omega G(\omega)|$

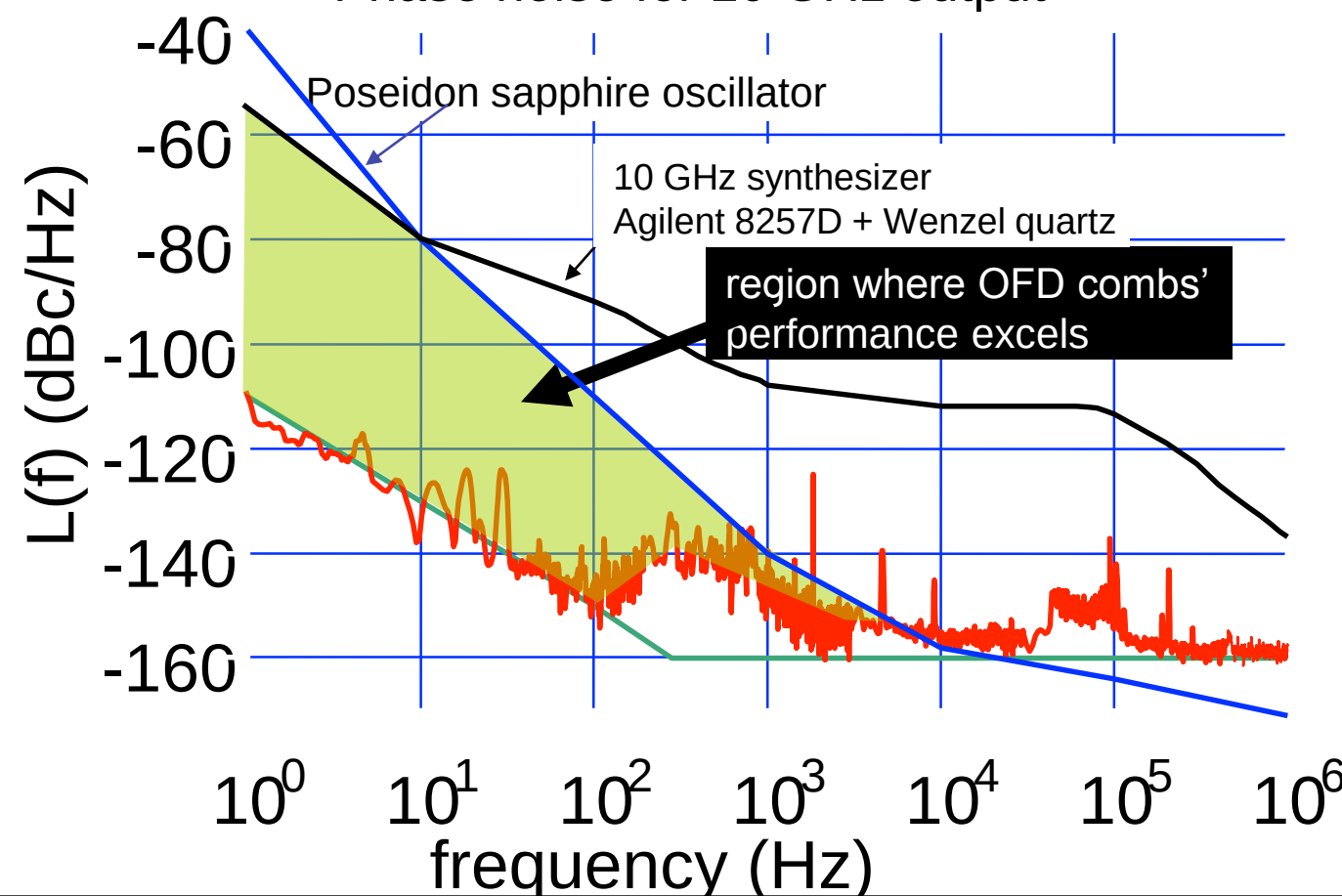


→ Error induced by phase noise $\sigma_{\Phi}^2 = \int_0^{+\infty} |\omega G(\omega)|^2 S_{\phi}(\omega) \frac{d\omega}{2\pi}$



T_c (s)	$2T$ (s)	Source 1 $\sigma_\phi(T_c)$ (mrad)	Source 2 $\sigma_\phi(T_c)$ (mrad)	Source 3 σ_ϕ (mrad)	Best Source $\sigma_a(T_c)$ (m.s ⁻²) / shot	Best Source $\sigma_a(1s)$ (m.s ⁻² .Hz ^{-1/2})
0.25	0.1	1.2	3.5	2.2	3×10^{-8}	1.5×10^{-8}
10	2	22	8.8	4.6	1.1×10^{-9}	3.6×10^{-9}
10	5	55	22	10	6.0×10^{-11}	9.1×10^{-10}
10	10	110	44	20	3.0×10^{-11}	4.5×10^{-10}

Two Optical Frequency Dividers Phase noise for 10 GHz output



- Raman phase noise w
- Best known μ wave so
- Expected performance

ons (σ_ϕ) and to the acceleration
three different sources assuming
ion time. (Source 1: Premium;

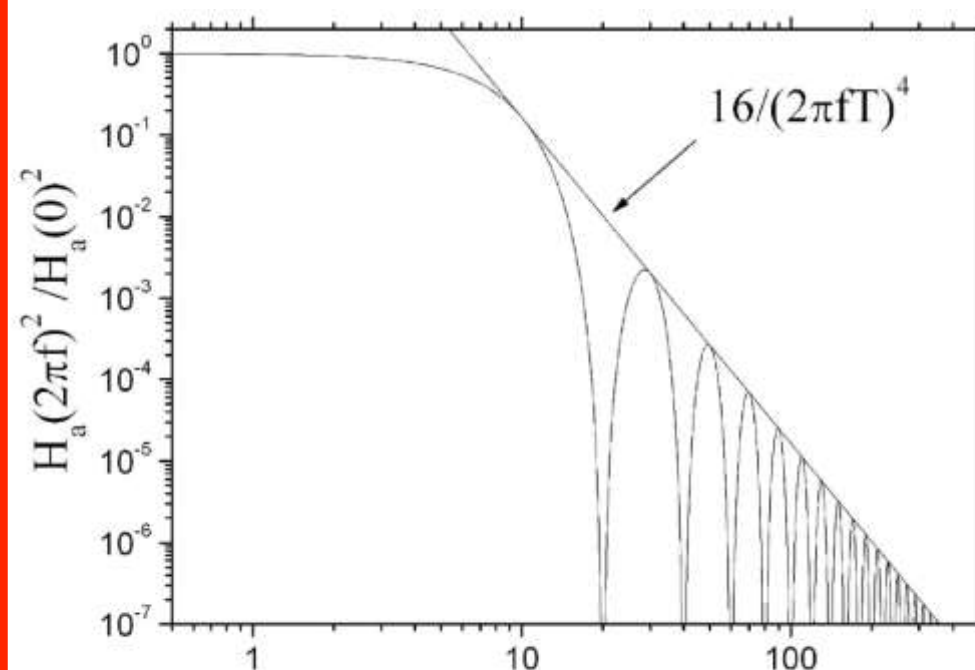
on time' (Source 1: Premium;
three different sources assuming
us (σ_ϕ) and to the acceleration

It is now possible to calculate the signal for a “noisy” acceleration spectrum

→ Introduce the sensitivity function for acceleration

$$g_a(t) = 2 \lim_{\delta_a \rightarrow 0} \frac{\delta P_a(\delta_a, t)}{\delta_a}$$

$$g_s(t) = \frac{1}{k_{eff}} \frac{d^2 g_a(t)}{dt^2}$$



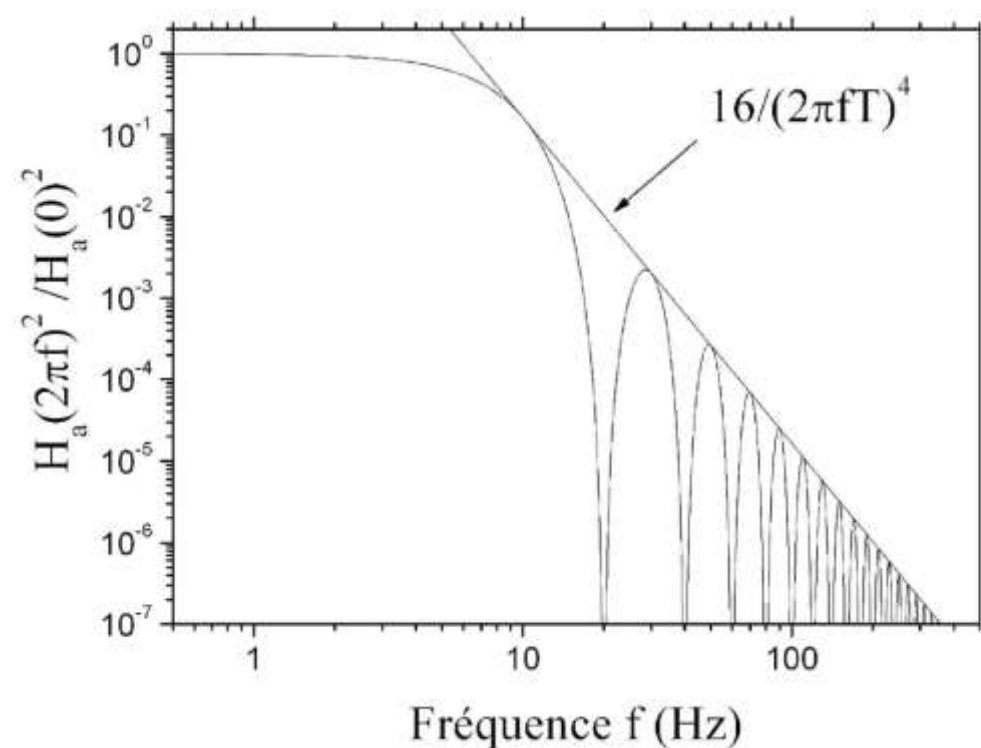
$$|H_a(\omega)|^2 = \frac{k_{eff}^2}{\omega^4} |H(\omega)|^2$$

$$\sigma_{\Phi}^2(\tau_m) = \frac{k_{eff}^2}{\tau_m} \sum_{n=1}^{\infty} \frac{|H(2\pi n f_c)|^2}{(2\pi n f_c)^4} S_a(2\pi n f_c)$$

Fréquence f (Hz)



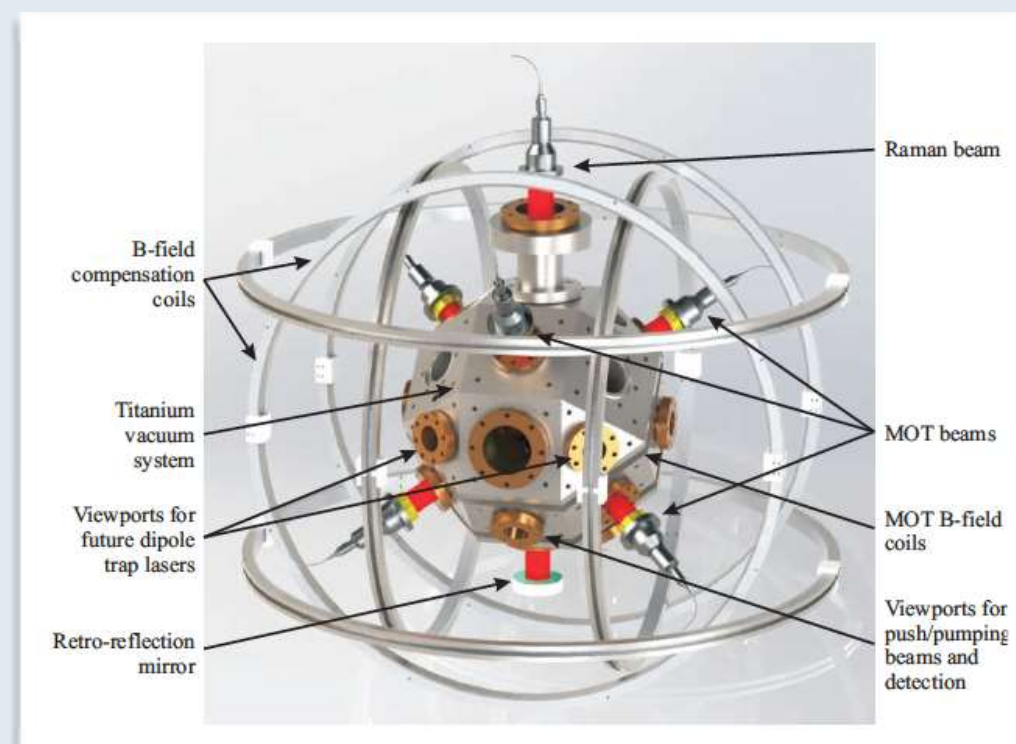
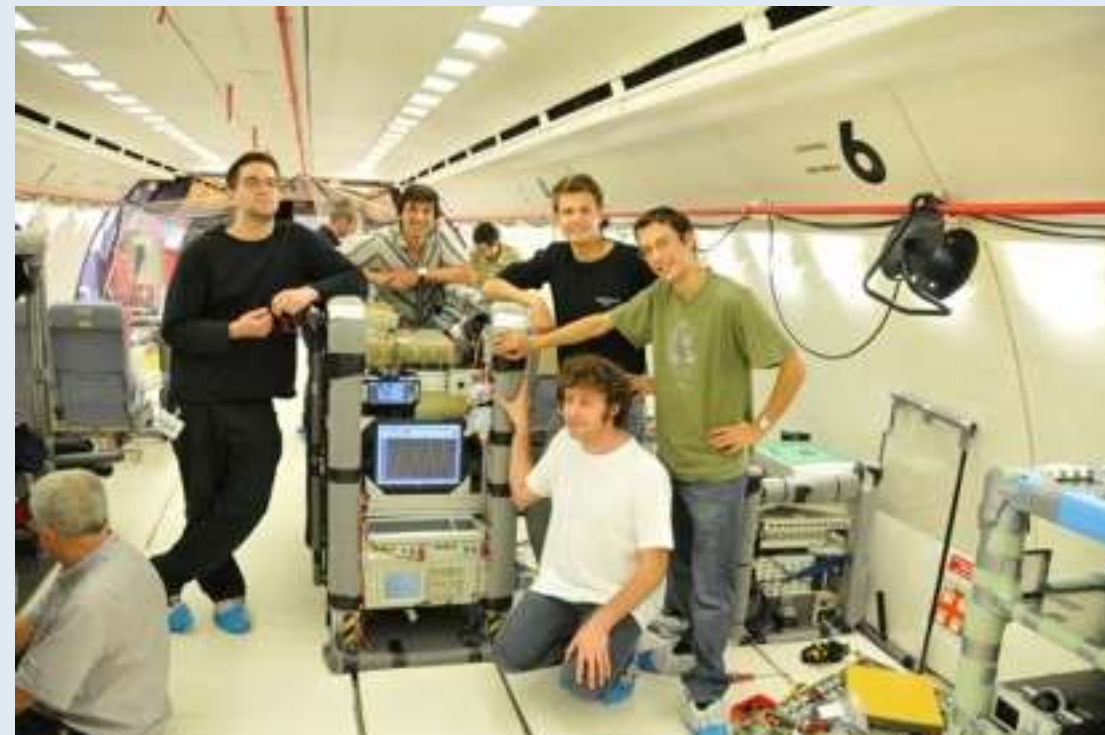
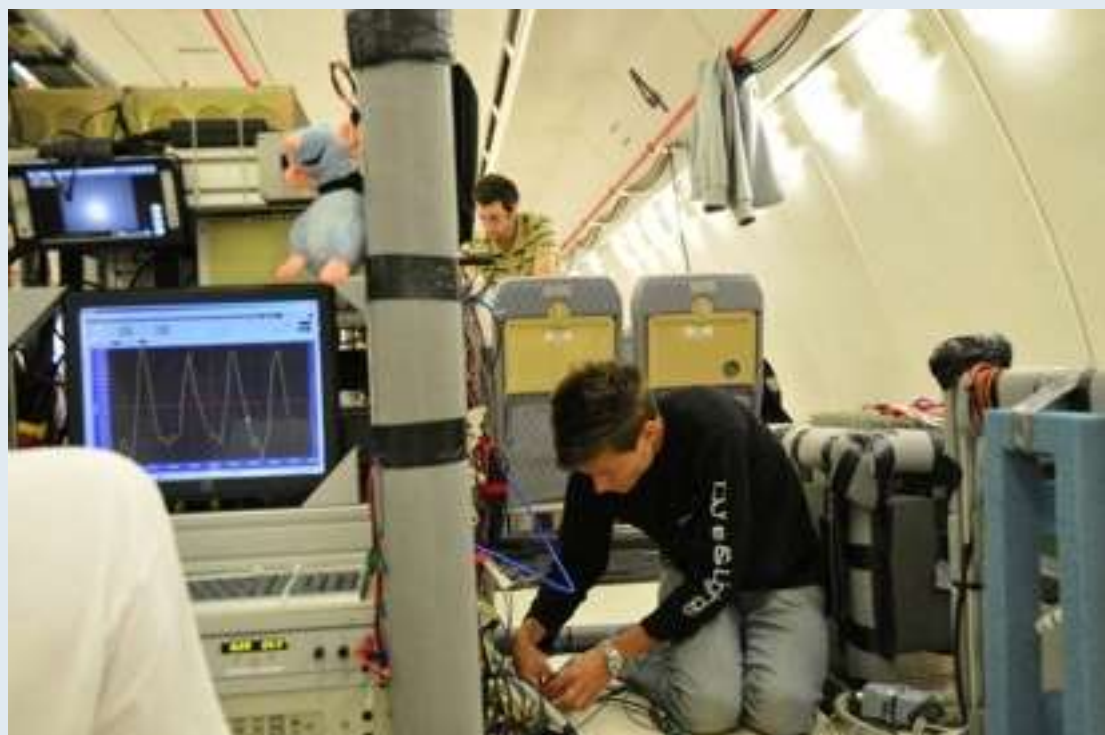
Integration time

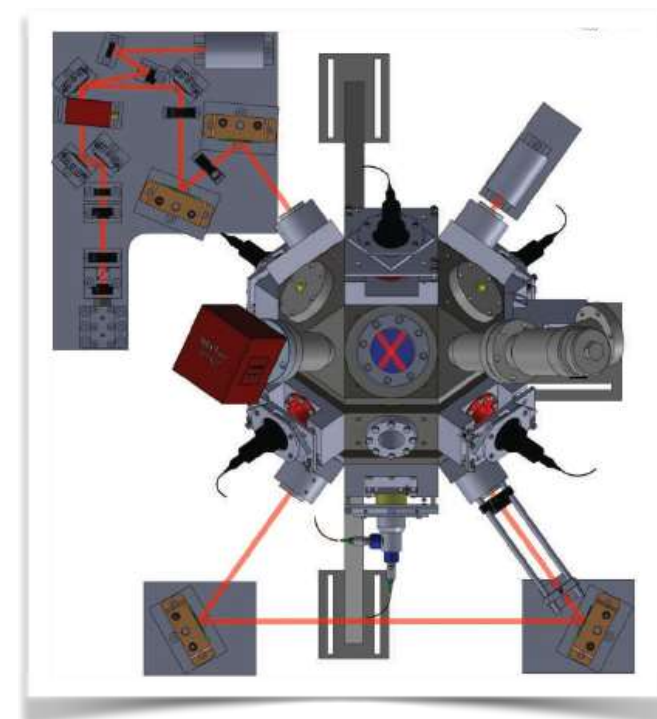
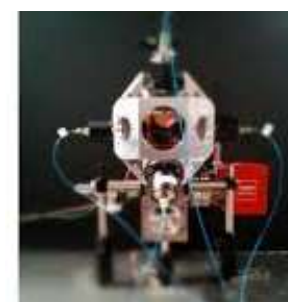
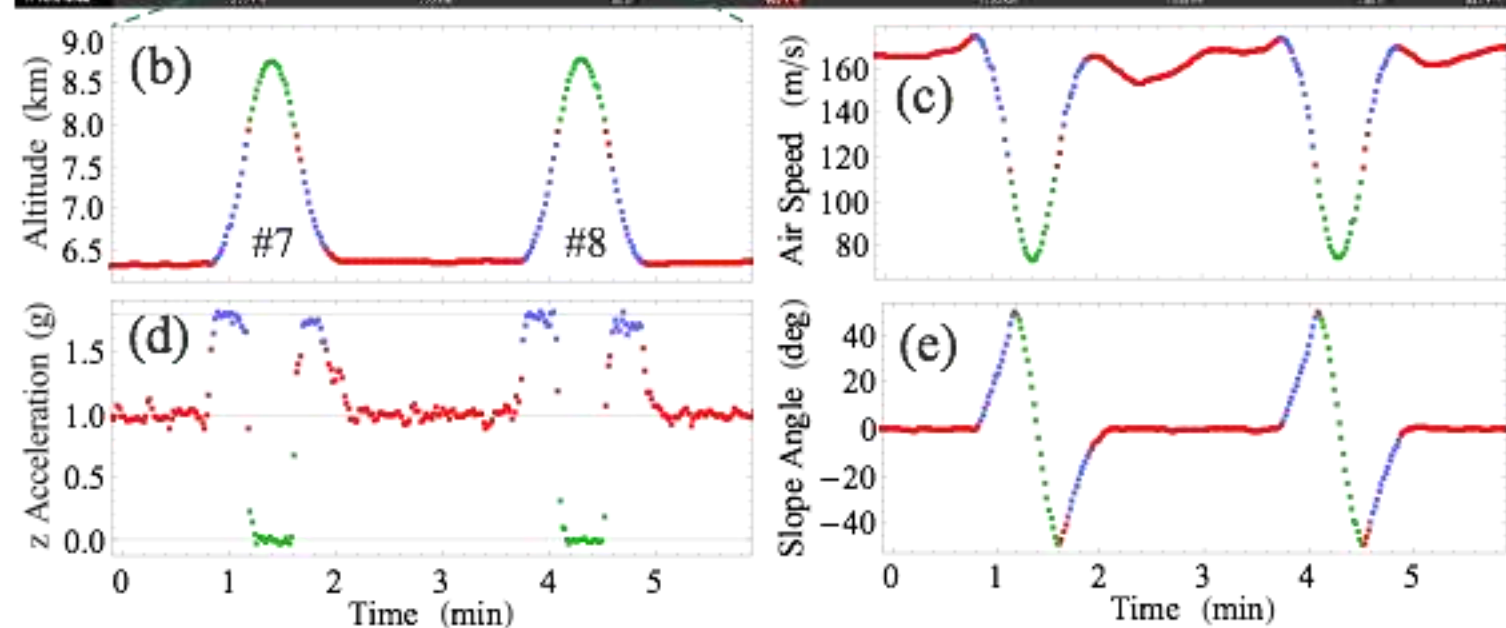
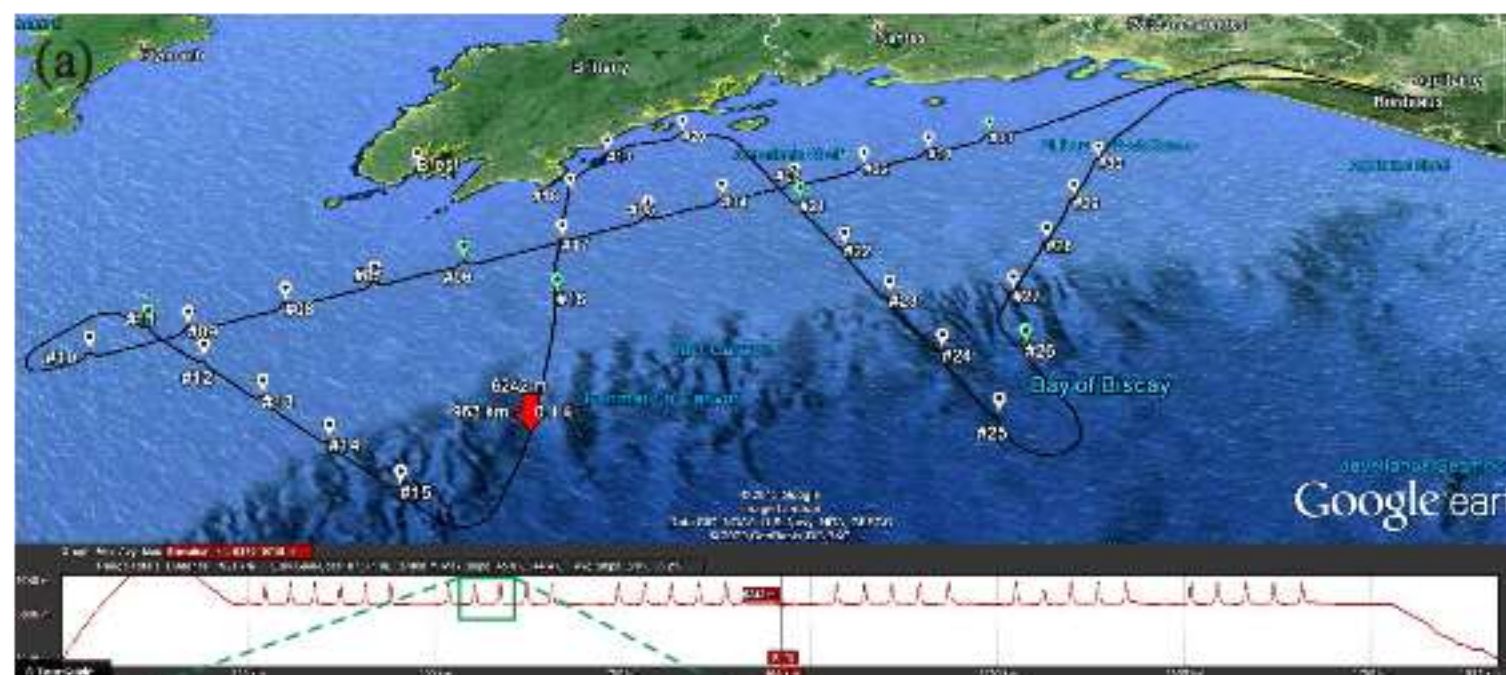


$$|H_a(\omega)|^2 = \frac{k_{eff}^2}{\omega^4} |H(\omega)|^2$$

$$\sigma_{\Phi}^2(\tau_m) = \frac{k_{eff}^2}{\tau_m} \sum_{n=1}^{\infty} \frac{|H(2\pi n f_c)|^2}{(2\pi n f_c)^4} S_a(2\pi n f_c)$$

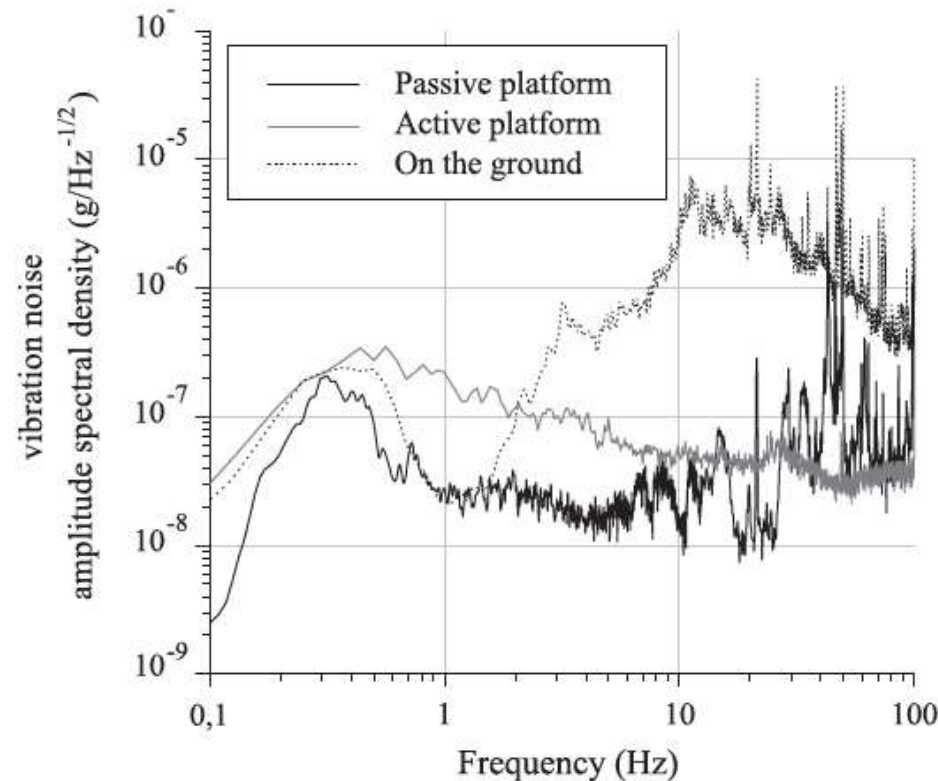
P. Cheinet, et al., "Measurement of the sensitivity function in time-domain atomic interferometer", IEEE Trans. on Instrum. Meas. 57, 1141, (2008)





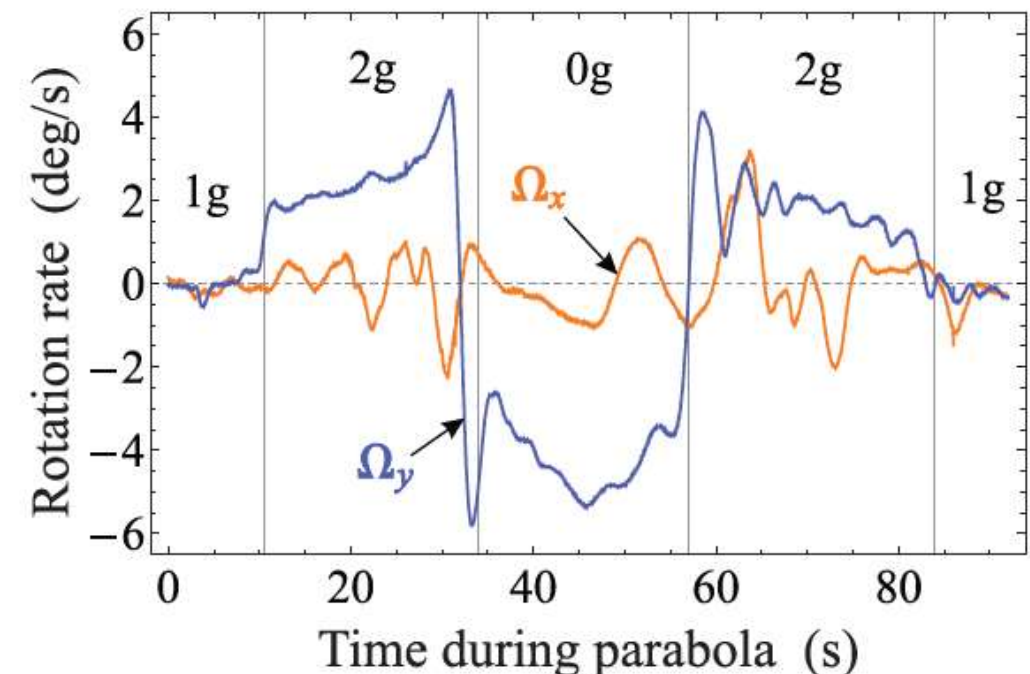
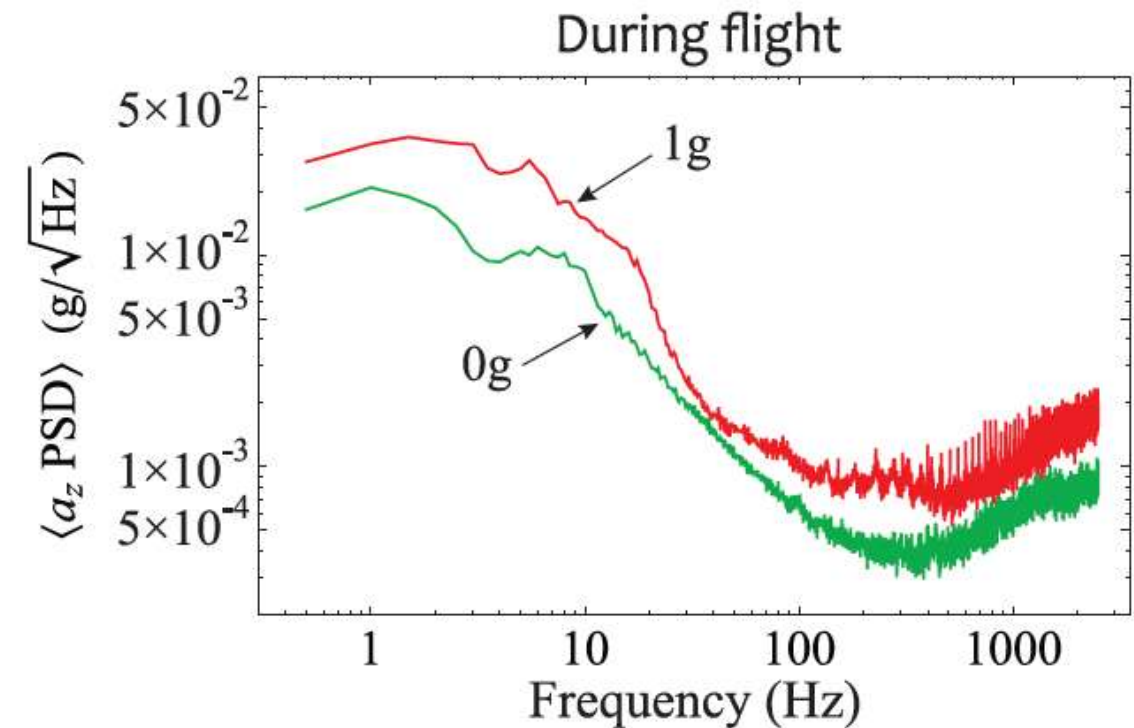
iXAtom

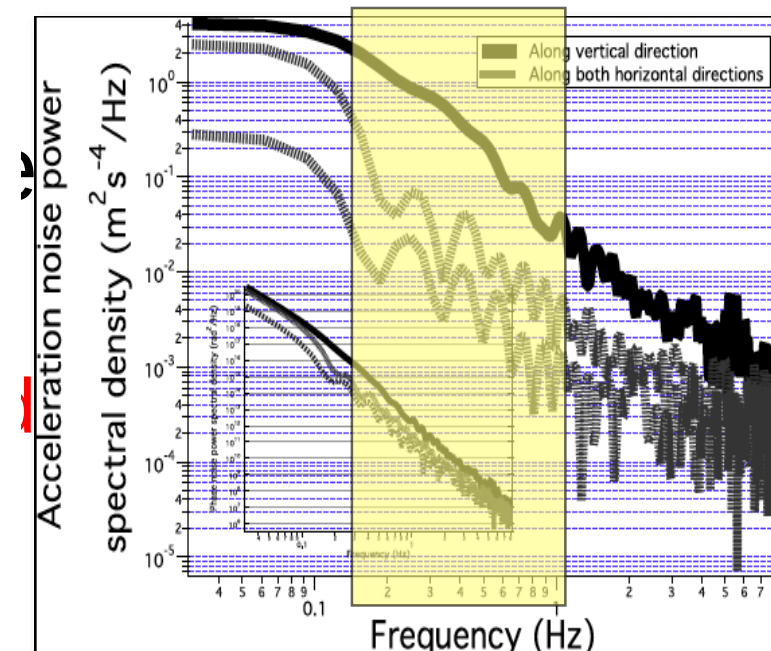
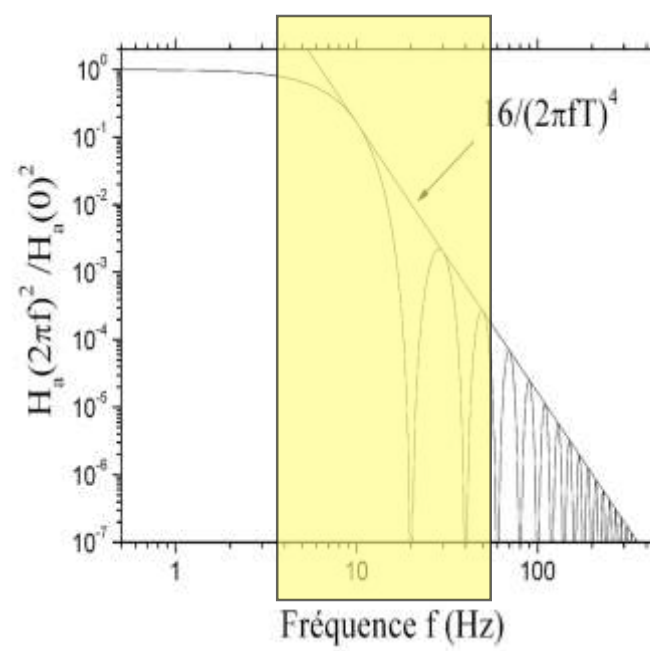
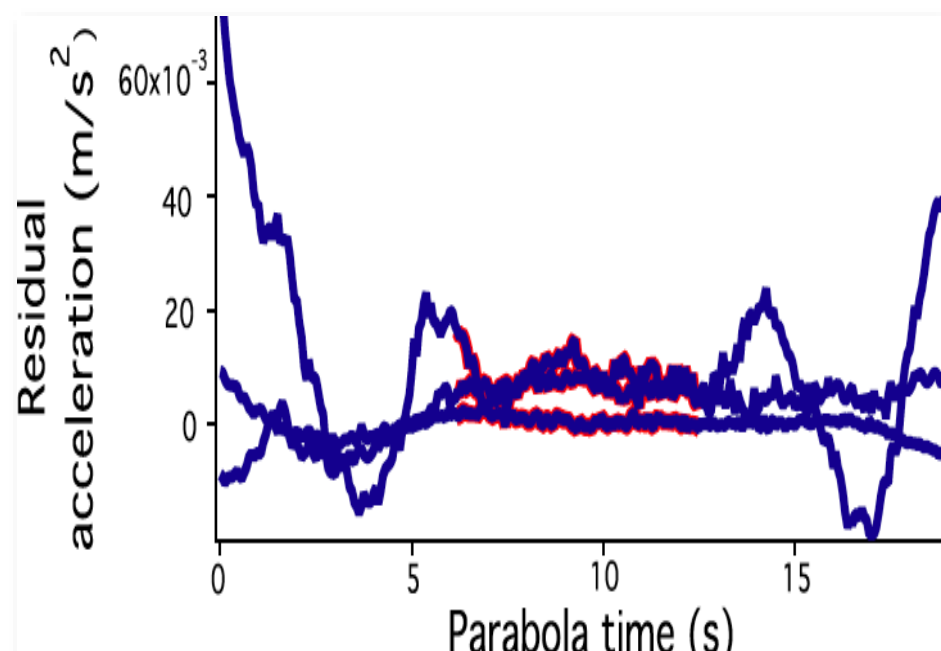
- Strong vibrations (10^5 times larger than on ground)



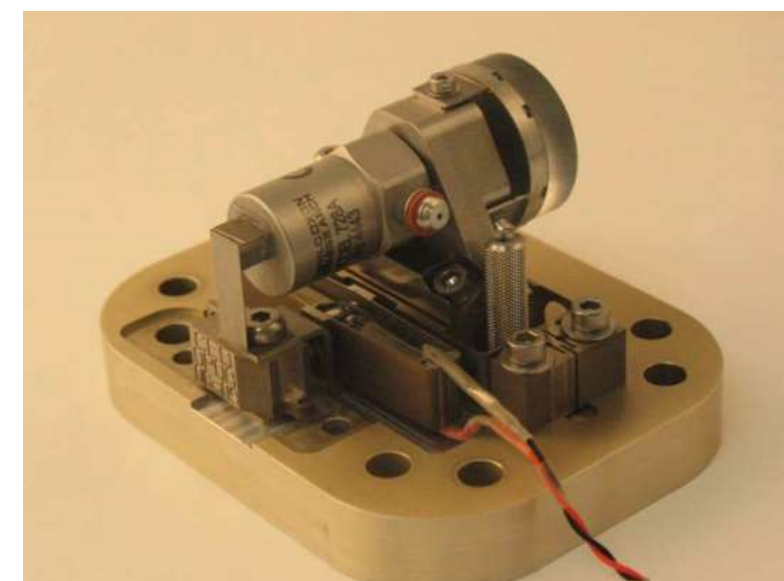
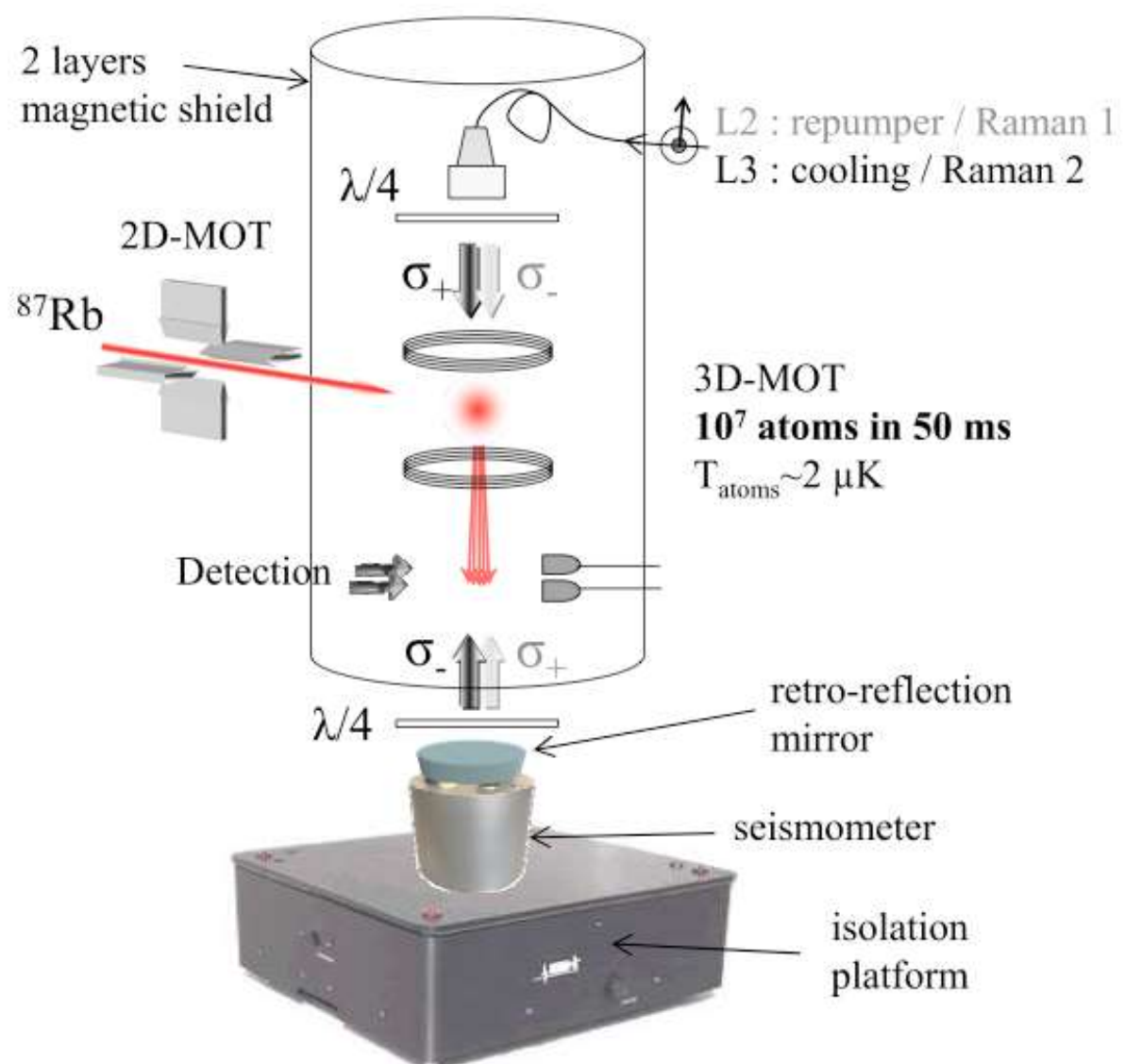
J. Le Gouët et al, Appl. Phys. B **92**, 133 (2008)

- High rotation rates (10^3 times larger than on ground)
- Large temperature fluctuations (20 C)
- Requires robust and automated systems
- Time constraints (only 3 or 6 flights per year)

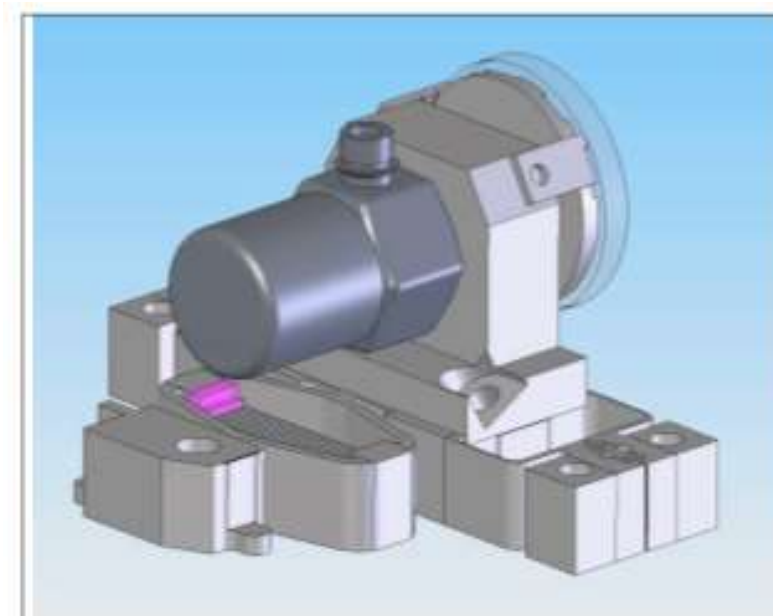




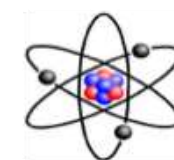
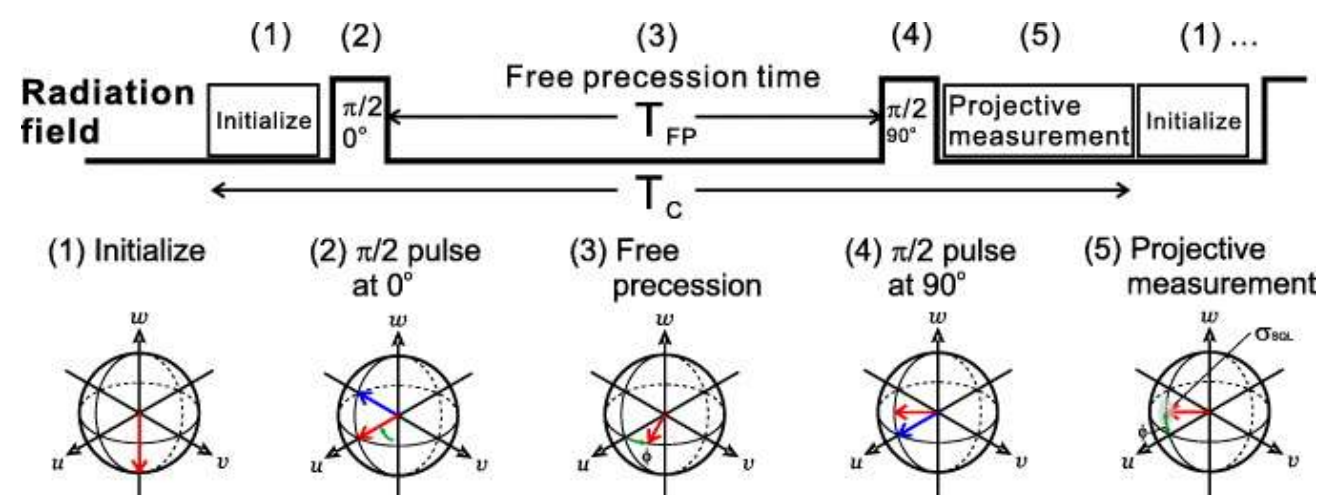
- One frequency range needs to be cancelled (directly averaged in interference signal)



Accelerometer wilcoxon 728 A



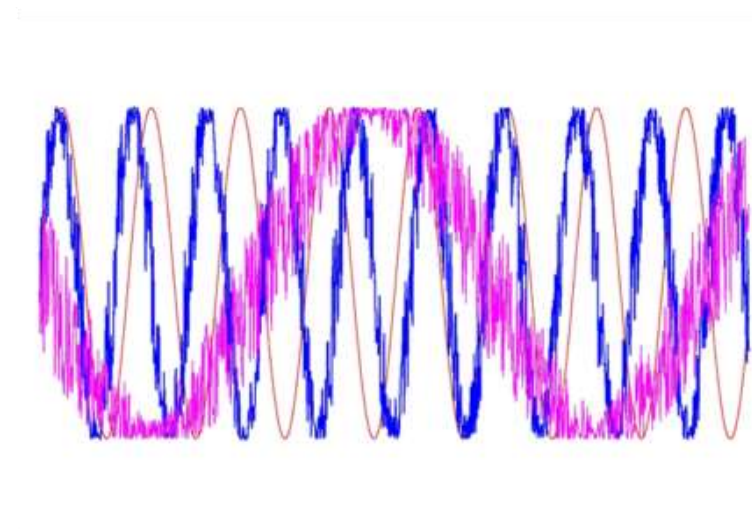
CLOCKS AND SENSORS ACT AS “INTEGRATORS” FOR THE NOISE IN A CERTAIN BAND.



ϕ_{at}



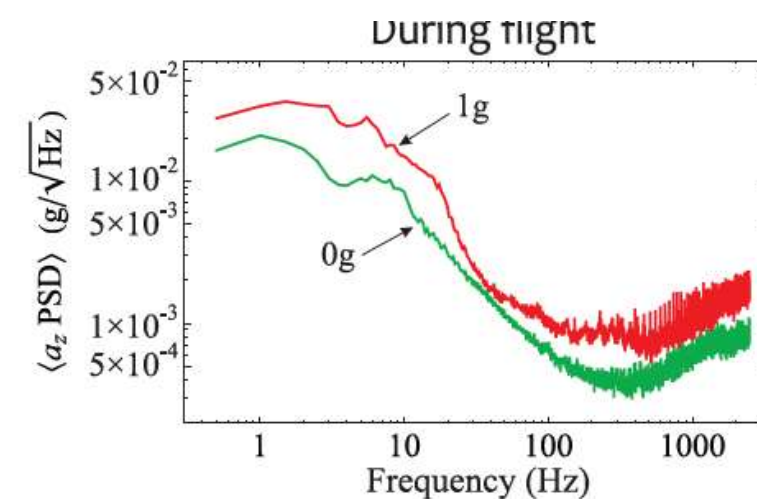
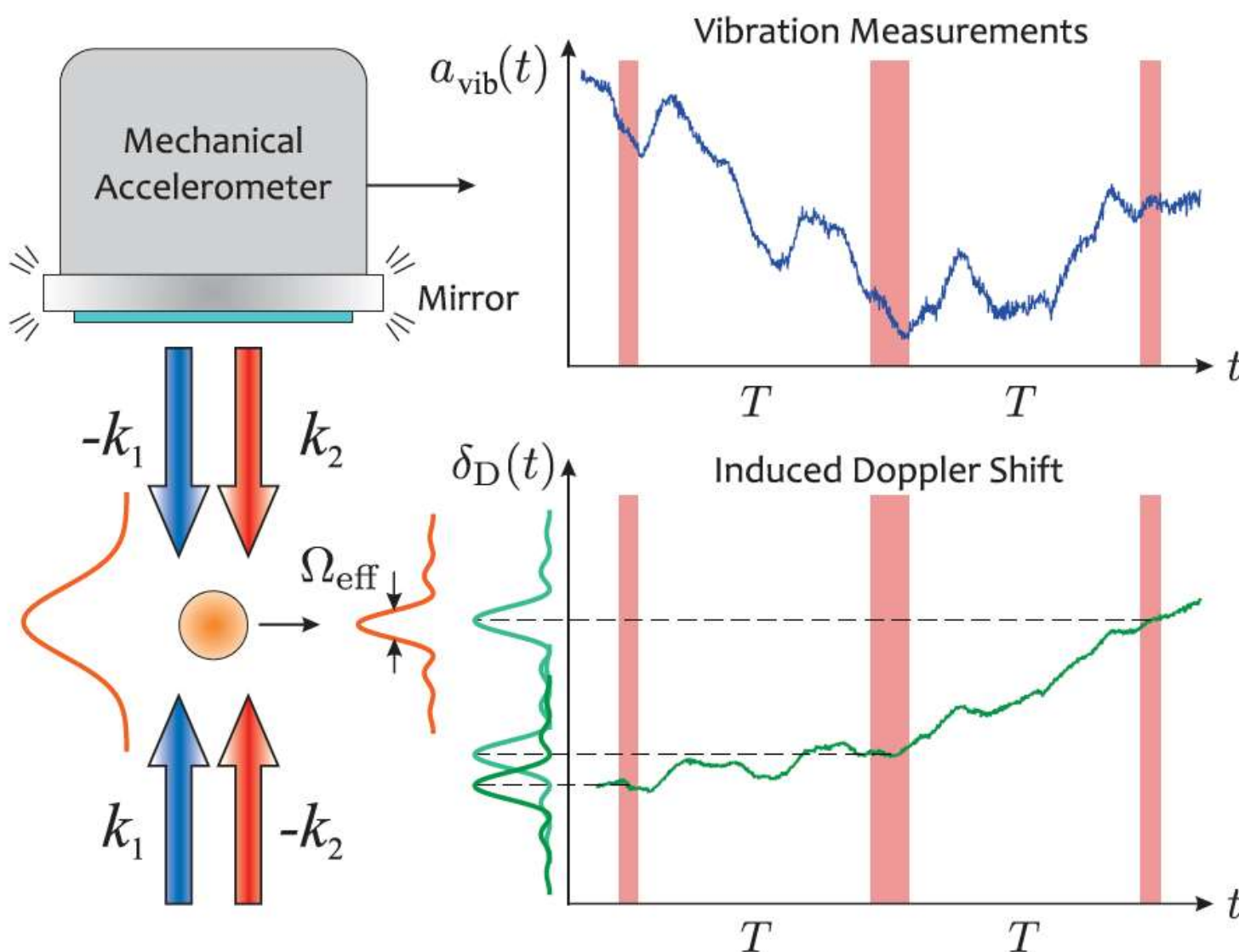
ϕ_{LO}



- ⇒ In clocks, we basically “track” the difference between the LO and the atomic natural frequency
- ⇒ Drift and/or noise should not lead to ambiguity ($\Delta\phi > \pi/2$).

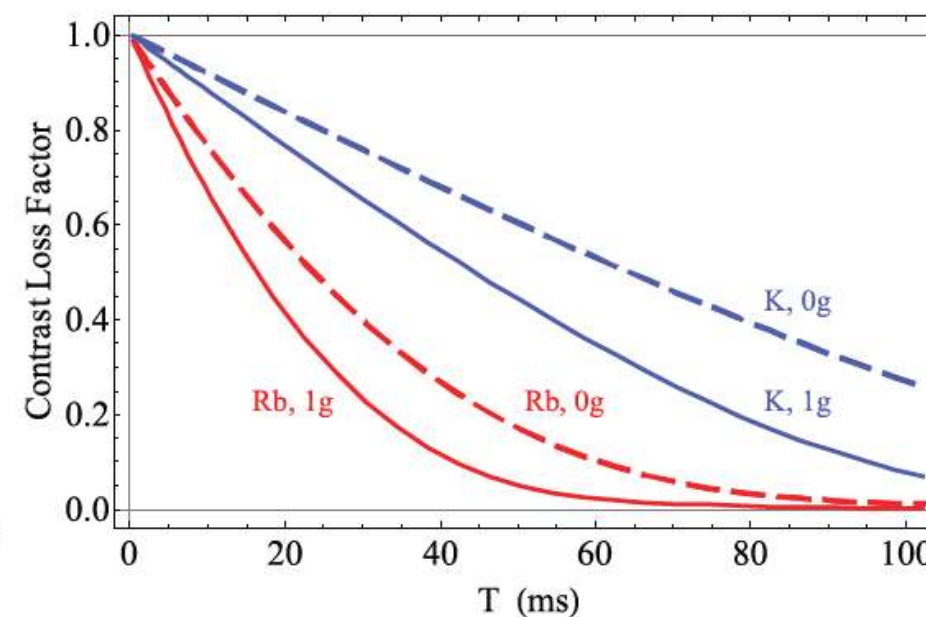
- Vibration level 10^5 times at a quiet ground location
- Doppler shift follows random walk between Raman pulses

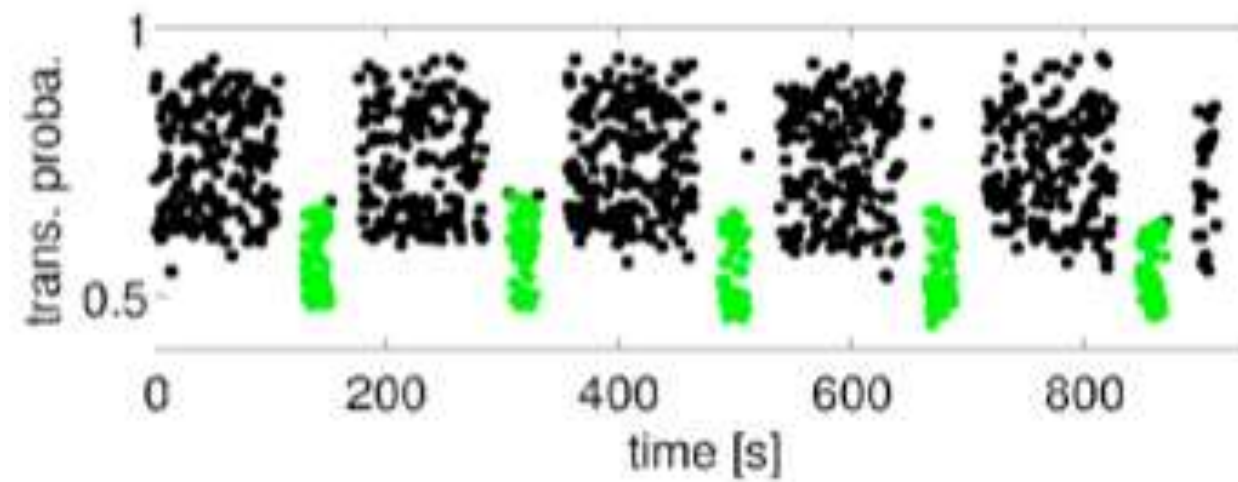
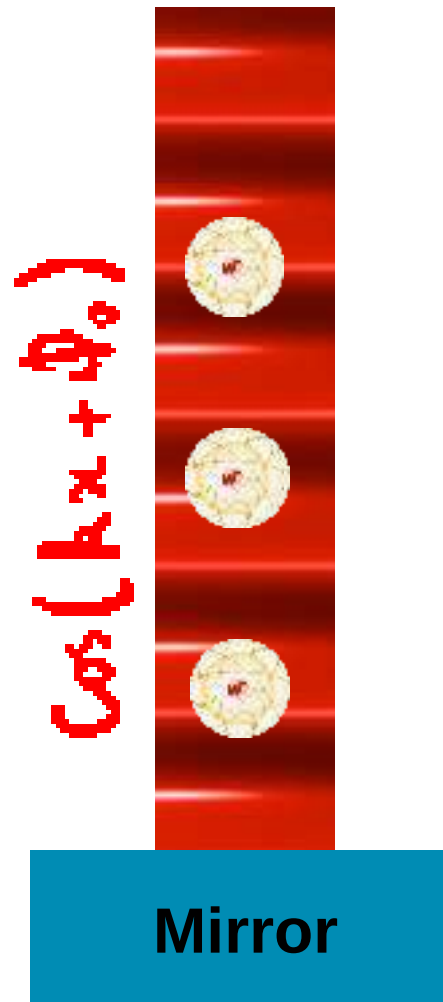
$$\delta_D(t) = k_{\text{eff}} \cdot \int a_{\text{vib}}(t) dt$$

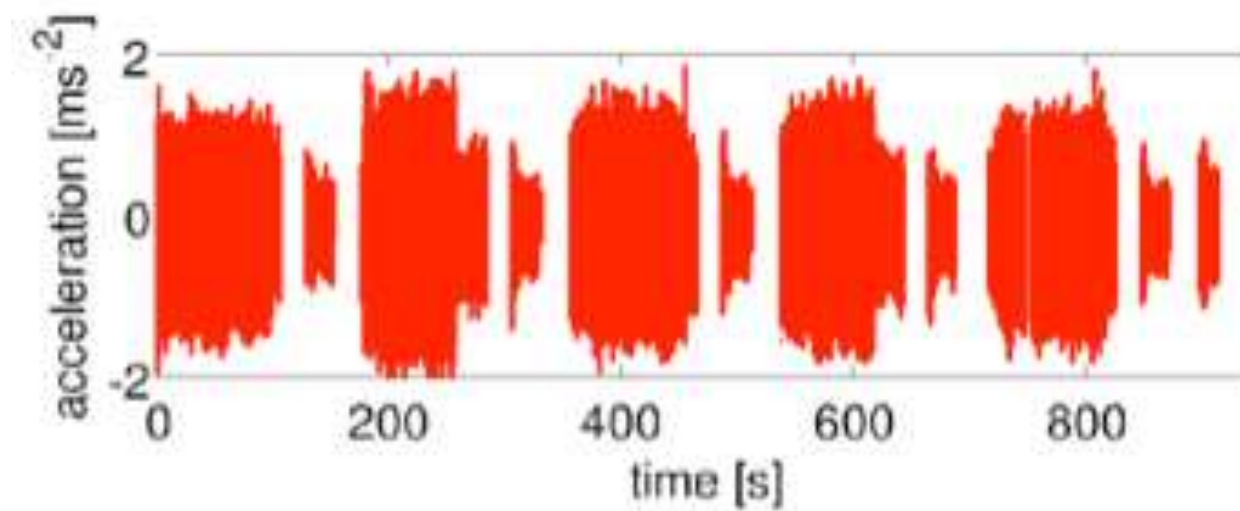
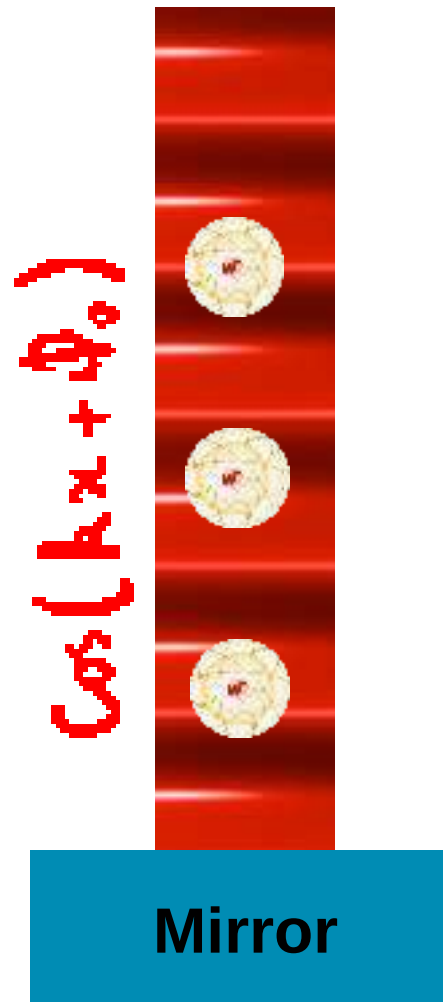


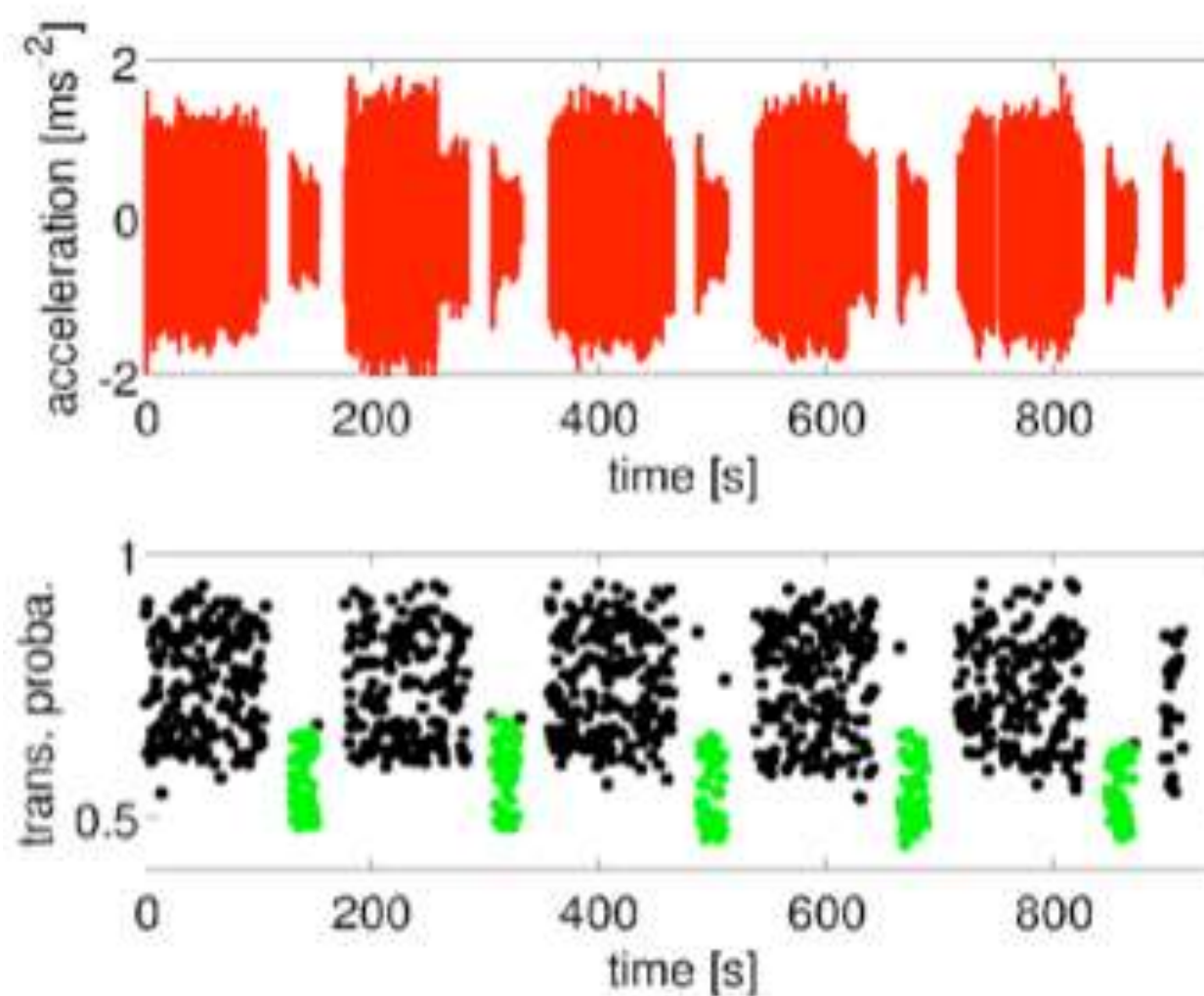
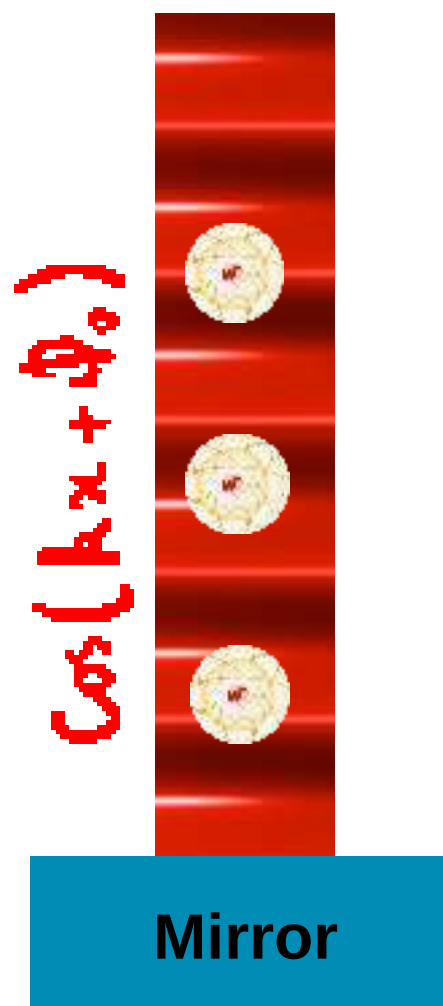
$$|\delta_D(t_p) - \delta_D(t_p - T)| \ll \Omega_{\text{eff}}$$

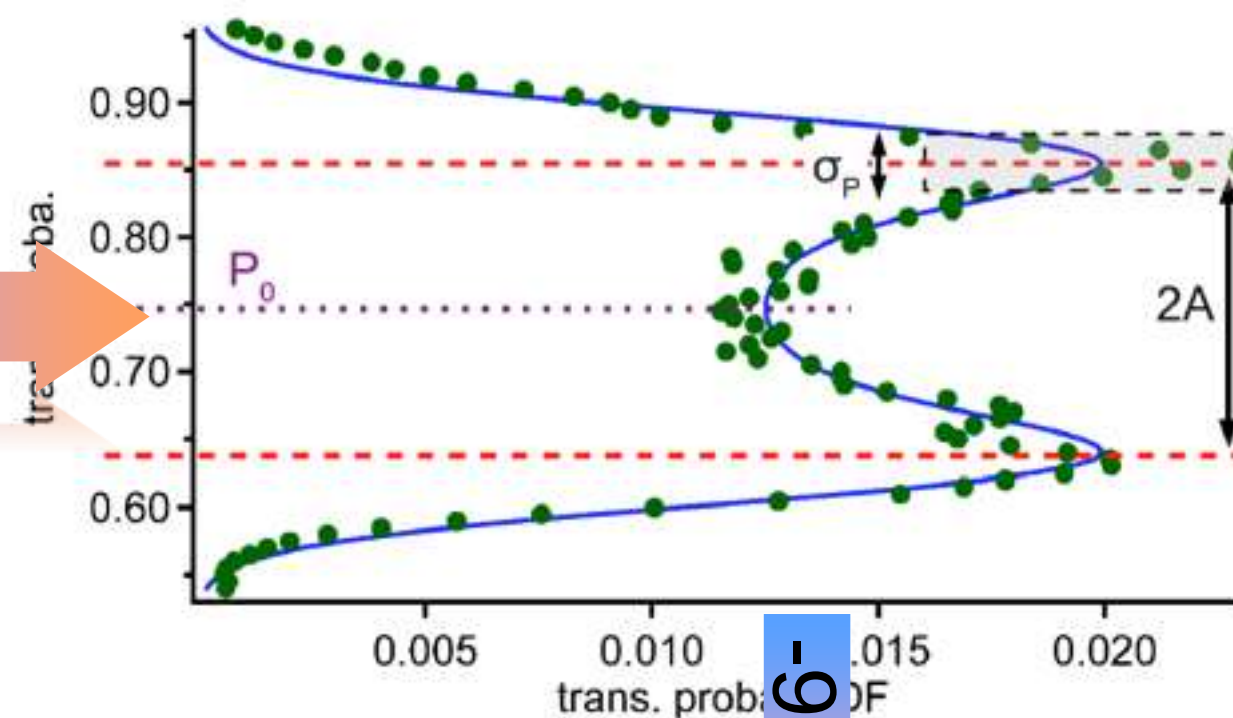
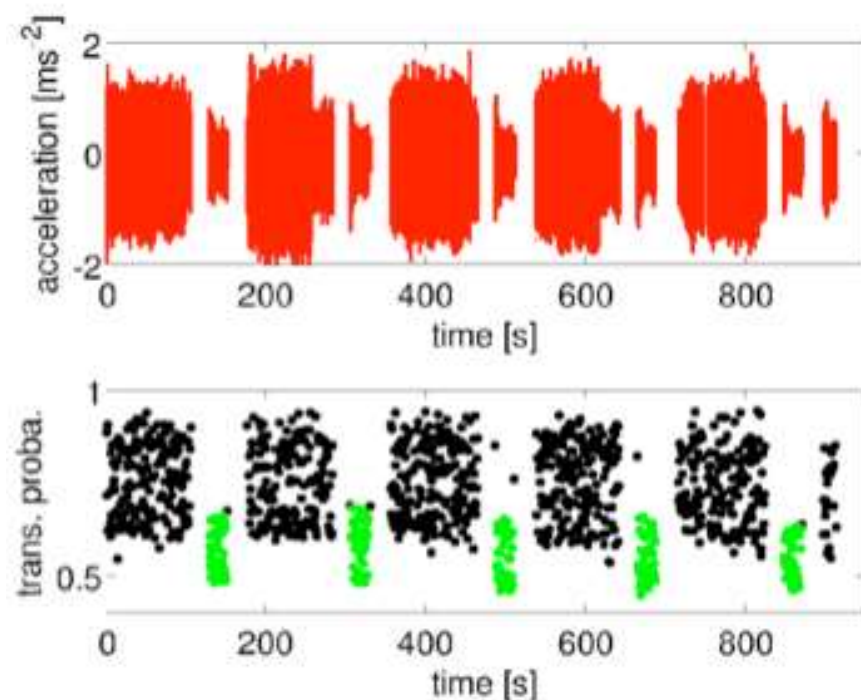
$$\Rightarrow \sigma_{\text{vib}} \ll \Omega_{\text{eff}} / k_{\text{eff}} T$$











06-

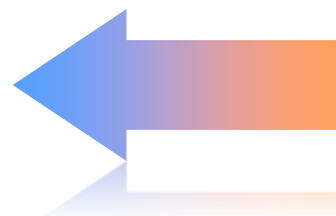
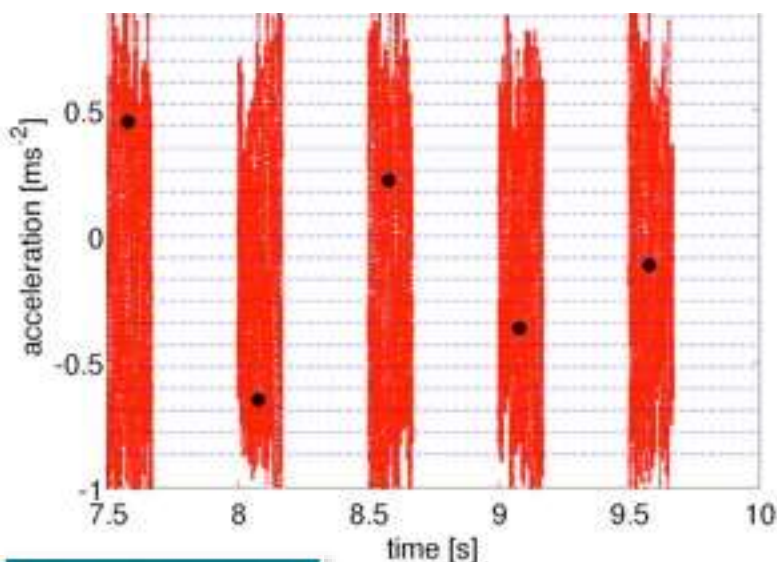
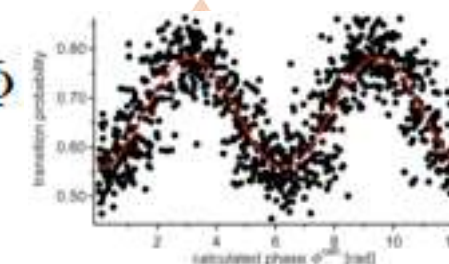


$$\Phi = a_m \times kT^2$$

$$P^{\text{AI}}(\Phi) = P_0 - A \cos \Phi$$

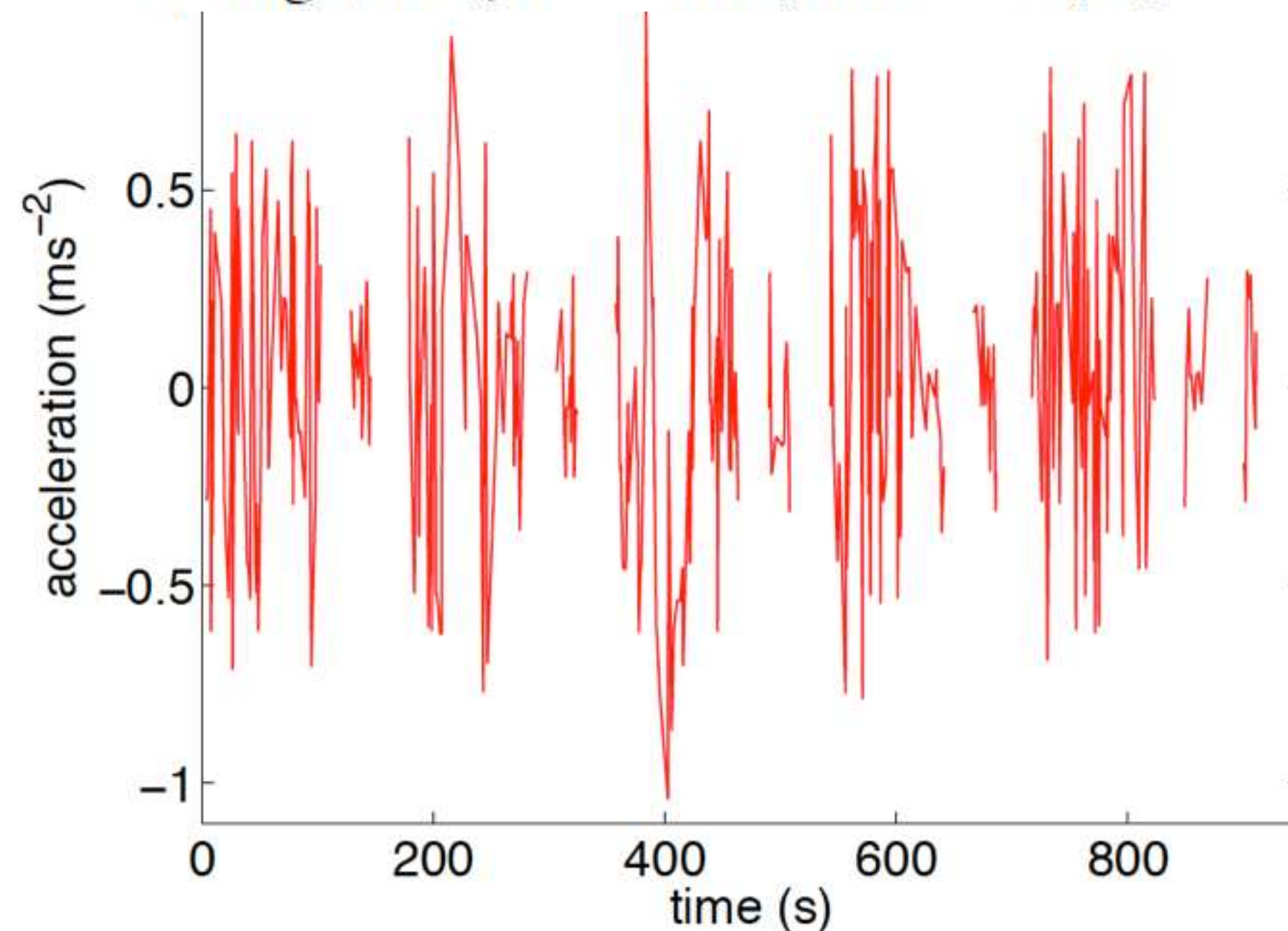
$$\text{SNR} = A/\sigma_P$$

$$P_0 = 0.747, 2A = 0.261 \text{ and } \text{SNR} = 4.7$$



Geiger et al., I.C.E.
project, Nature
Communication

$0.35 \text{ mg}/\sqrt{\text{Hz}}$ ($2T = 3 \text{ ms}$, $\text{SNR} = 4.7$, $T_c = 350 \text{ ms}$)

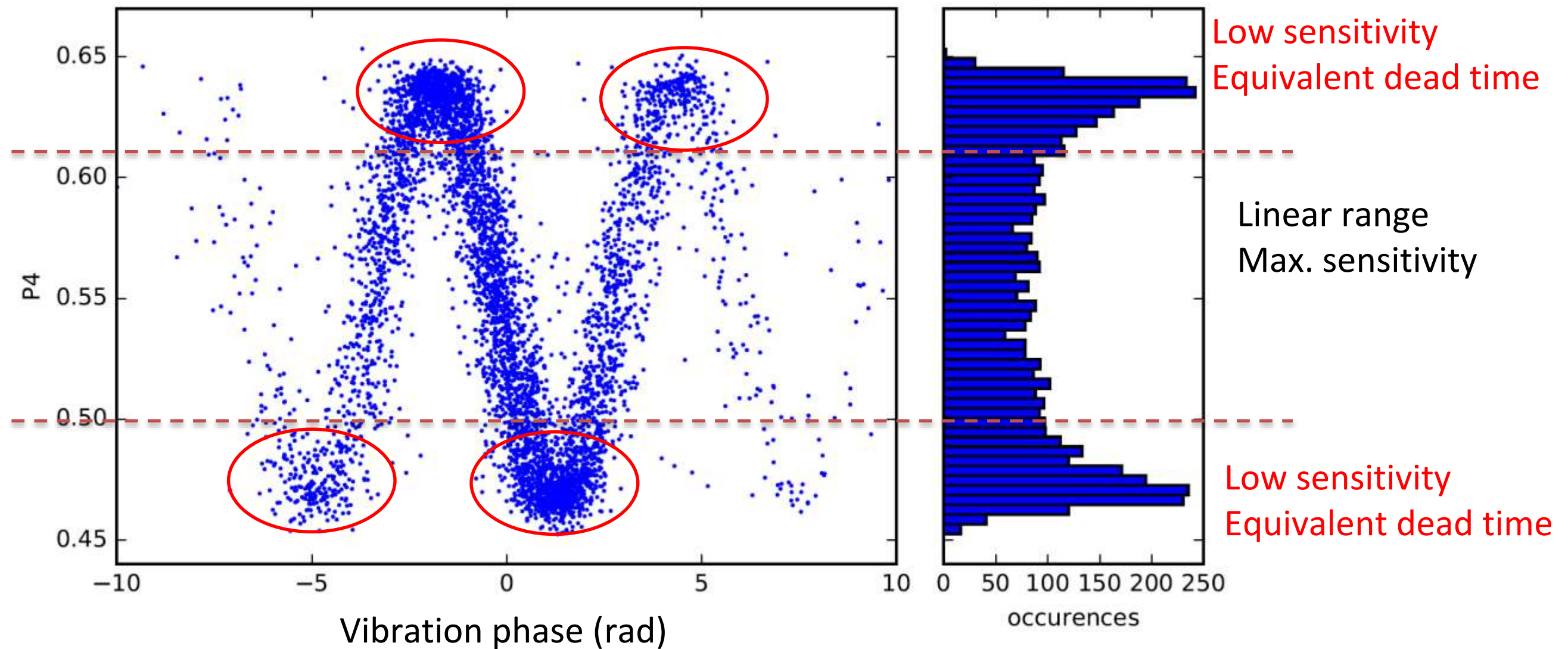


0-g time

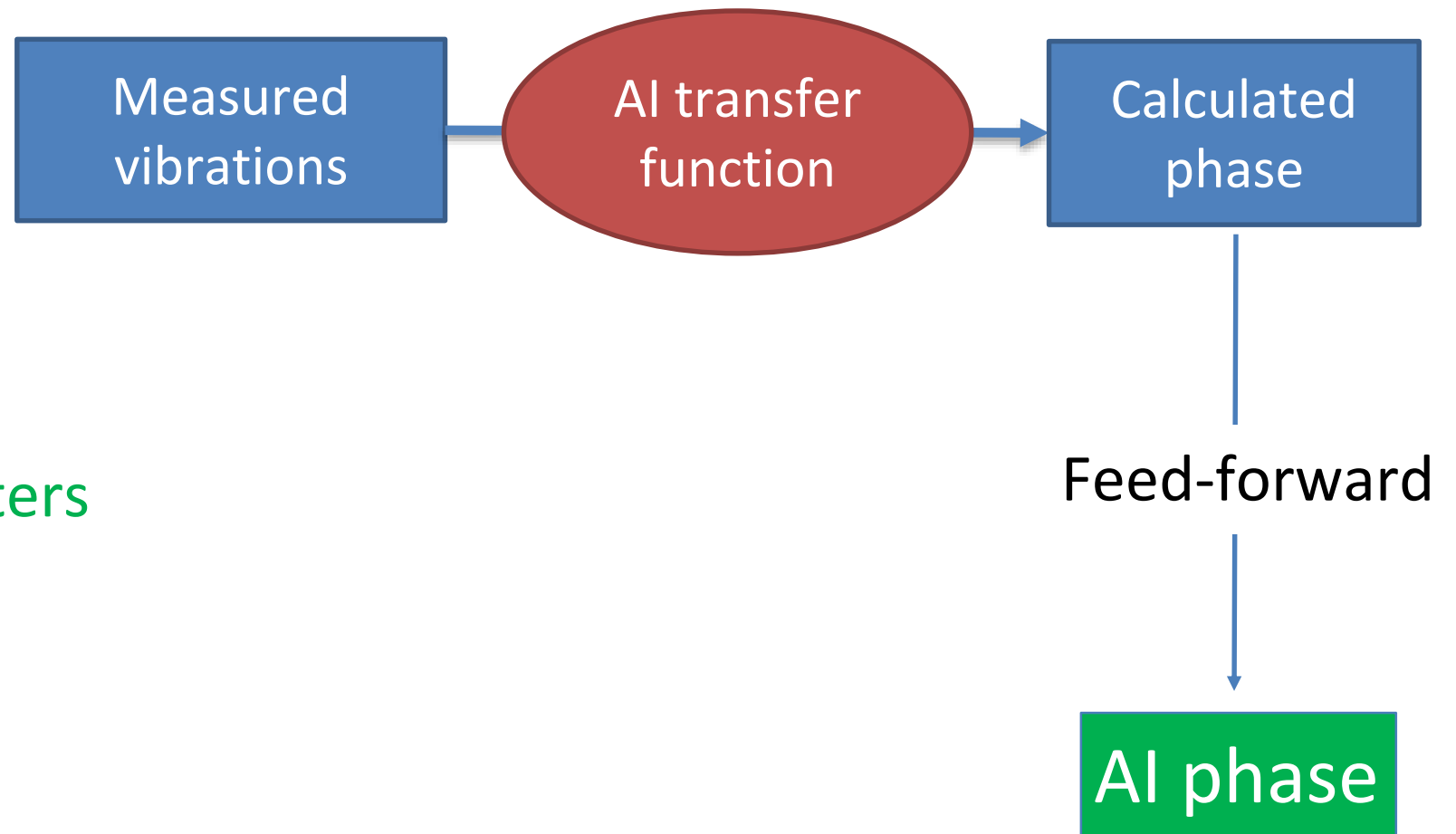
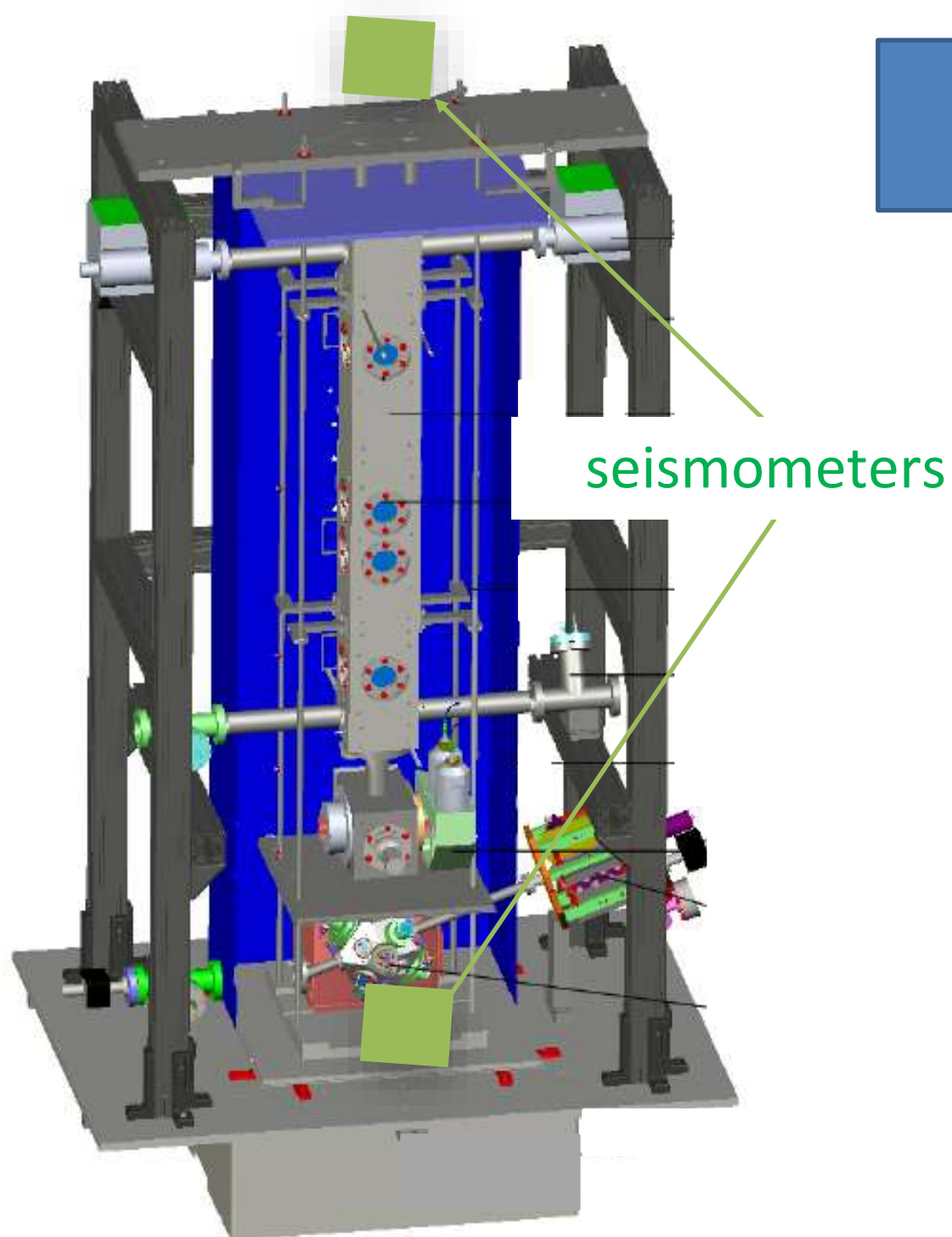
- Sensitivity (max interrogation time) limited by the linearity of the standard accelerometer (Sensorex SX46020 and IMI 626A03)

**Geiger et al., Nature
Comm. DOI:
10.1038/ncomms1479**

Operation in the linear regime

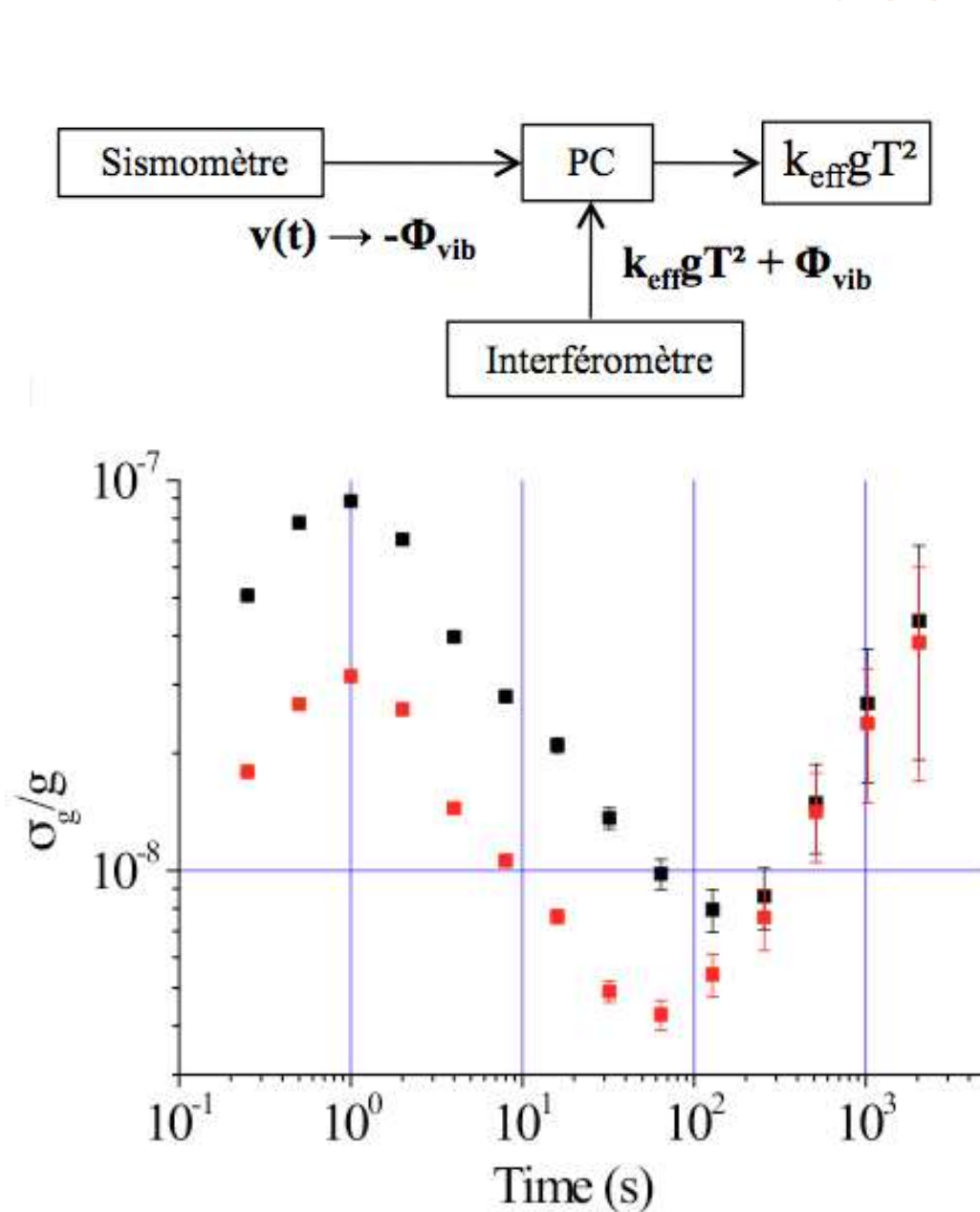


Vibration noise rejection

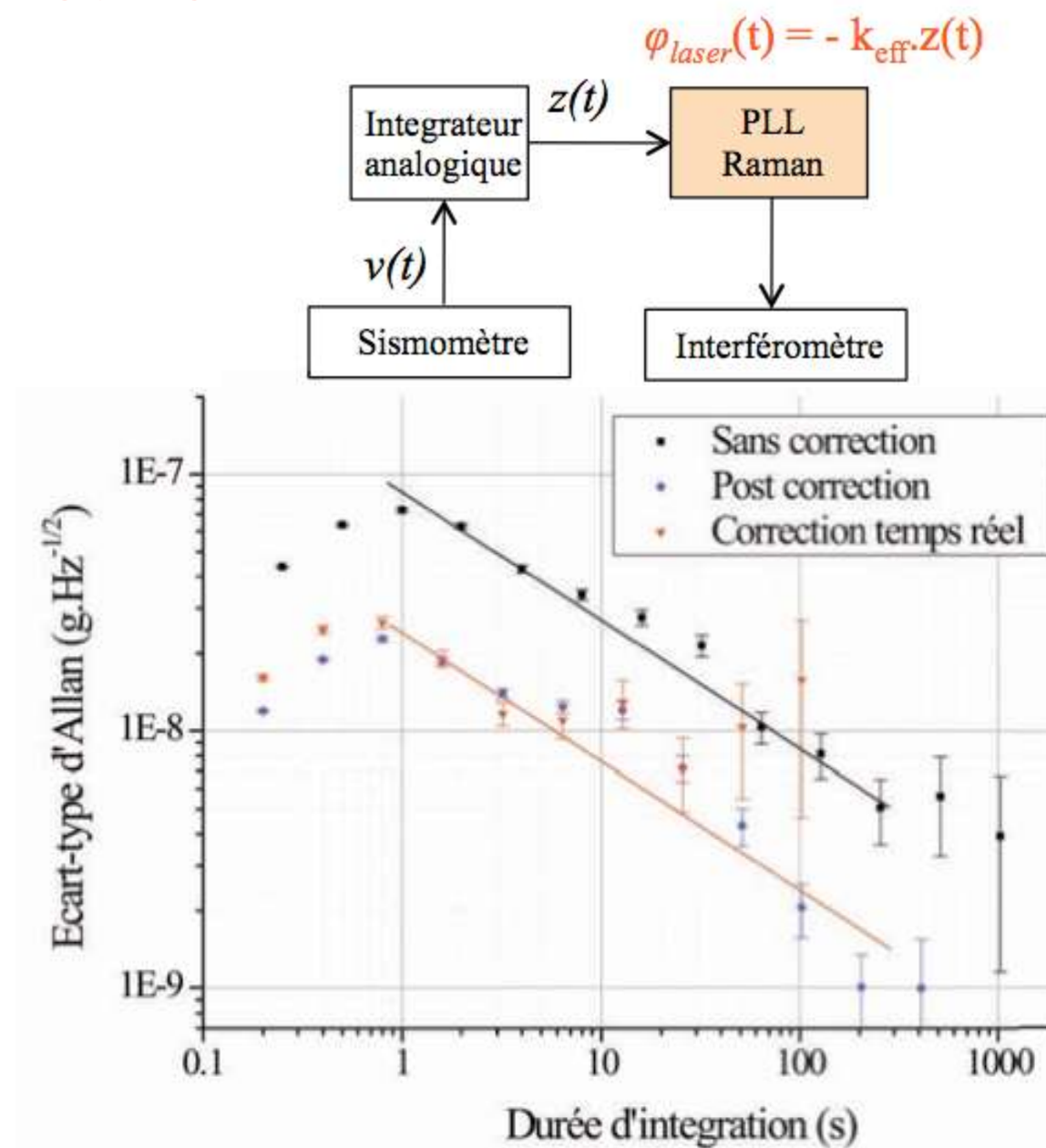


J. Lautier et al, App. Phys. Letters **105** 144102 (2014)

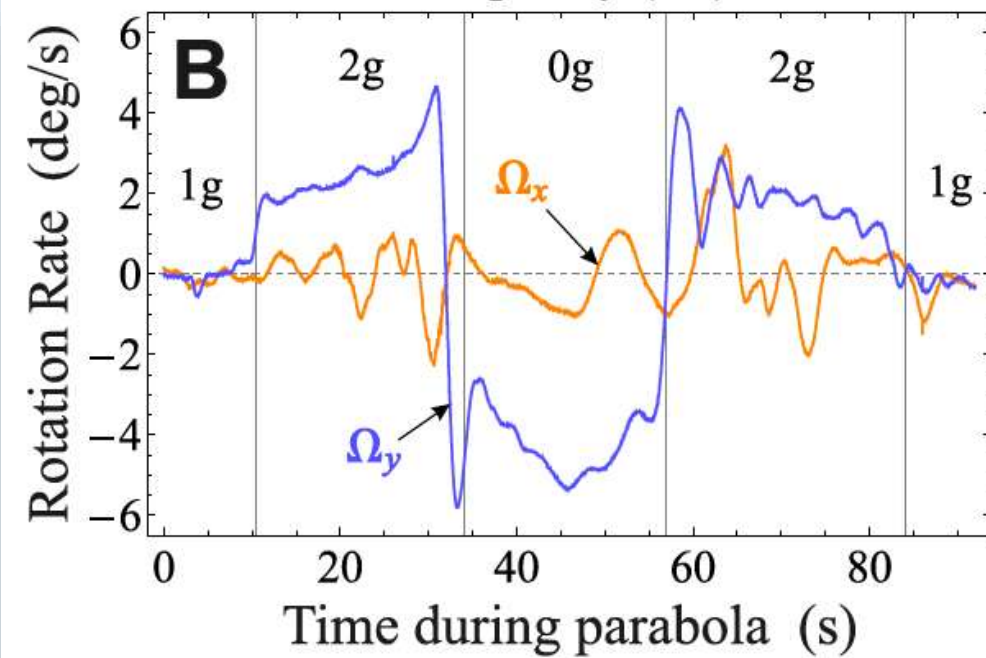
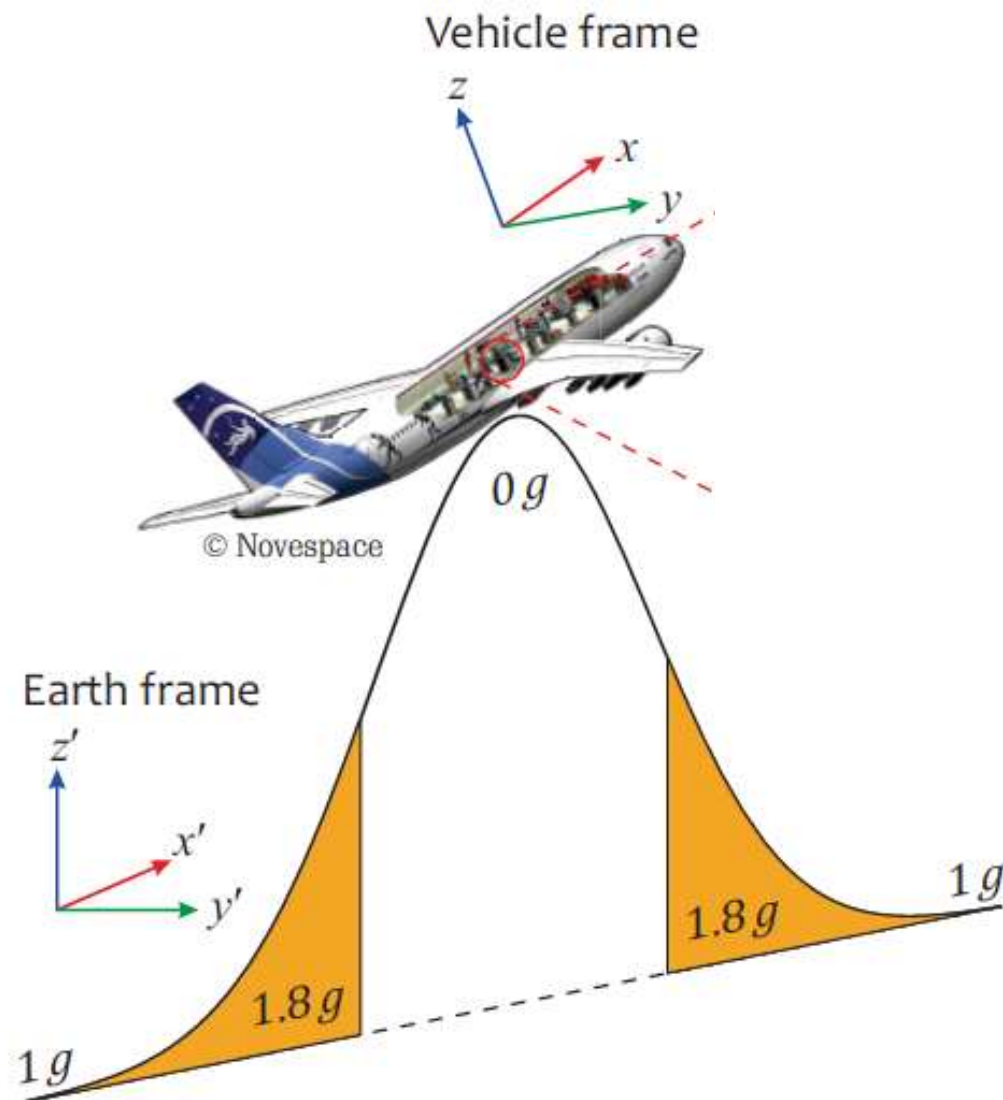
Passive/Active



Sensitivity improved from 80 to 30 ng/ $\sqrt{\text{Hz}}$



Sensitivity improved from 80 to 20 ng/ $\sqrt{\text{Hz}}$



Coriolis acceleration

- Additional phase shift
- Changes the interferometer

- Lagrangian for a body undergoing rotations and linear accelerations

$$L(\mathbf{r}, \mathbf{v}) = \frac{1}{2} M (\mathbf{v} + \boldsymbol{\Omega} \times \mathbf{r})^2 + M \mathbf{a} \cdot \mathbf{r}$$

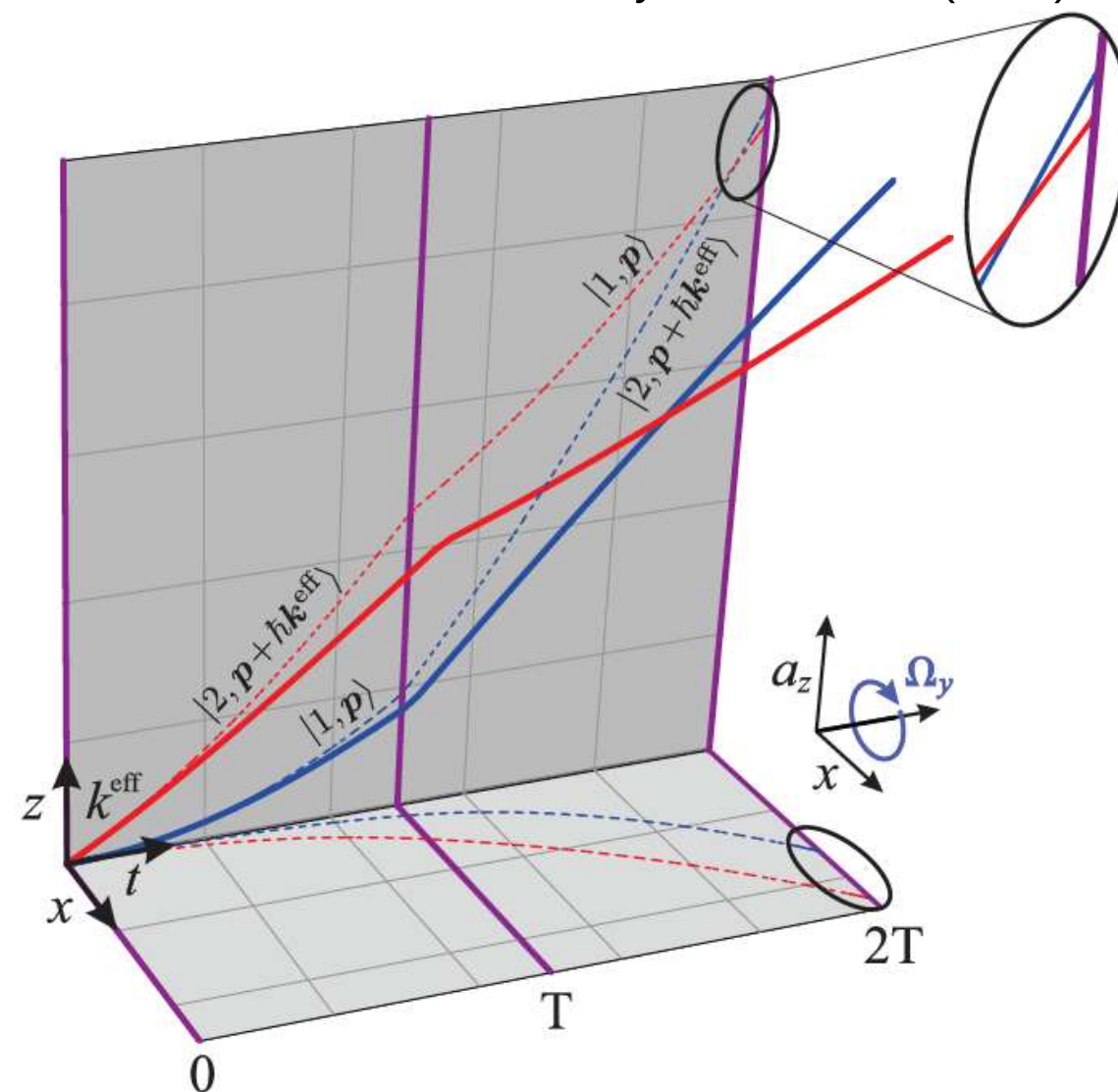
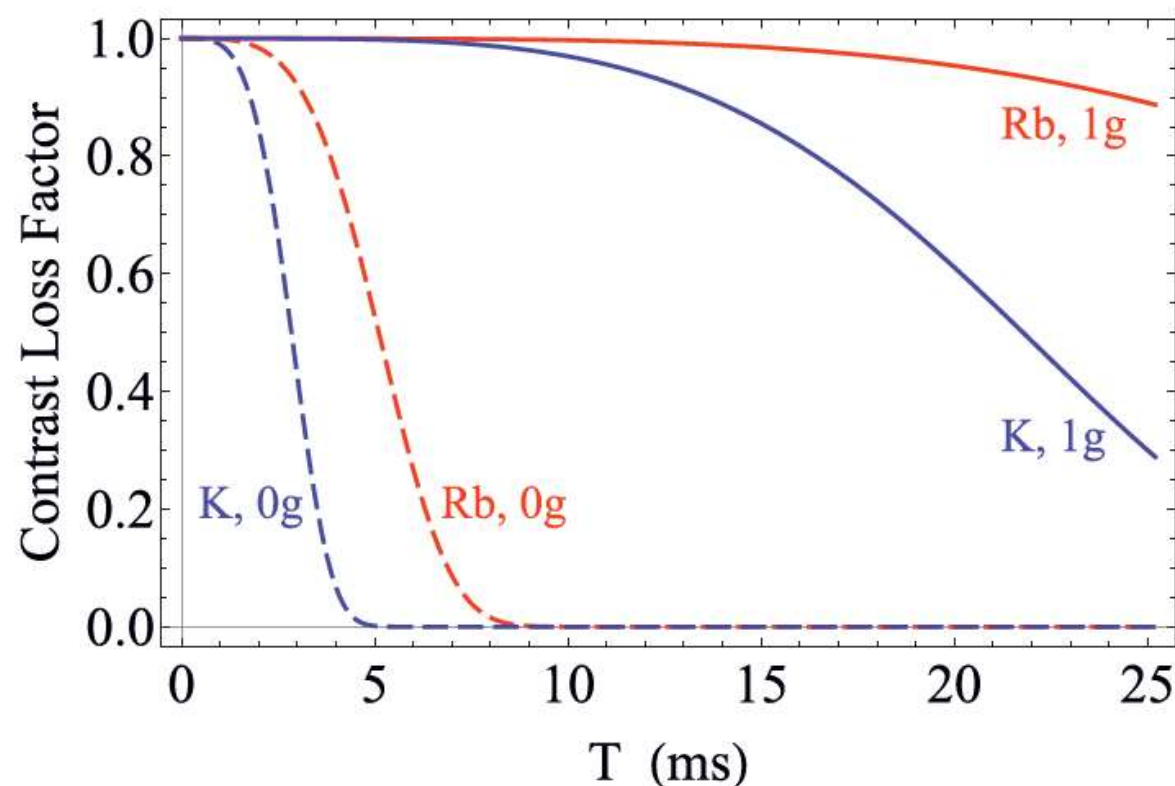
- Evaluate interferometer trajectories and insert into Roura model: New J. Phys. 16, 123012 (2014)

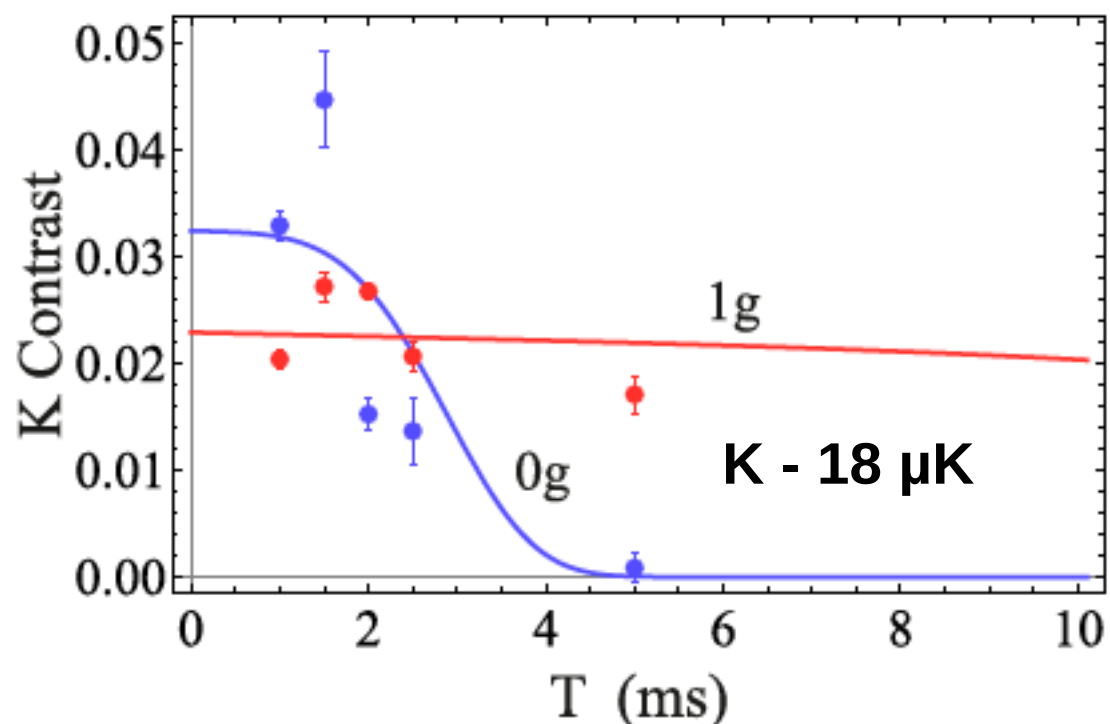
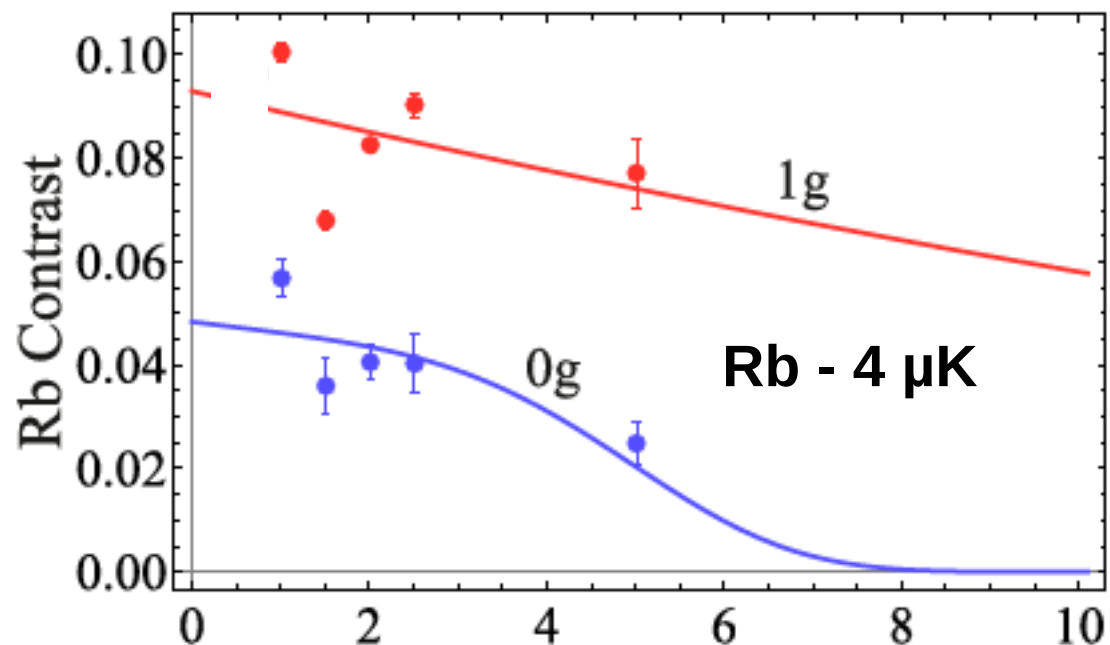
$$C \sim \exp[-(k_{\text{eff}} \sigma_v T)^2 (|\boldsymbol{\Omega}_T| T)^2]$$

Velocity spread

Interrogation time

Rotation rate
transverse to $\mathbf{k}_{\text{eff}}$





Rotation in the plane $\Omega_y \approx 5 \text{ deg/s}$

- **100 % contrast loss in $\approx 10 \text{ ms}$**
 - Limiting for navigation systems
 - Limiting for space mission in LEO
- **For 5 second interrogation time, rotation needs to be $\approx 6 \cdot 10^{-5} \text{ deg/s}$ (50 times than earth rotation)**
 - Rotation compensation of retroreflecting mirror

S. M. Dickerson, et al., Phys. Rev. Lett. 111, 083001 (2013).

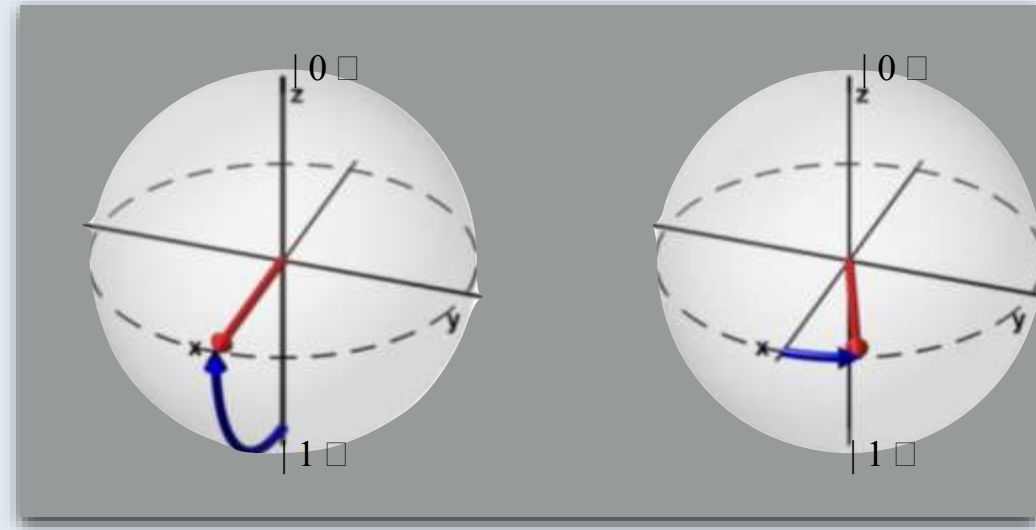
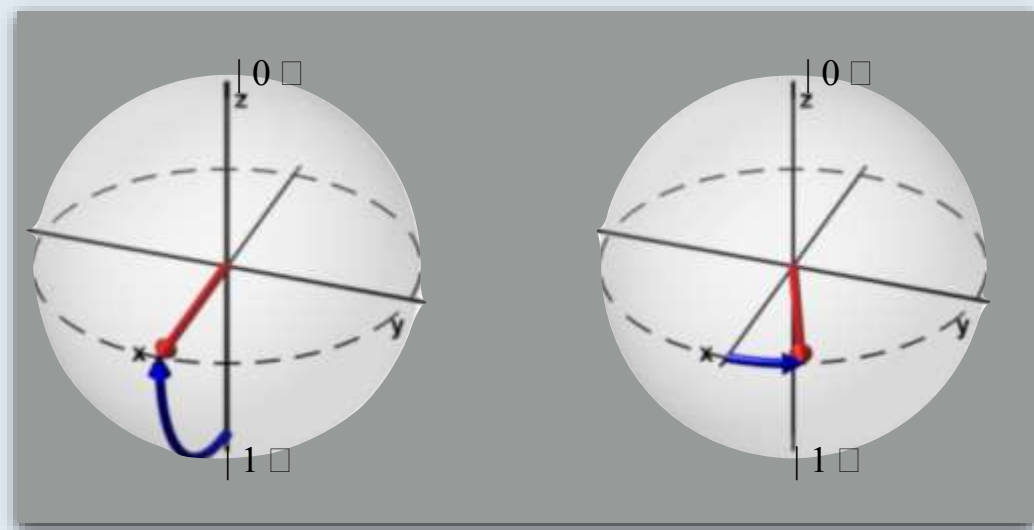
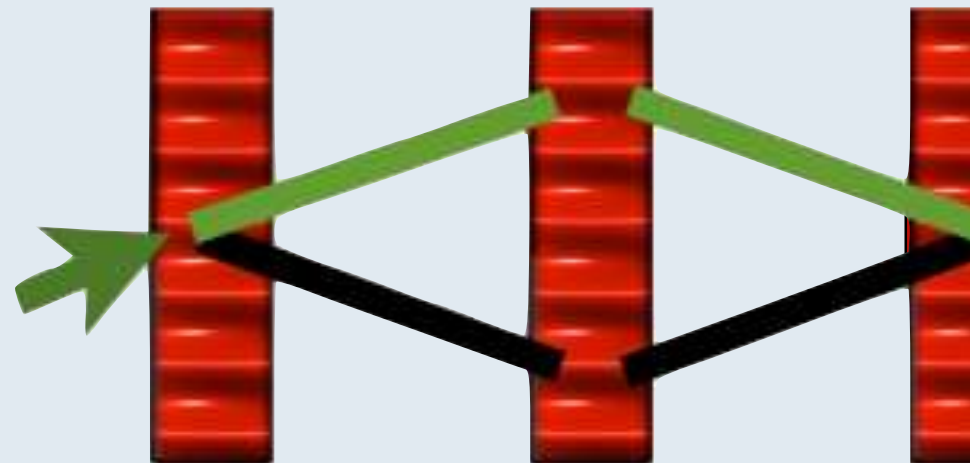
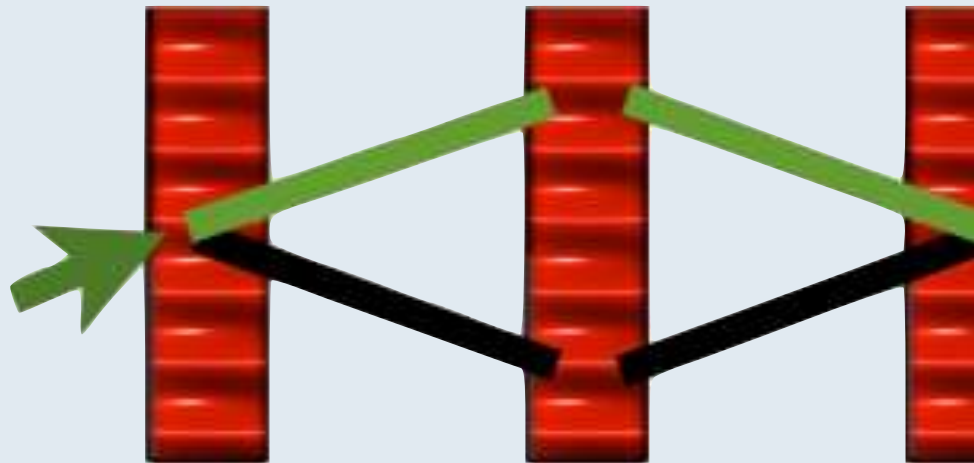
For navigation

- **Large rotation rates**
- **Large acceleration ($\pm 2 \text{ g}$)**
 - Limits T to 10 ms
 - Find locking methods to enhance measurement (T^2)

PROBLEM OF THE REPETITION RATE

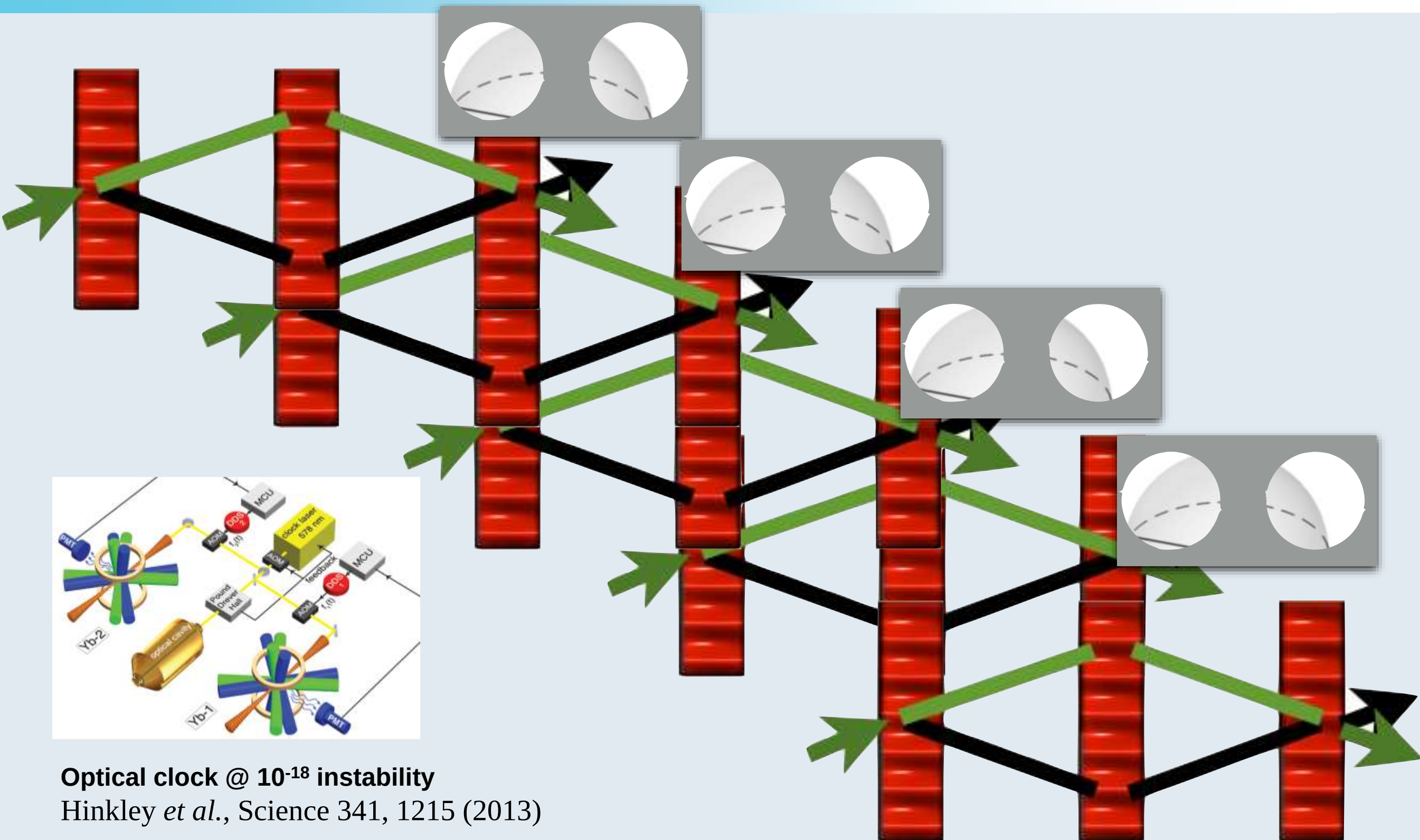


INTERLEAVING TWO CLOCKS MEASUREMENTS



repeat

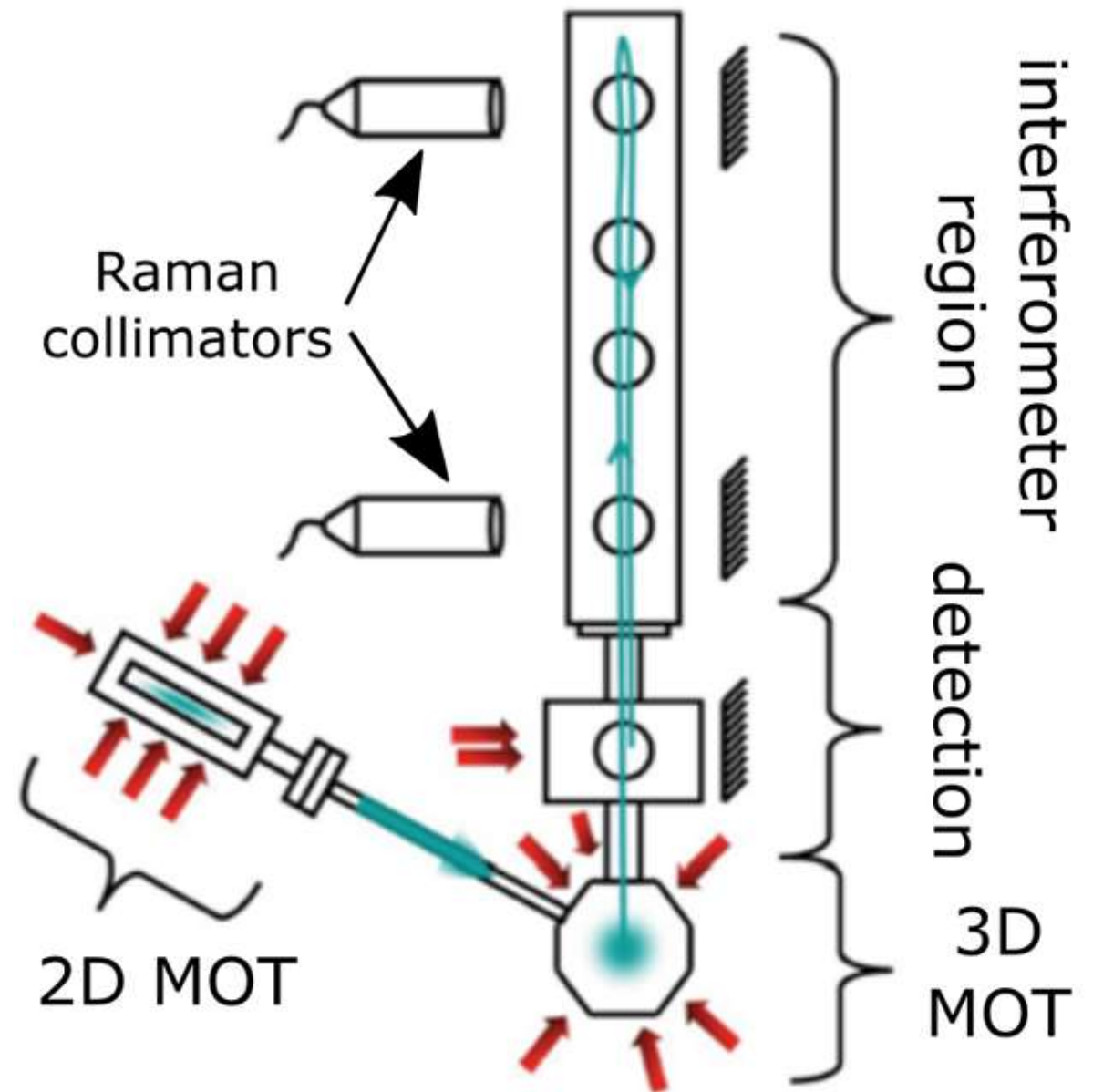
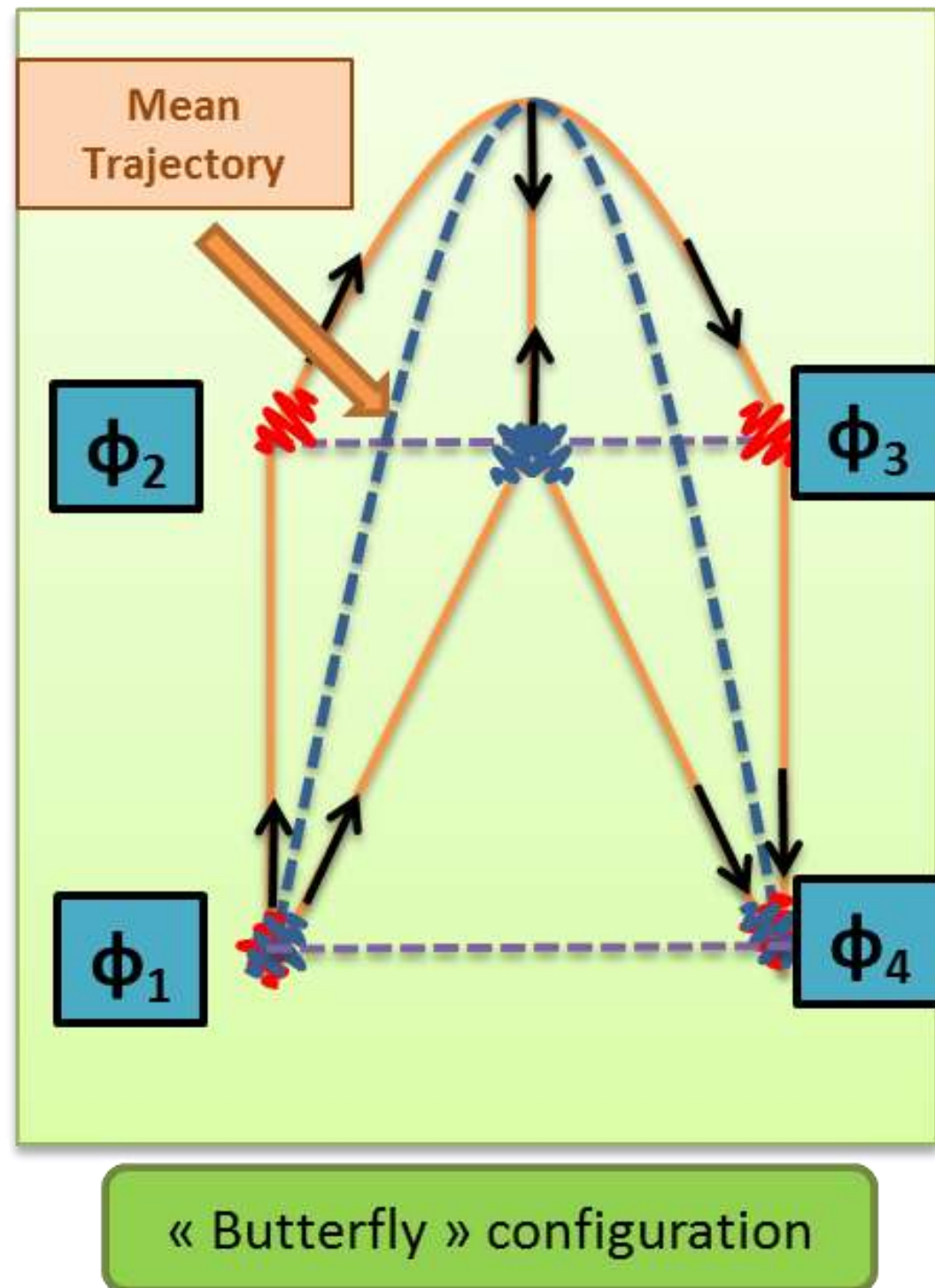
PROBLEM OF THE REPETITION RATE

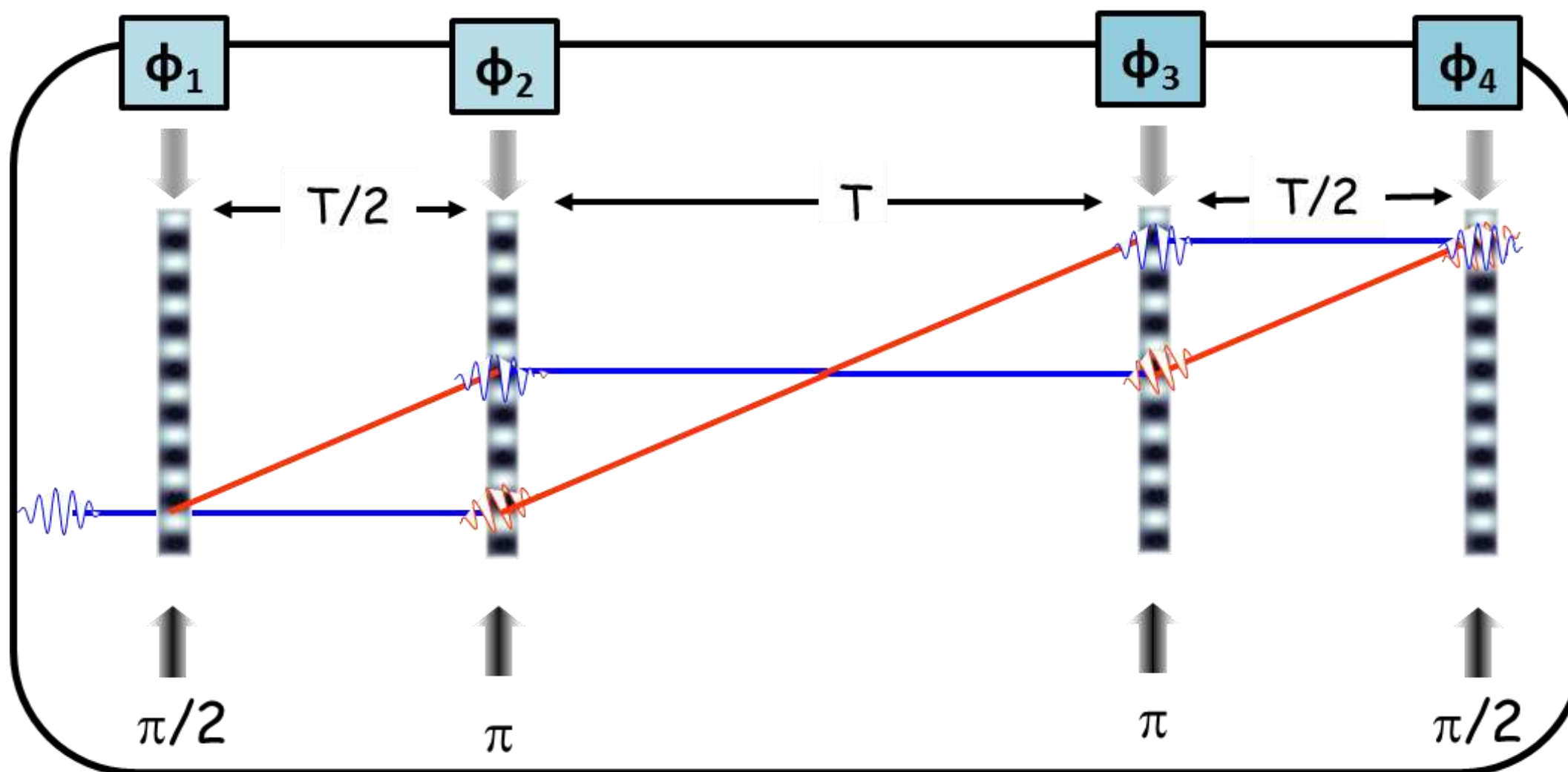


Optical clock @ 10^{-18} instability

Hinkley *et al.*, Science 341, 1215 (2013)

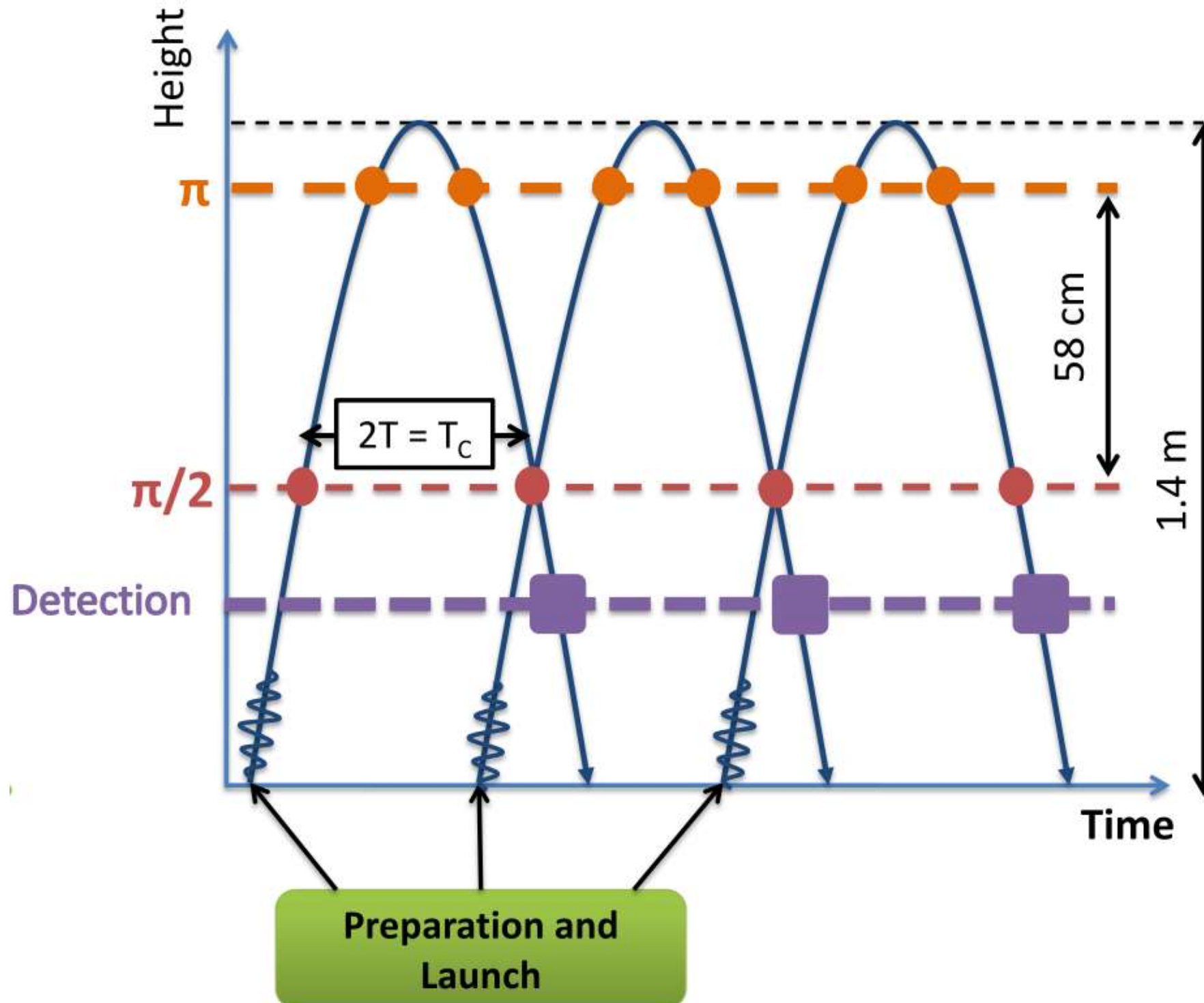
4-light pulse gyroscope



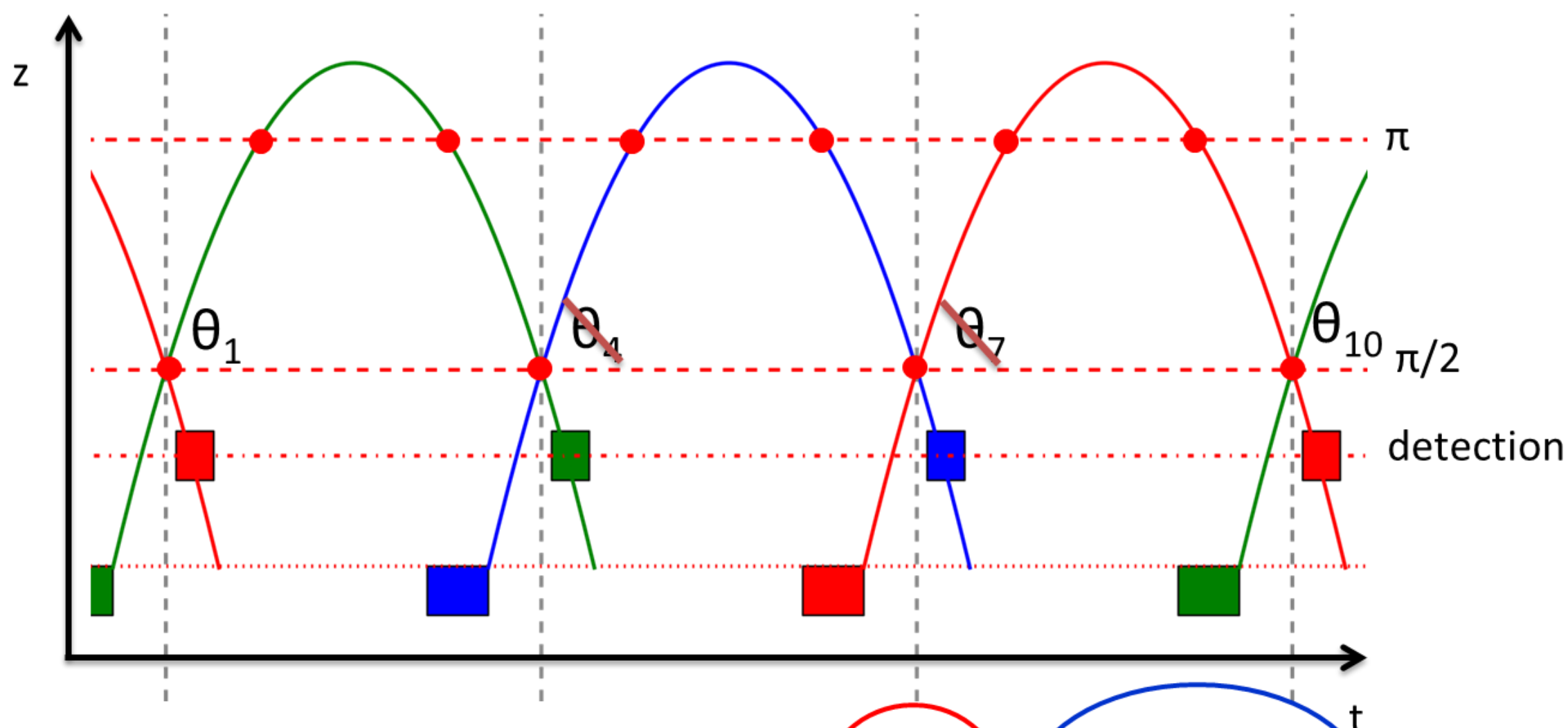


$$\Delta\phi = \phi_1 - 2\phi_2 + 2\phi_3 - \phi_4$$

Prepare the cold atoms and operate the AI in parallel



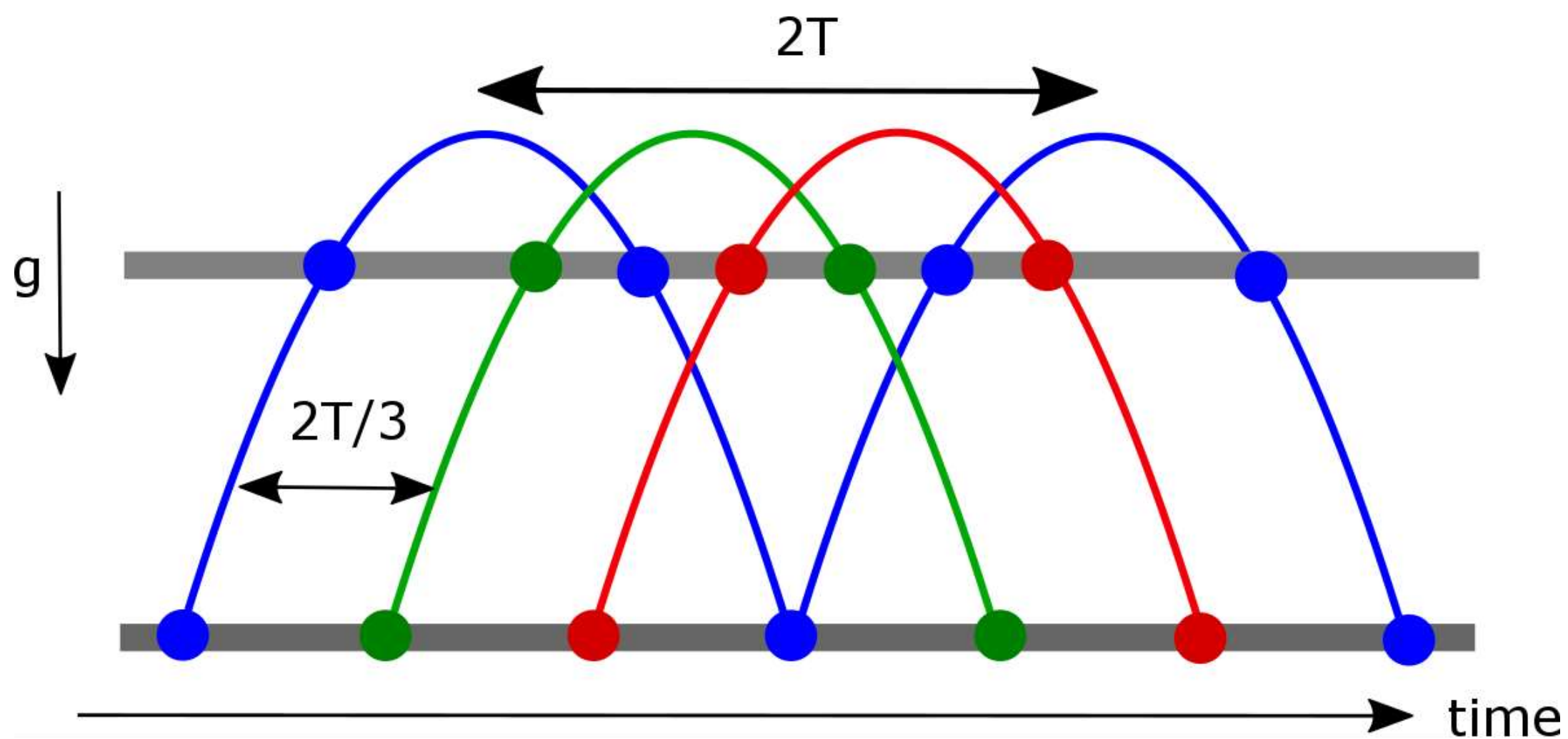
1. Rotation noise is canceled from shot to shot by the joint measurements



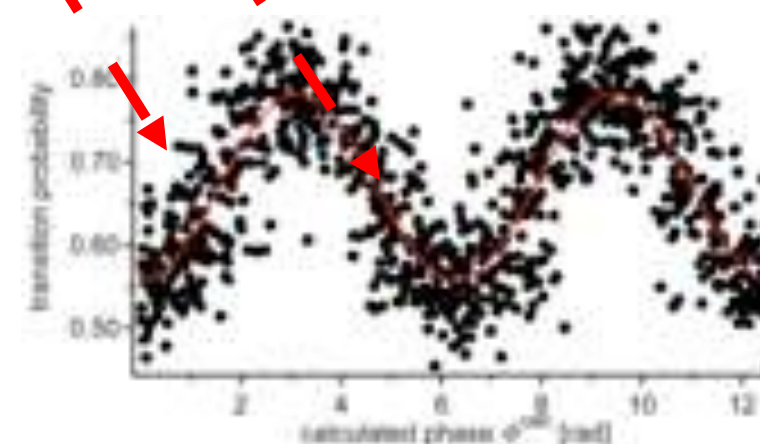
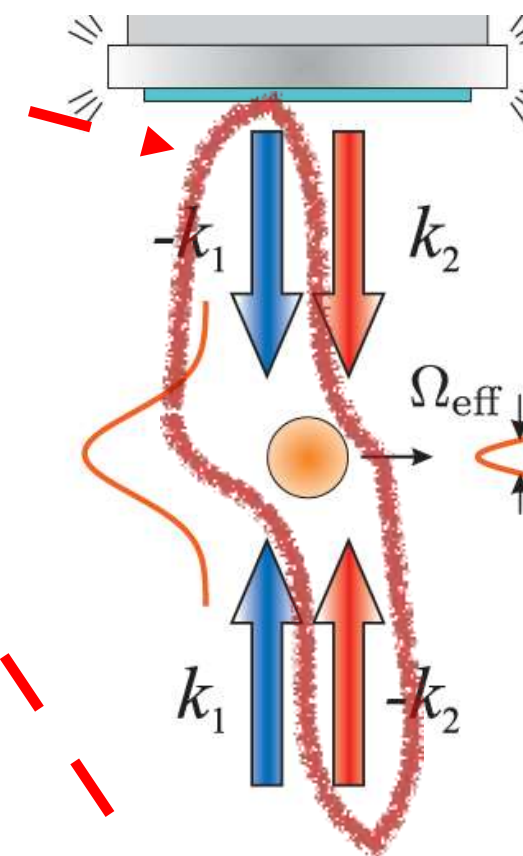
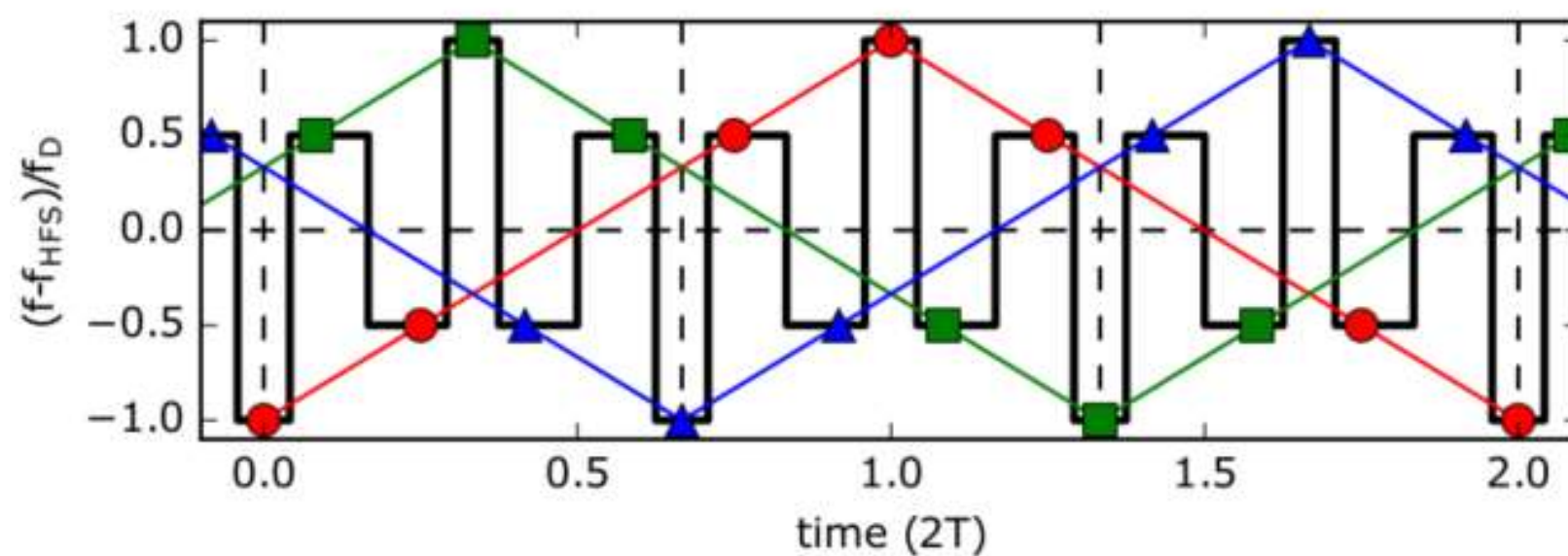
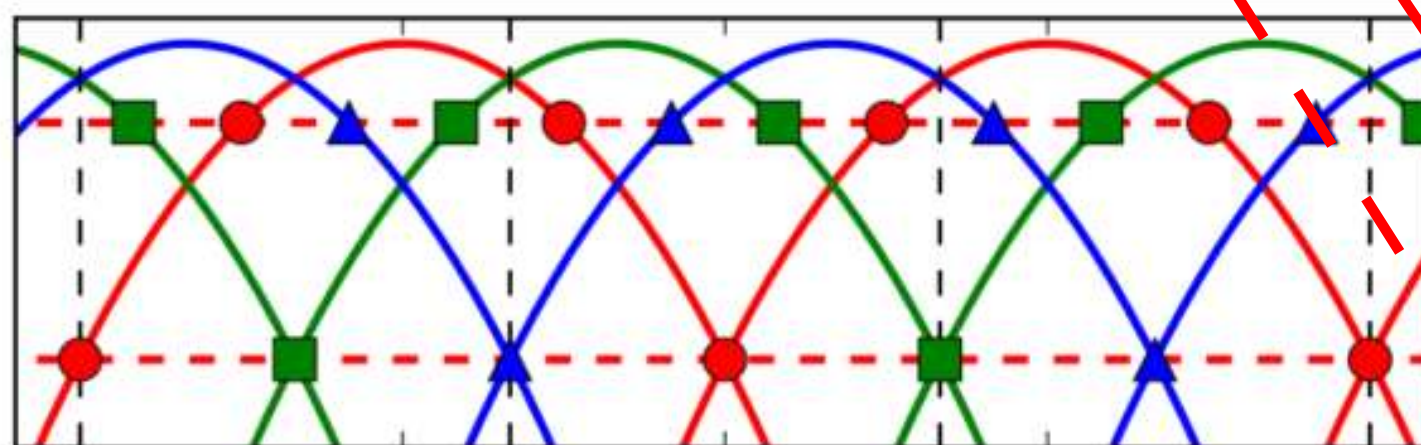
$$\Omega = \frac{1}{2NT} \sum_{i=1}^N (\theta_{3i+3} - \theta_{3i} + \delta\theta_i^{NC}) = \frac{\theta_{3N} - \theta_1}{2NT} + \frac{1}{2NT} \sum_{i=1}^N \delta\theta_i^{NC}$$

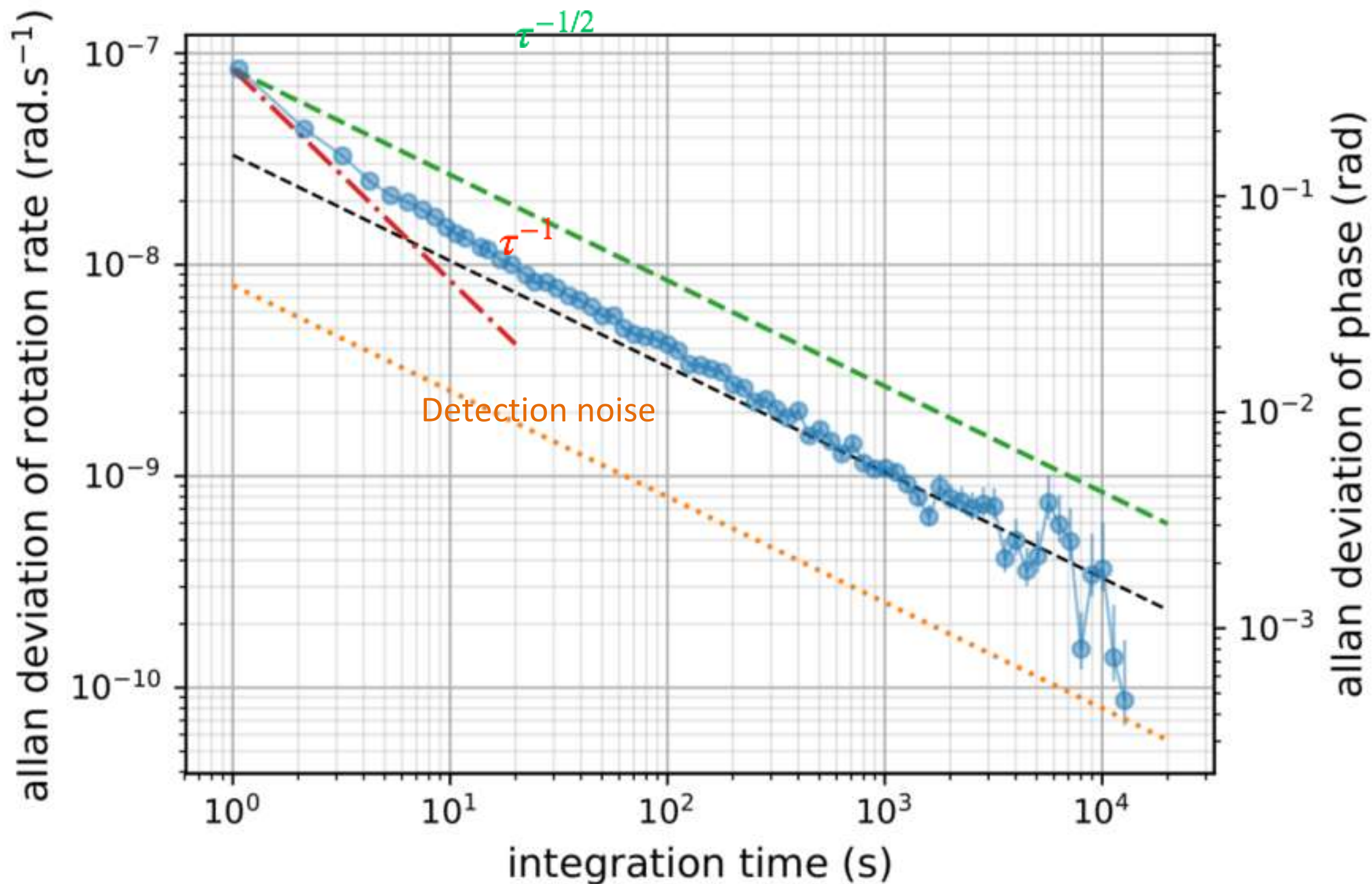
→ Integration as $\frac{1}{N}$

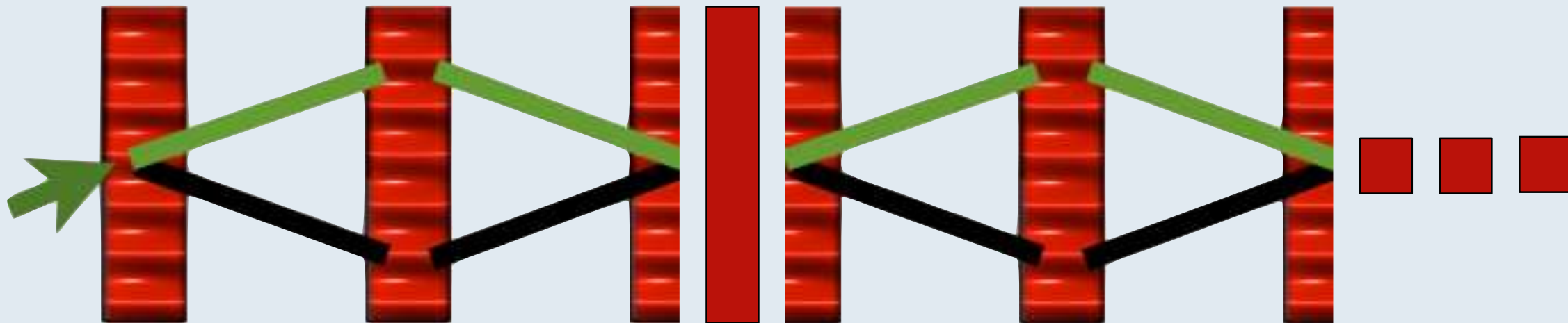
→ Integration as $\frac{1}{\sqrt{N}}$



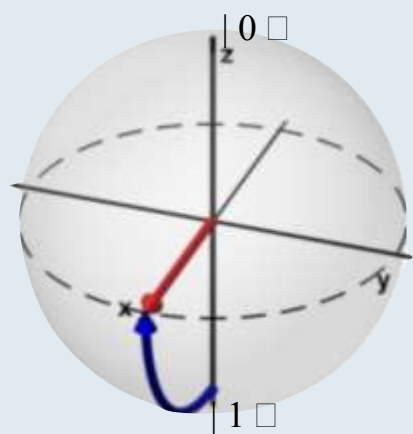
keff	+	-	+	-	+	-	+	-	+
fringe side	+	+	-	-	+	+	-	-	+



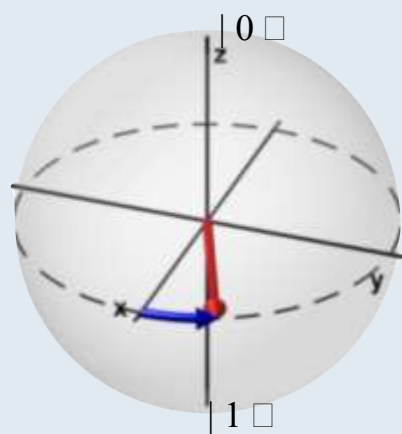




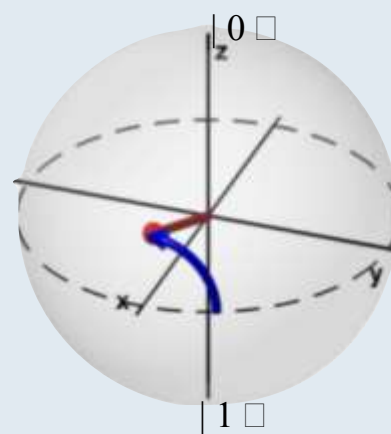
QND/weak measurement



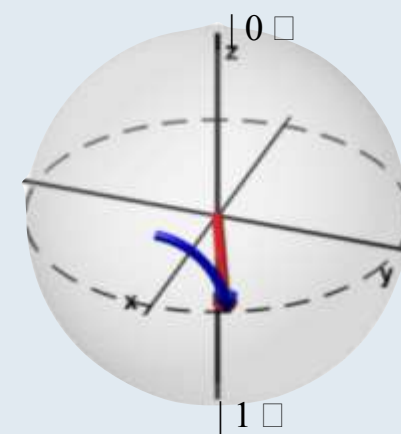
**prepare
CSS**



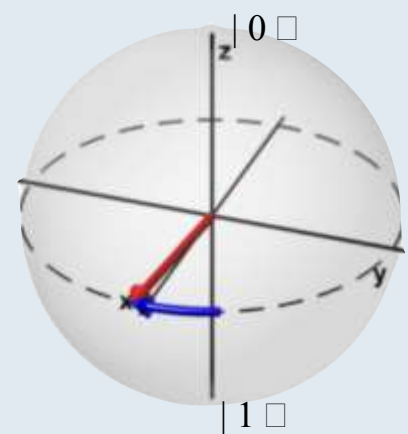
**free
precession**



**Coherence
preserving
measurement**

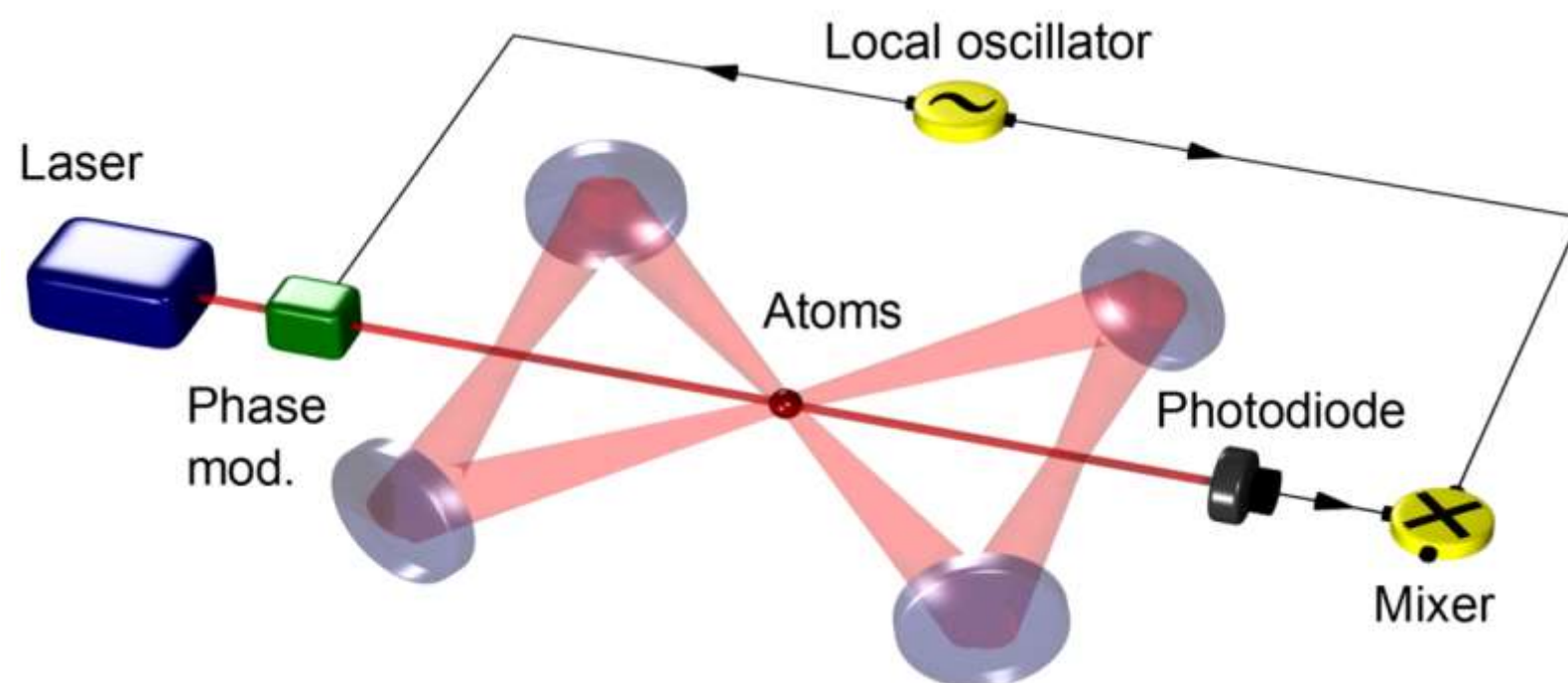


reinsert

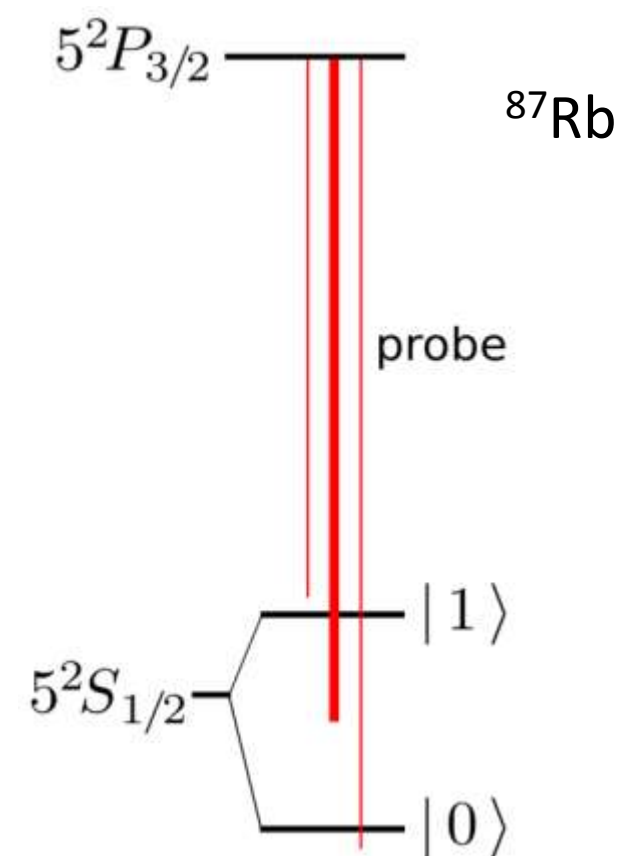


**Feedback
on the LO
phase**

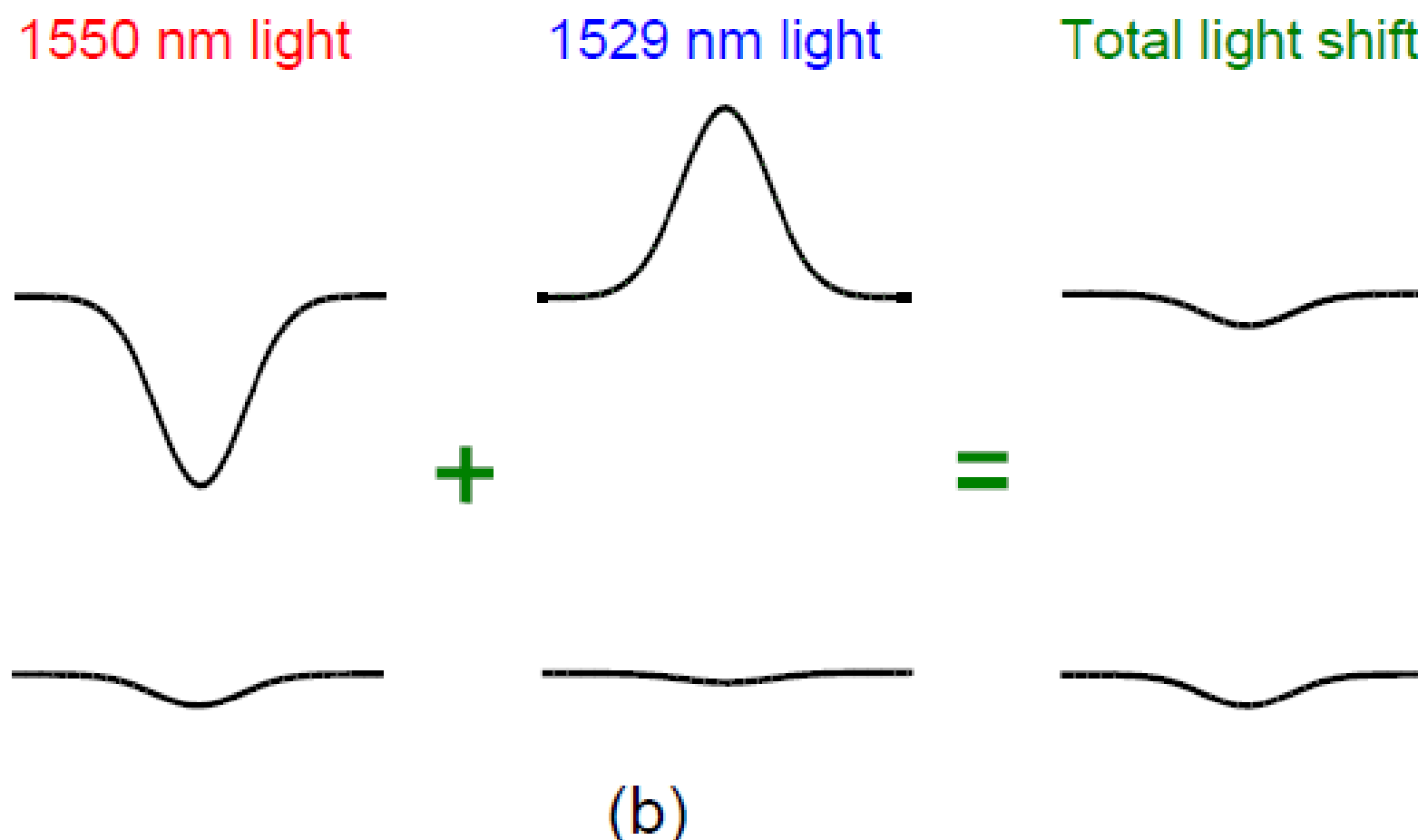
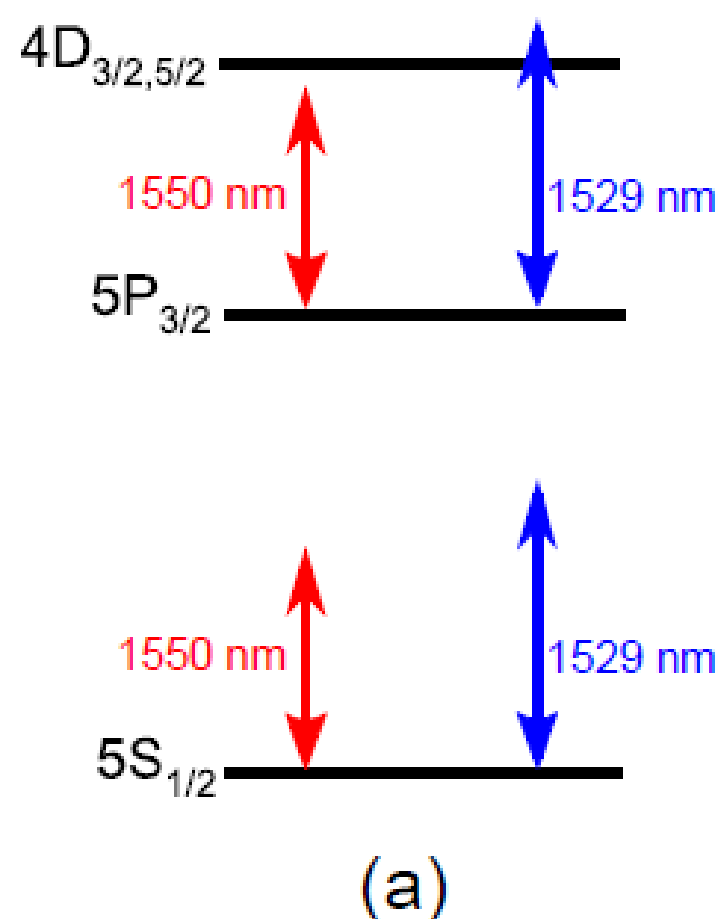
repeat



5×10^5 atoms @ 10 μ K in
the cavity enhanced
dipole trap

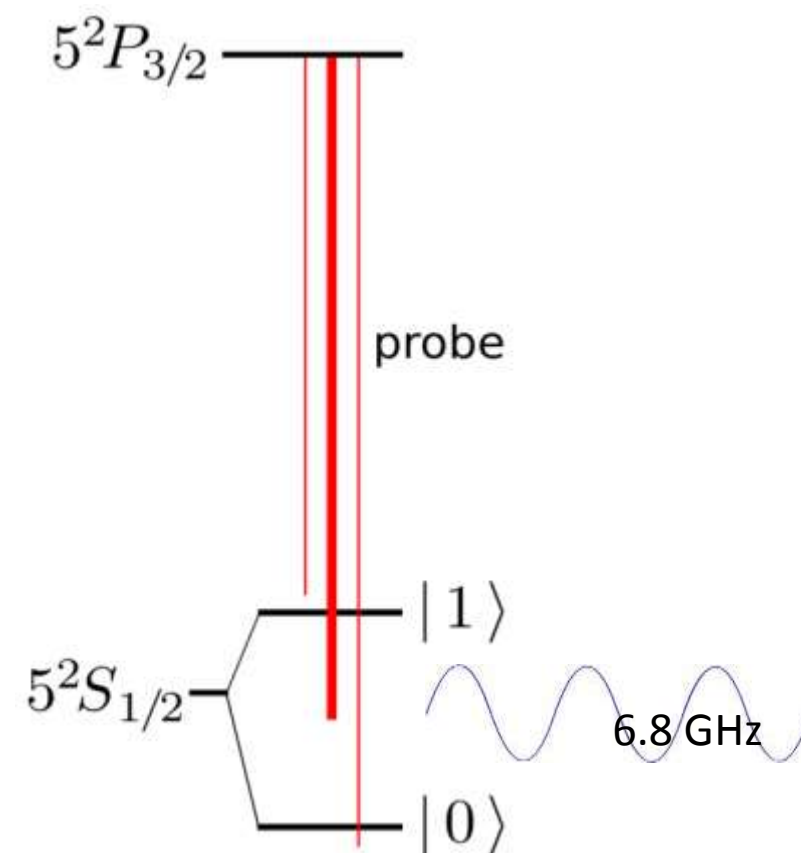
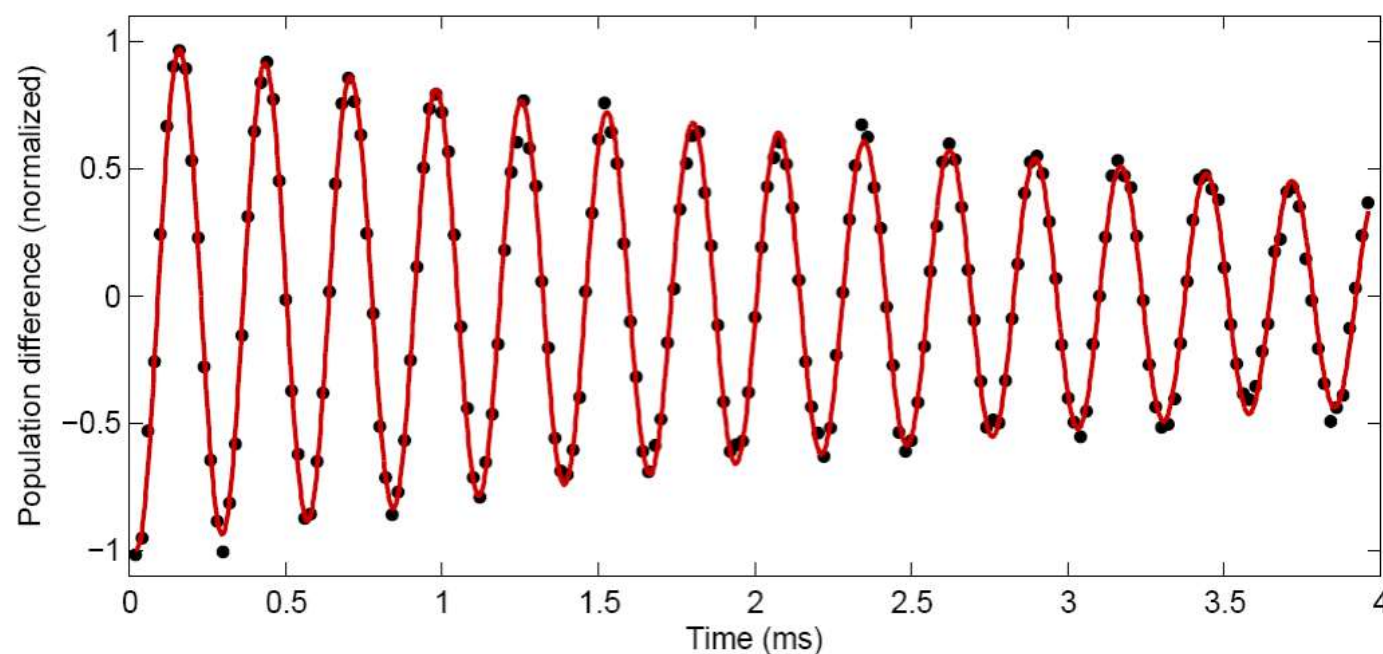


Frequency modulation spectroscopy to
measure non-destructively $N_1 - N_0$
to which is mapped the atom-LO phase



New laser stabilization scheme for cavity locking of 1529 nm laser

Kohlhaas et al., Opt. Lett. 37, 1005 (2012)



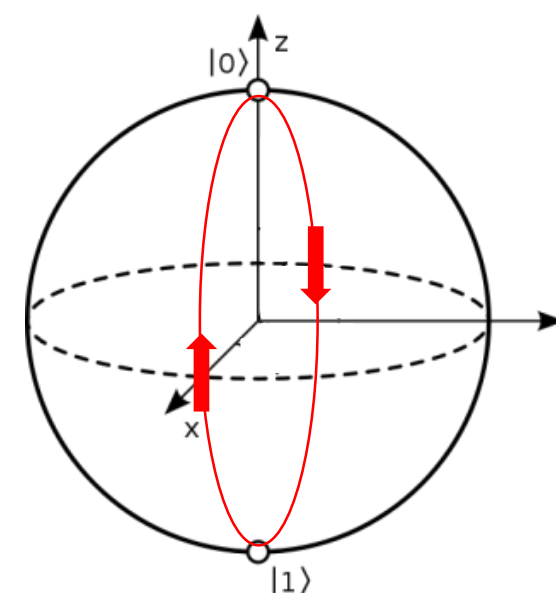
Probe pulse every 20 μ s

45% residual coherence after 200 measurements

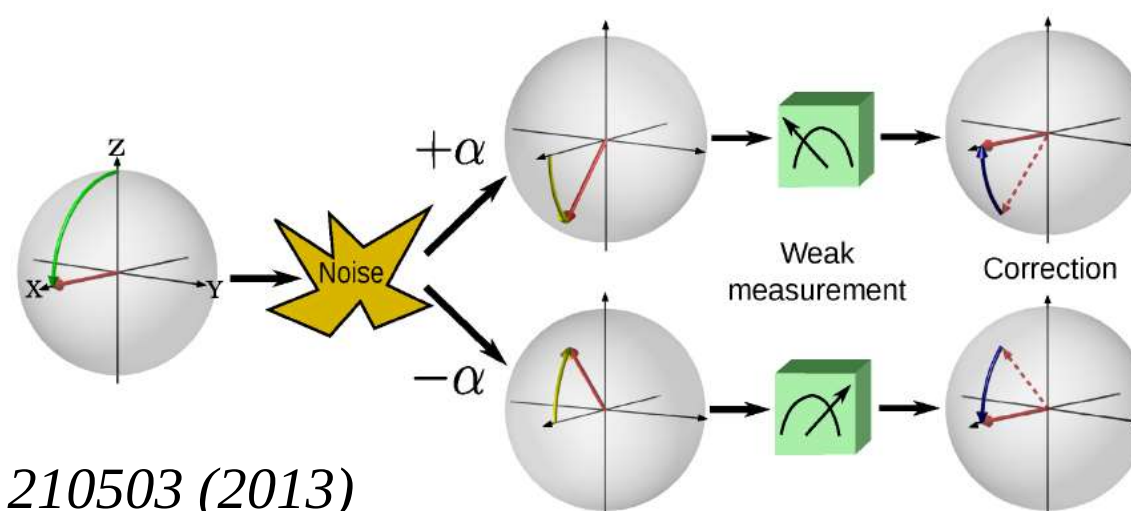
Decay dominated by spontaneous emission

→ measurement preserves coherence!

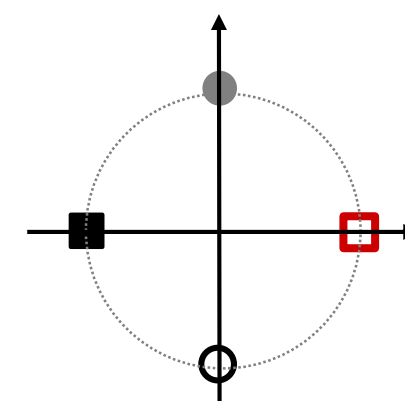
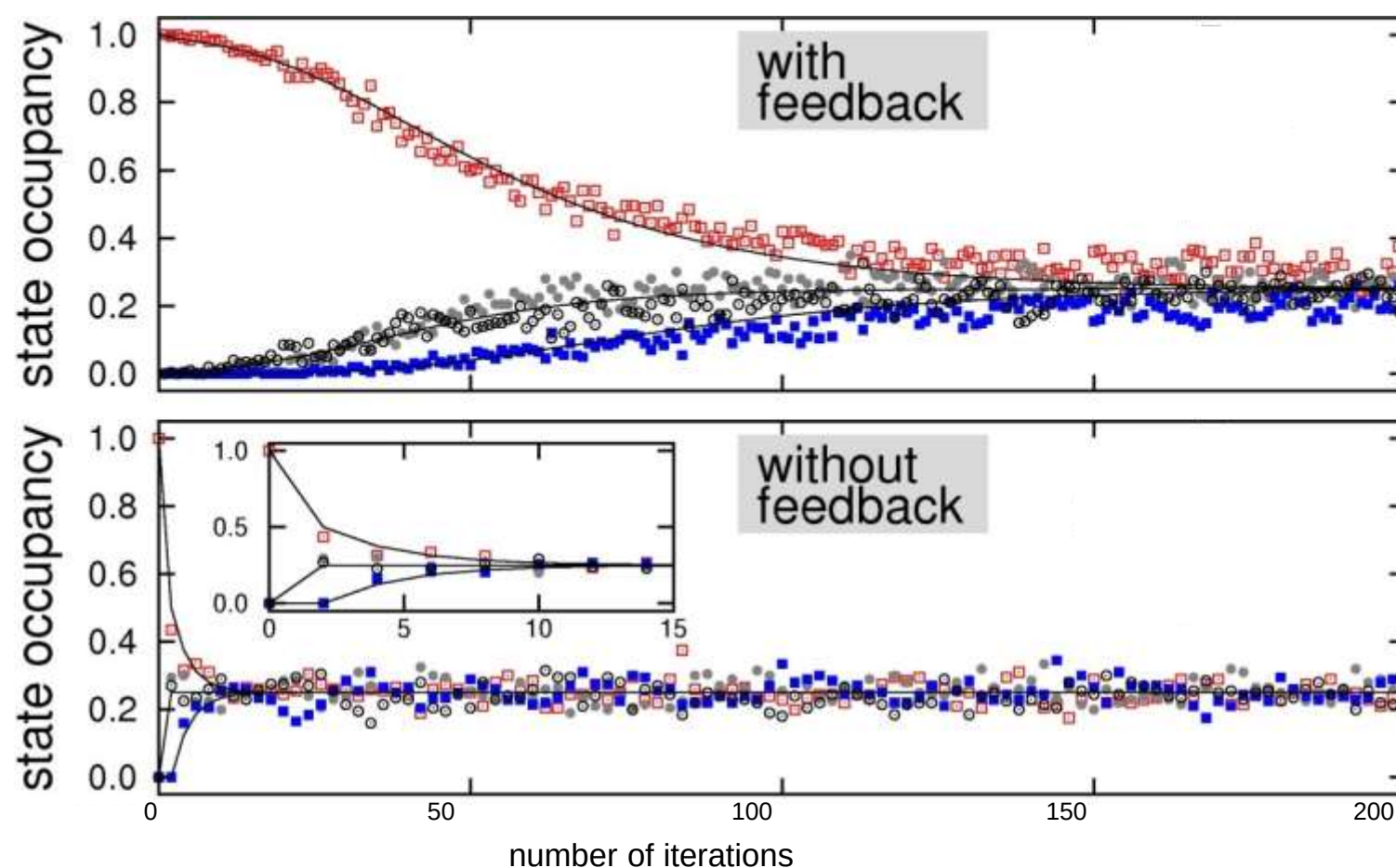
Bernon S. et al., NJP 13 (2011)

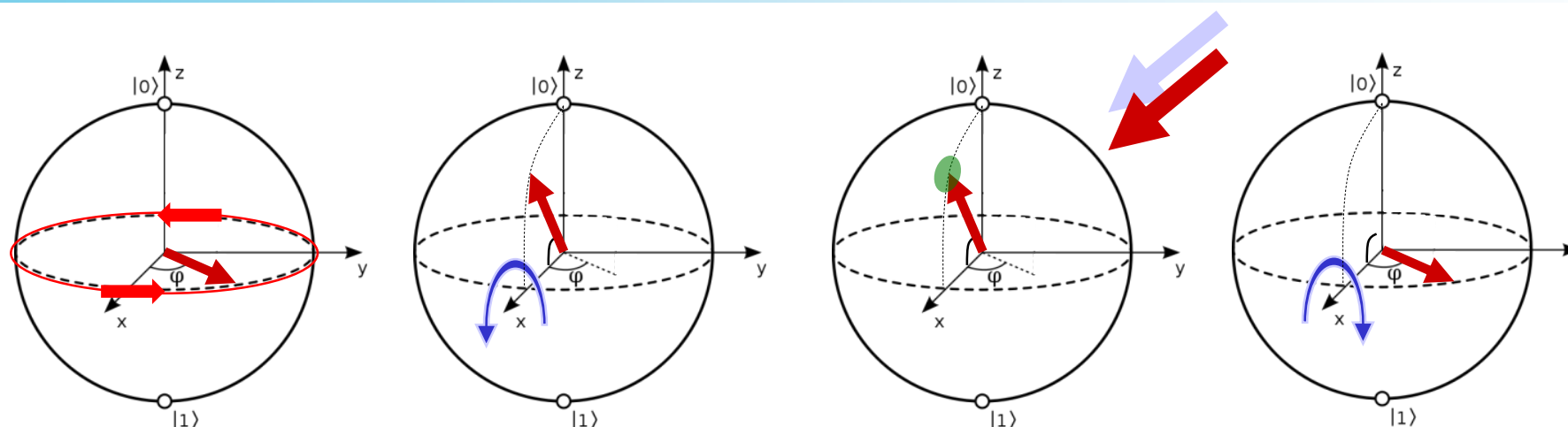


PROTECTING A COHERENT SPIN STATE AGAINST DECOHERENCE



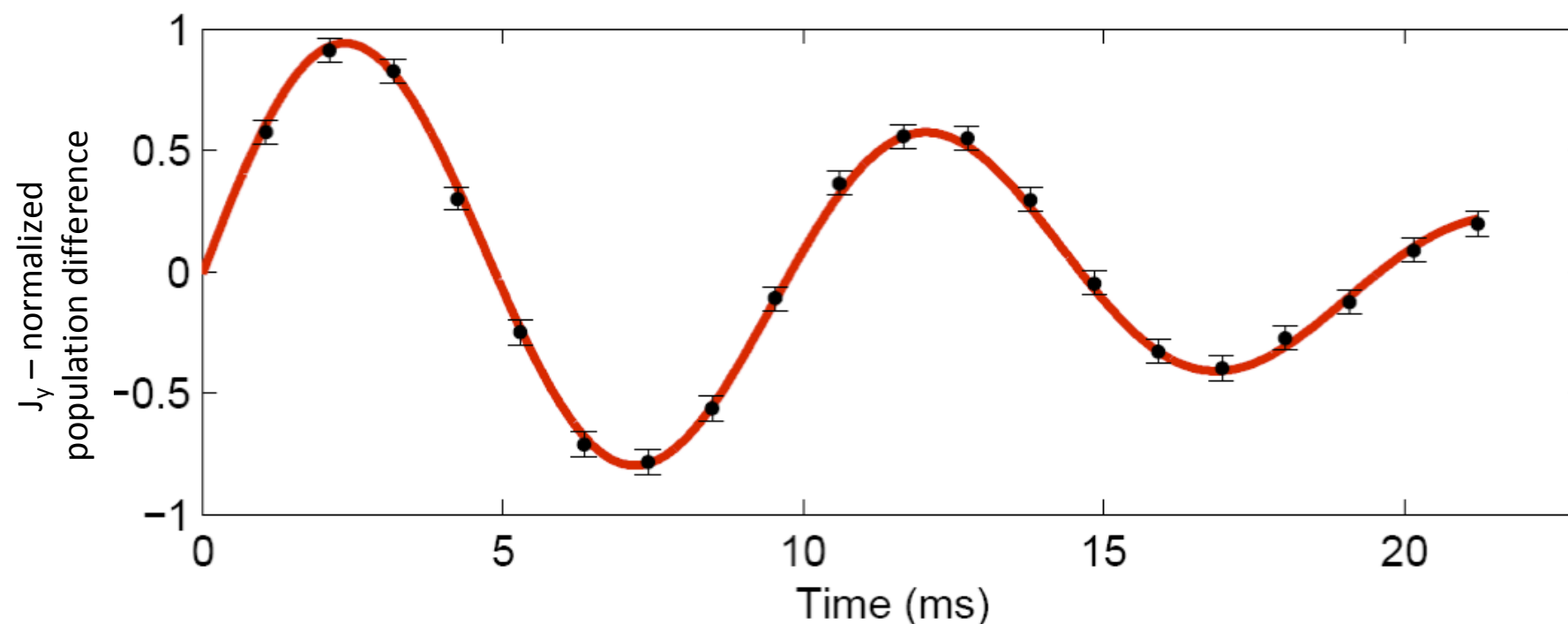
Vanderbruggen, T. et al., PRL 210503 (2013)





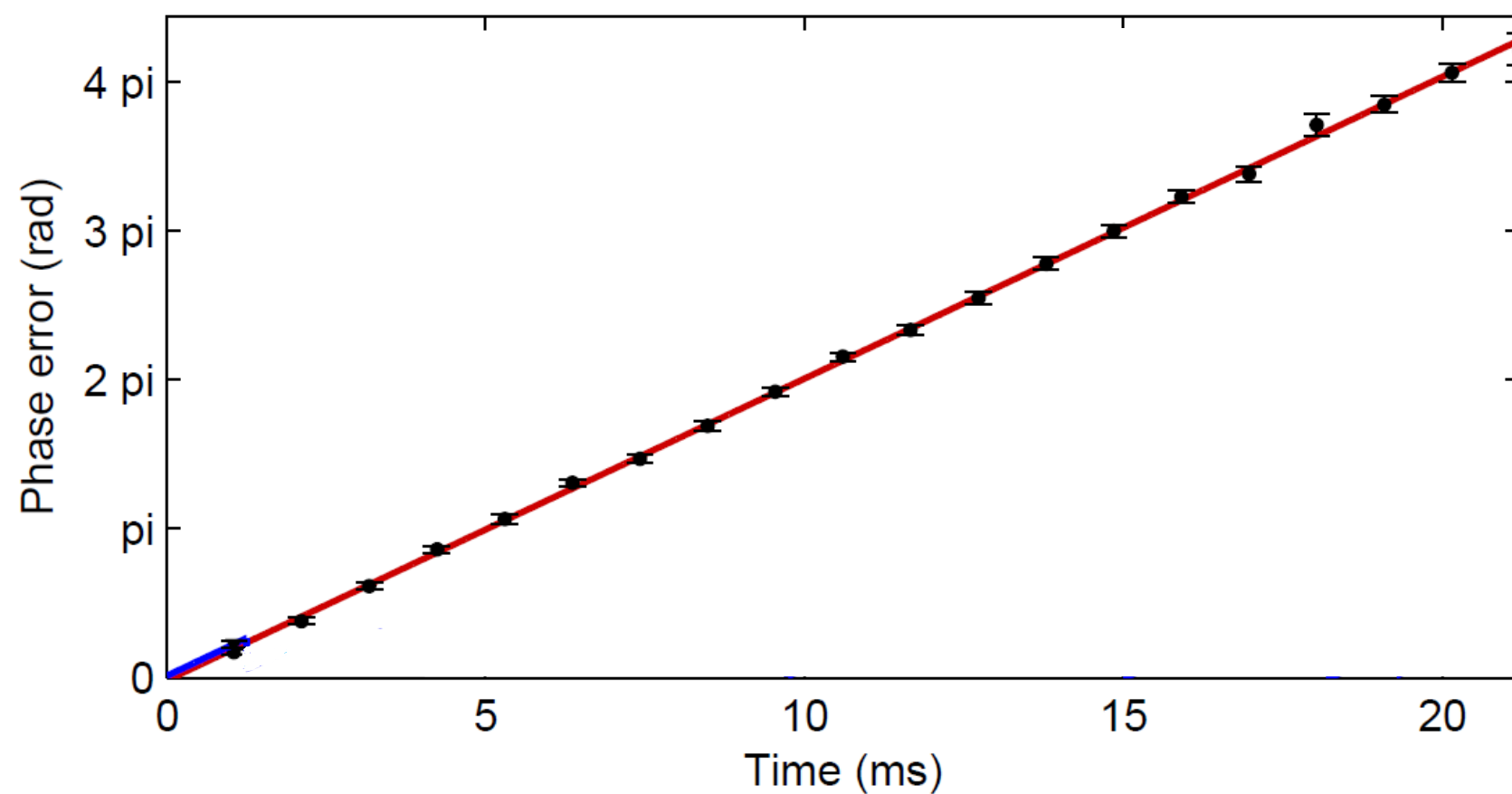
1. 100 Hz Frequency offset on LO

2. repeat weak measurement of the relative phase



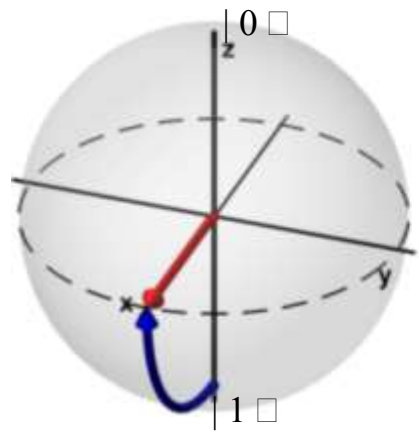
Measure in real time
phase in atomic
interferometer

Decay by
inhomogeneous
lightshift in trap

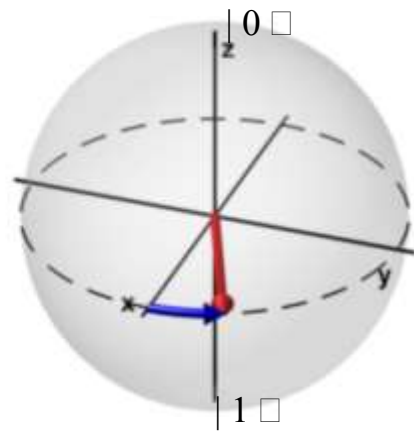


Well beyond the $[-\pi/2; \pi/2]$ inversion region

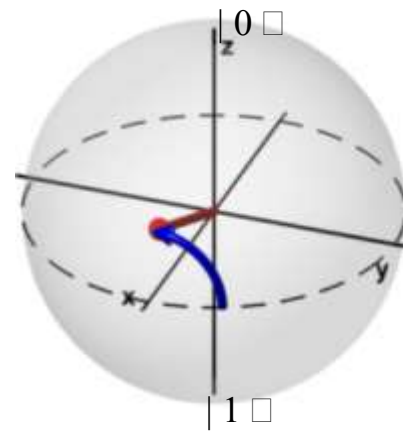
HOW TO KEEP THE PHASE DRIFT SMALL



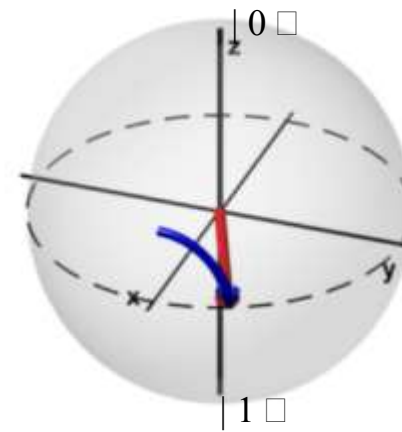
prepare
CSS



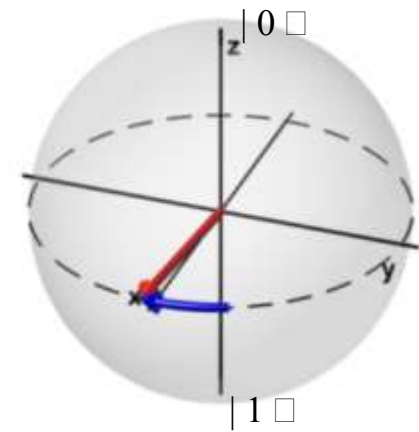
free
precession



Coherence
preserving
measurement



reinsert

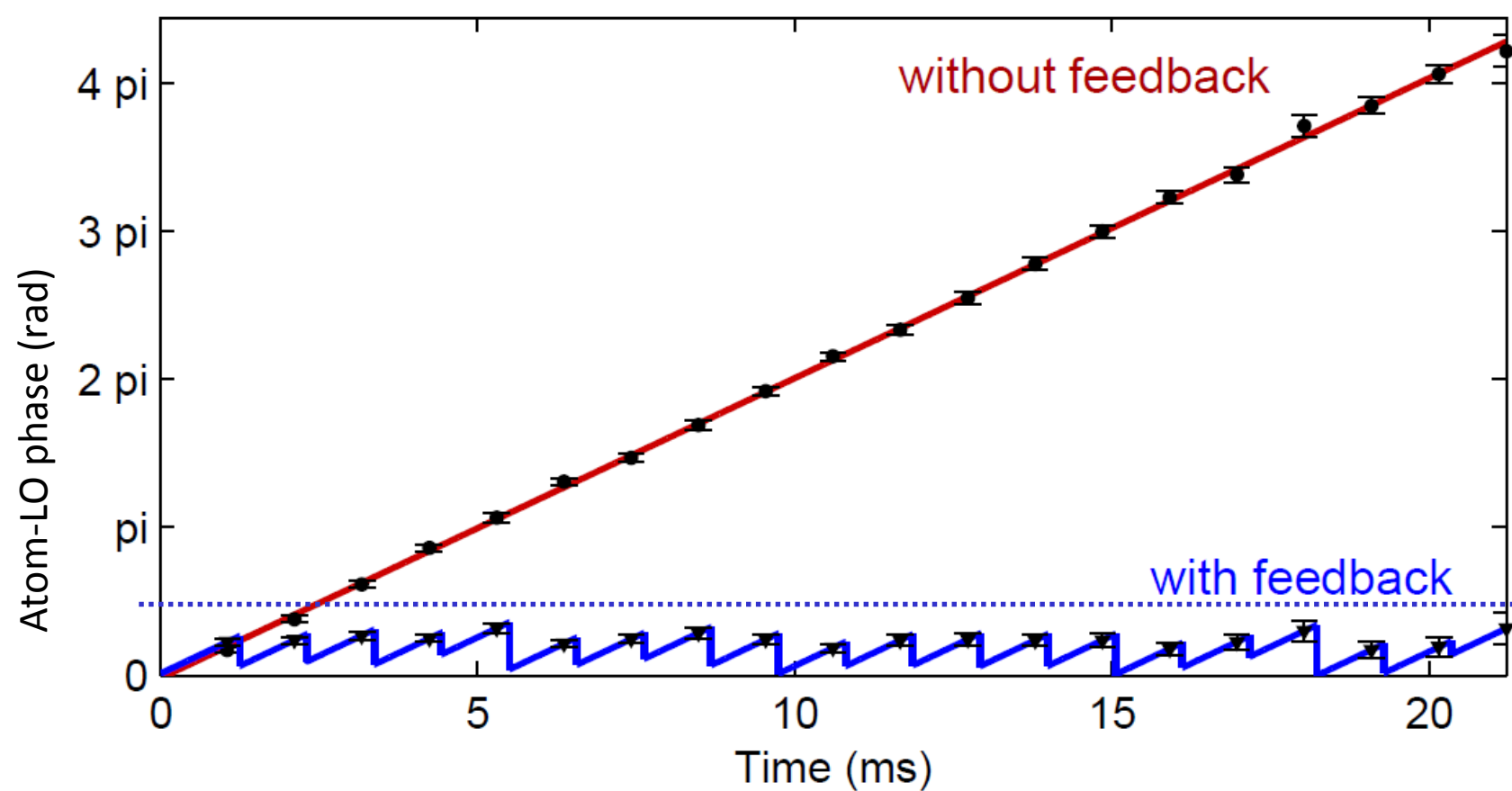


Feedback
on the LO
phase

repeat

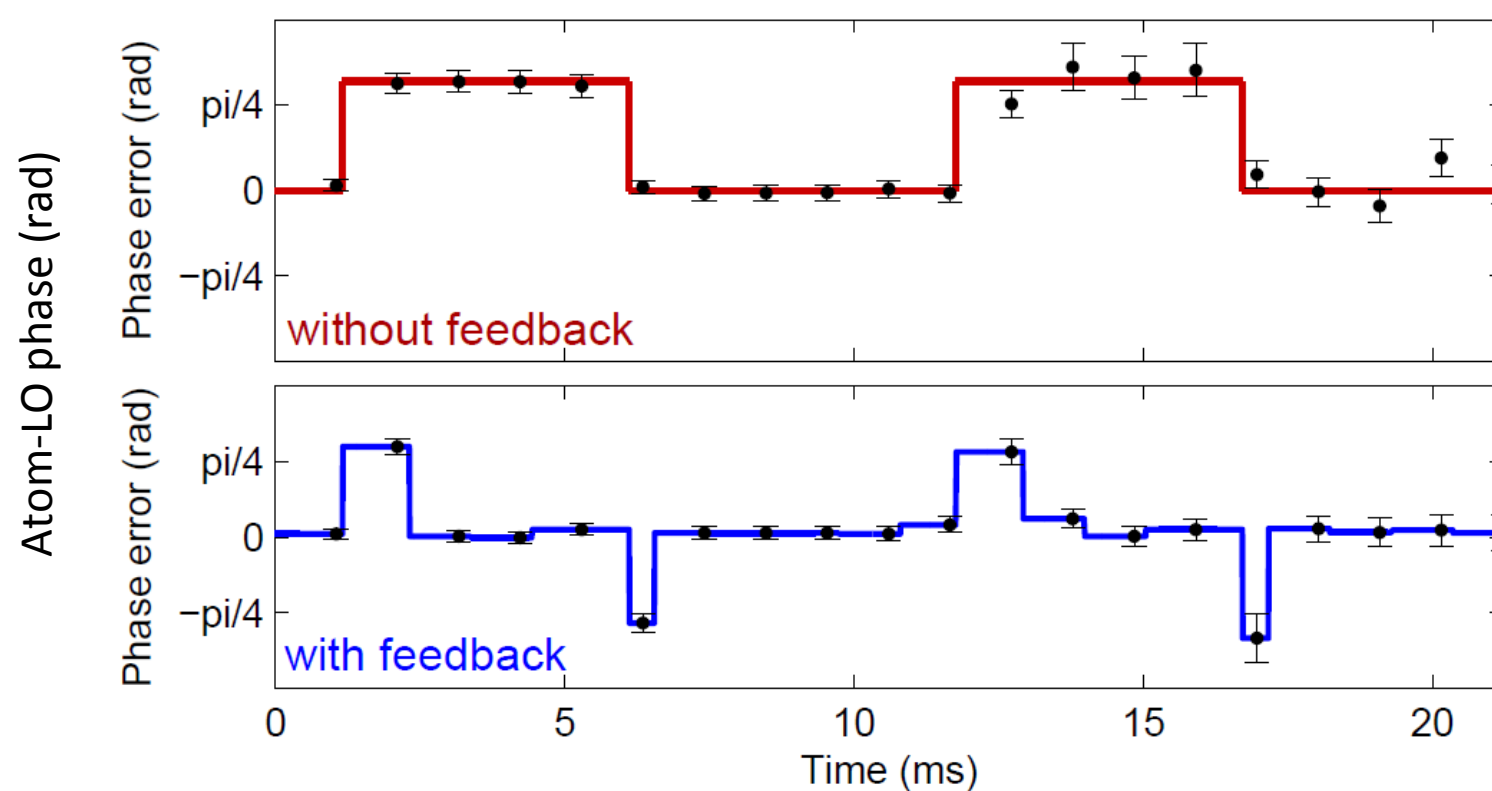
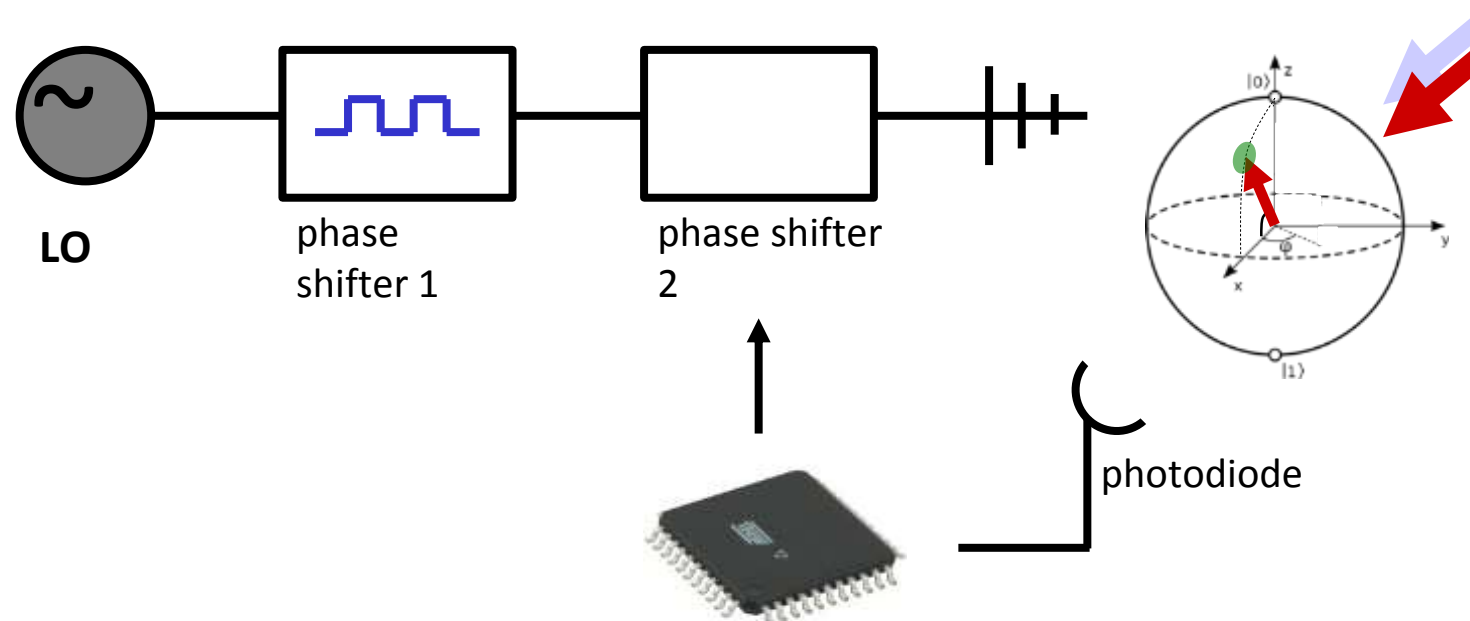
Phase corrections saved for final evaluation

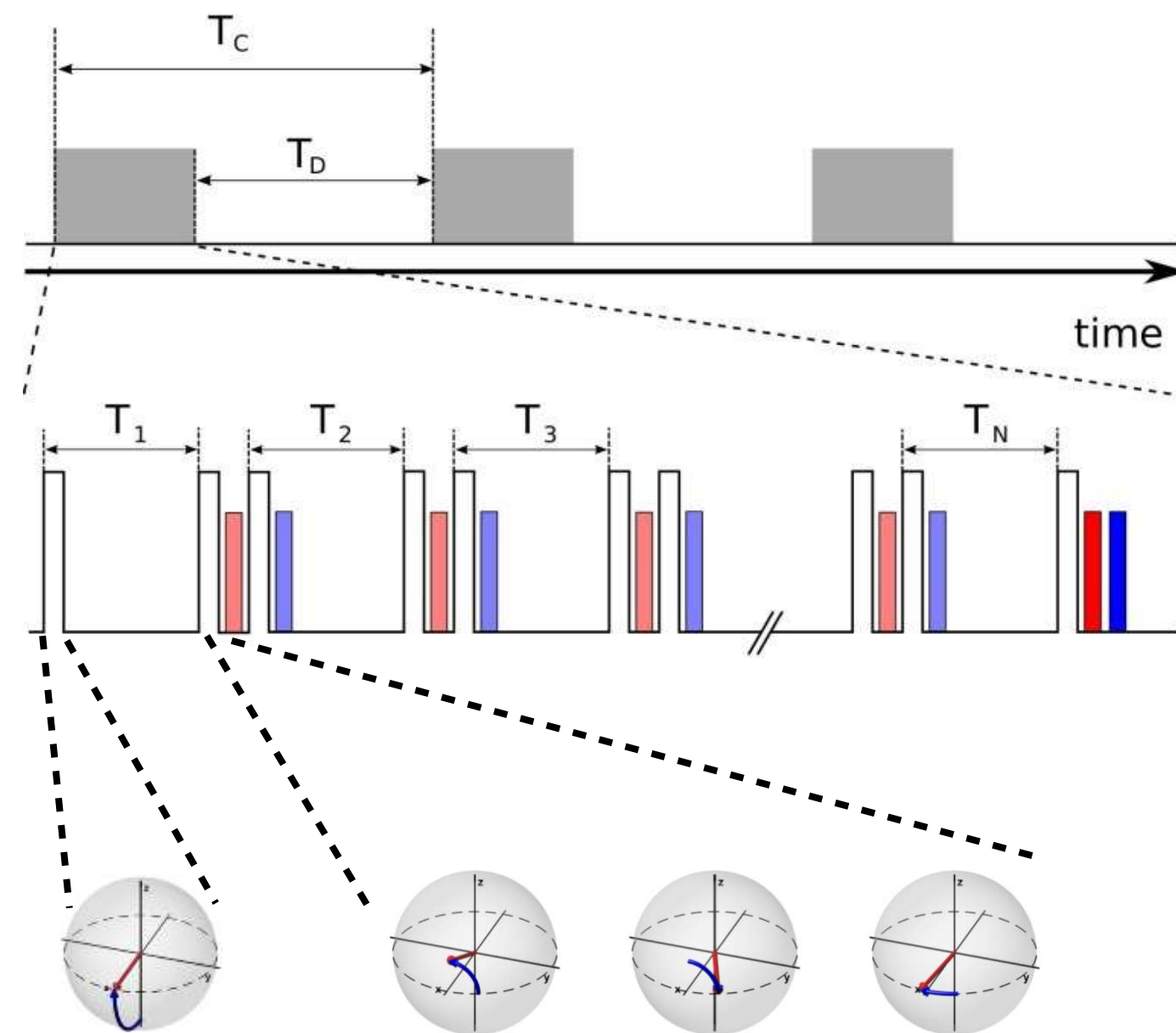
Classical oscillator is stabilized on superposition state



Feedback maintains phase in inversion region

RECONSTRUCTION OF THE PHASE EVOLUTION WITH ARBITRARY NOISE

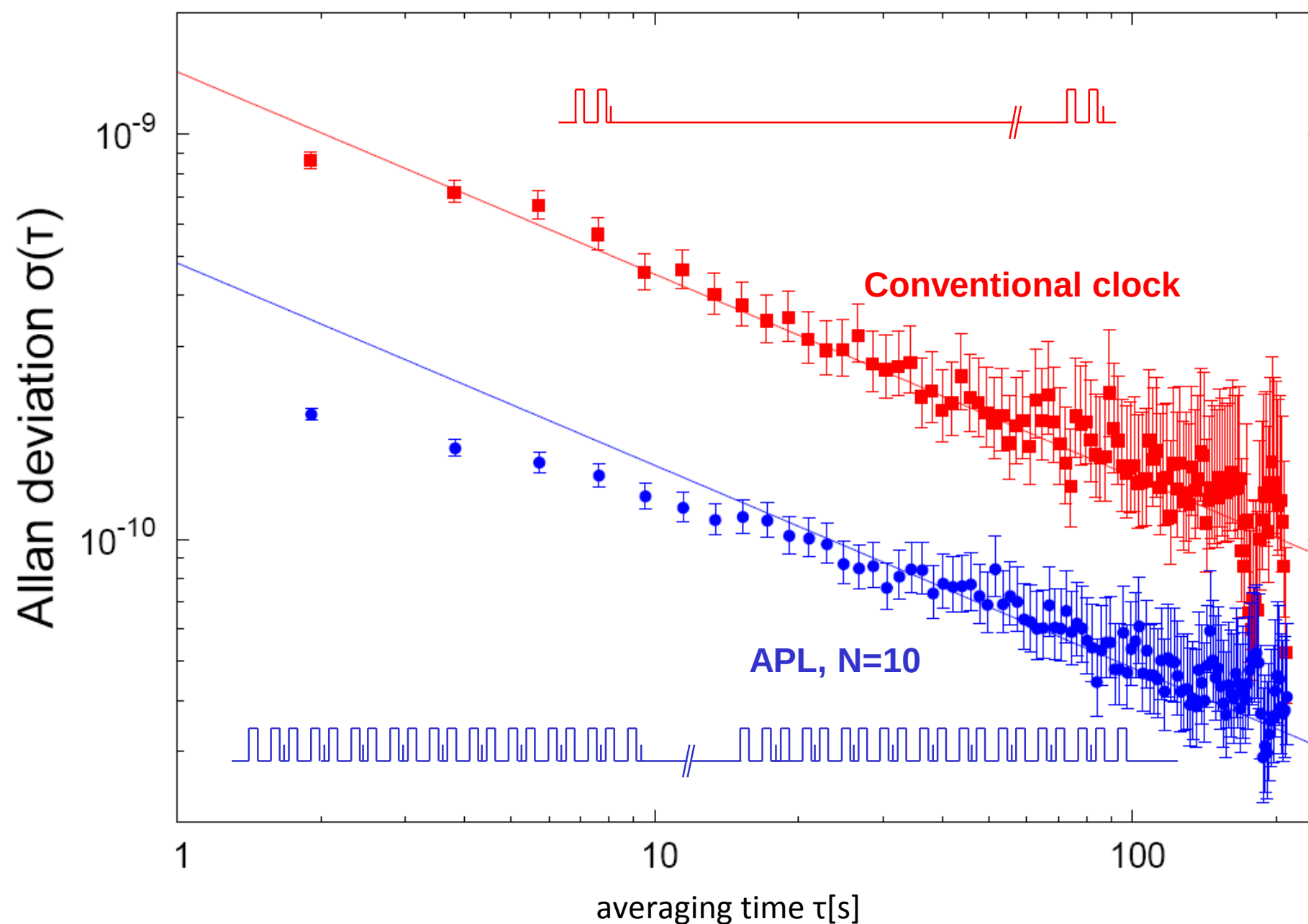




1. atomic state preparation

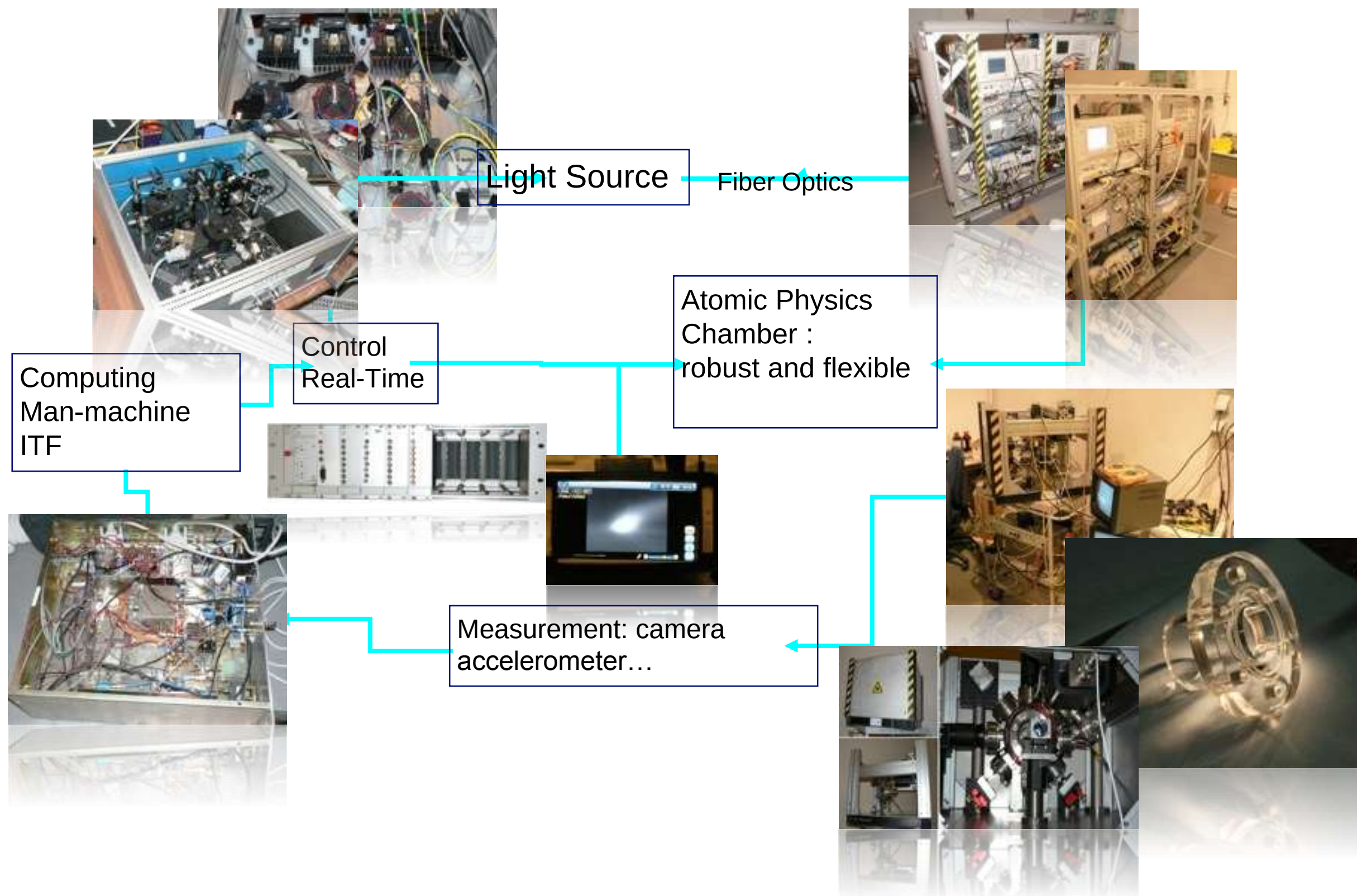
2. N -cycles of **coherence preserving measurement of J_z** and **phase correction on the LO** to stay in the inversion region for the differential phase

3. final destructive measurement, **calculation of the total differential phase cumulated** on the extended interrogation time, frequency correction on the LO

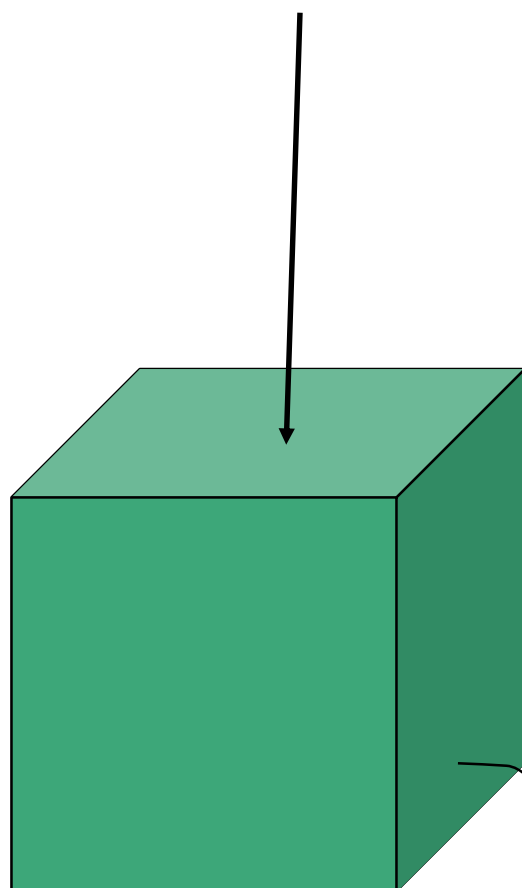


- white frequency noise on the LO, $\varphi_{\text{rms}}=430$ mrad @ 10 ms
 - 10 measurements / phase corrections, $T=1$ ms (blue)
 - comparison with “standard” clock with $T=1$ ms (red)
 - better Allan deviation by 5 dB
- BUT: T is not limited to 1 ms by the noise on the LO...**

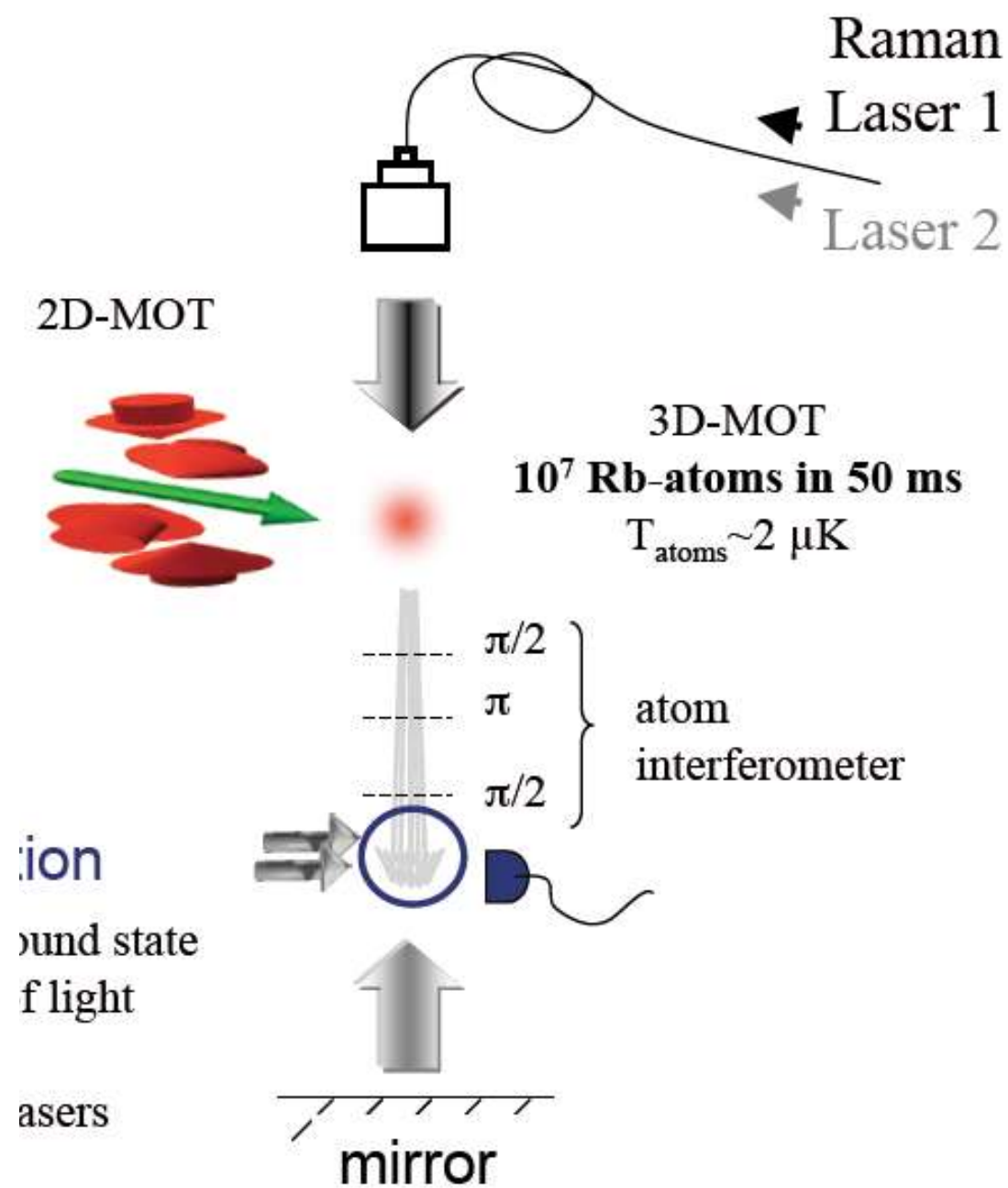
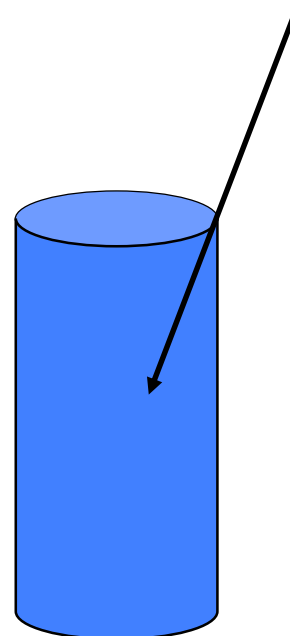
LECTURE 3 : APPLICATION TO GEOPHYSICS, NAVIGATION, FUNDAMENTAL PHYSICS

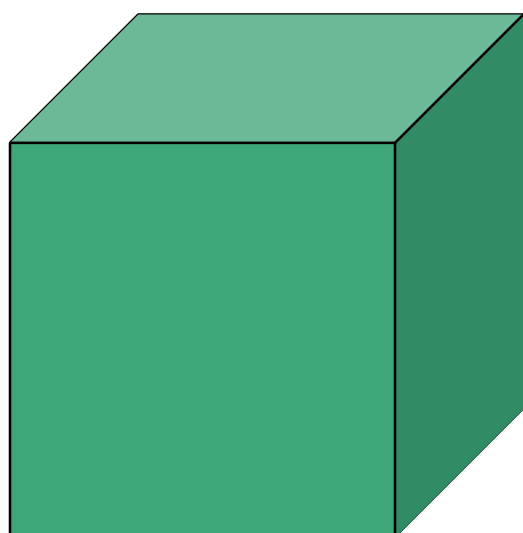


LASER SYSTEM



SENSOR HEAD





Laser manipulation is key for the atom interferometer

- many frequencies required for laser cooling (780 nm for Rb, 767 nm for K, 850 nm for Cs ...)
- Need precise control of frequency and intensity

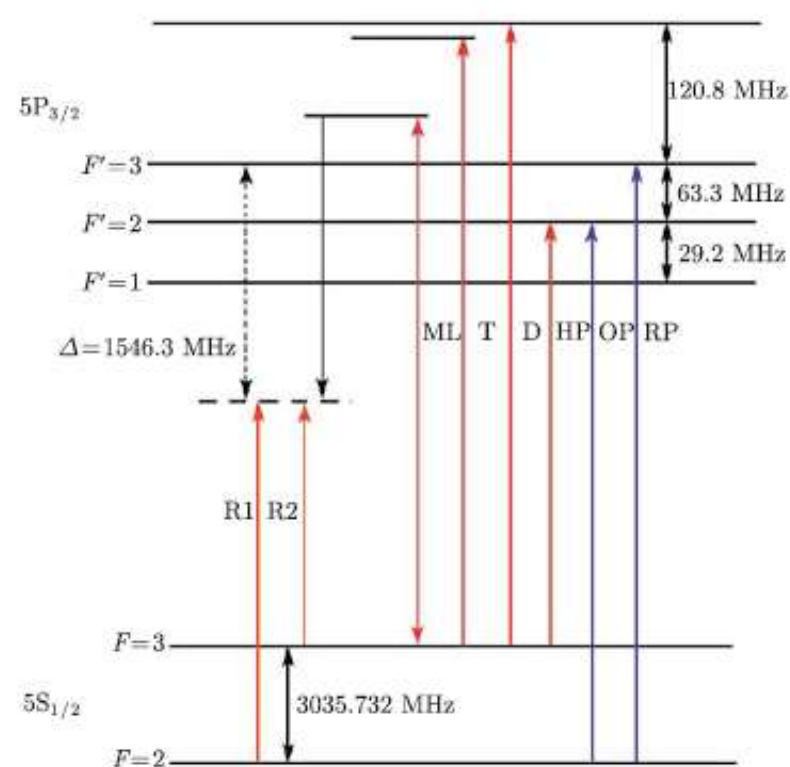
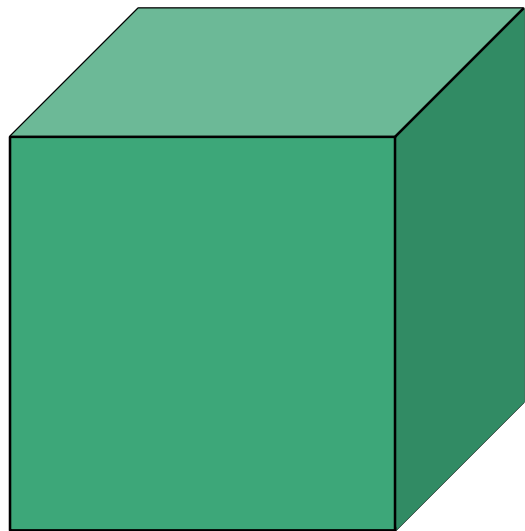
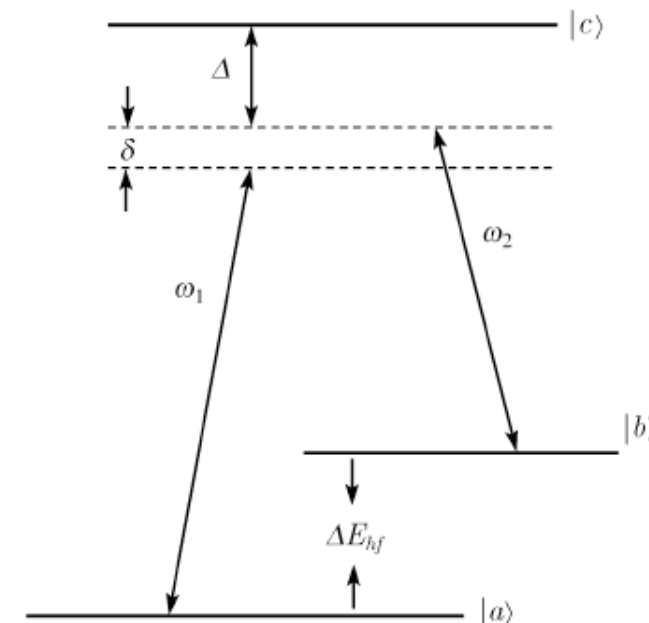
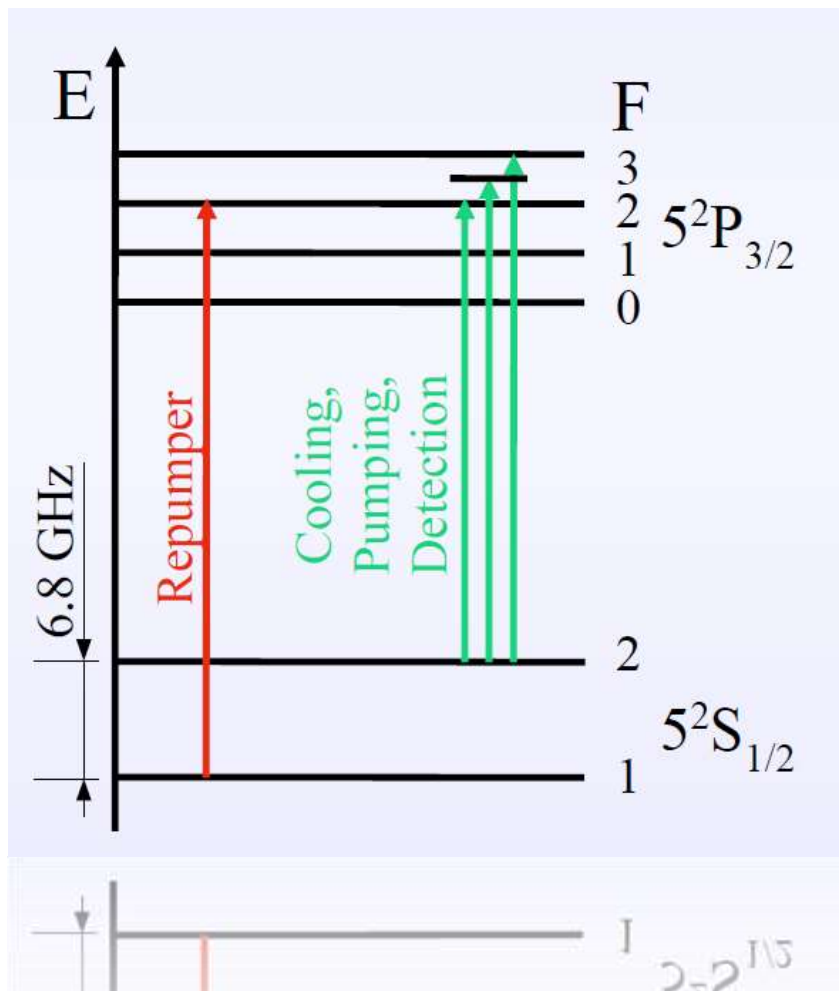


Fig. 4 Laser frequency assignment and energy levels of ^{85}Rb . R1, R2: Raman laser beams, ML: Main Laser; T: Trapping laser, D: Detection laser; OP: Optical pump laser; HP: Hyperfine pump laser, RP: Repumping laser.

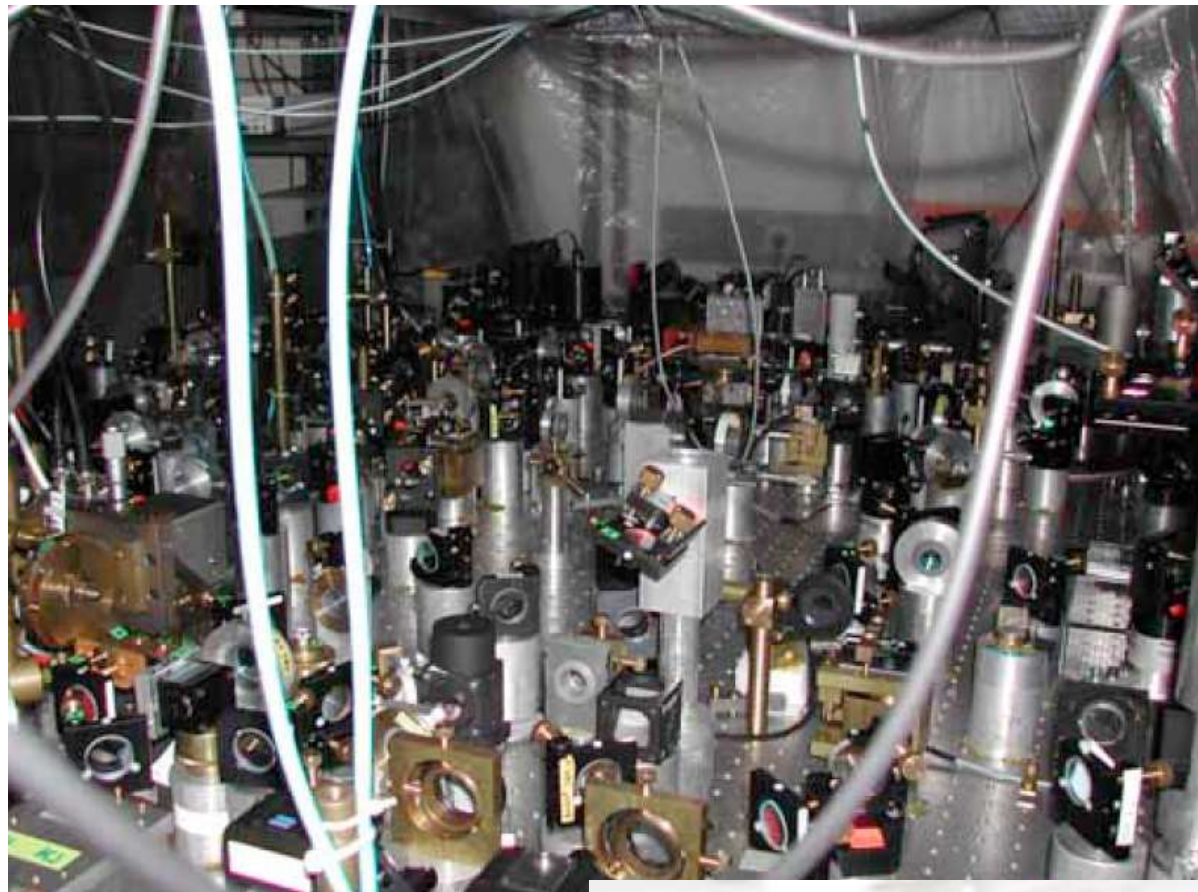


Laser manipulation is key for the atom interferometer

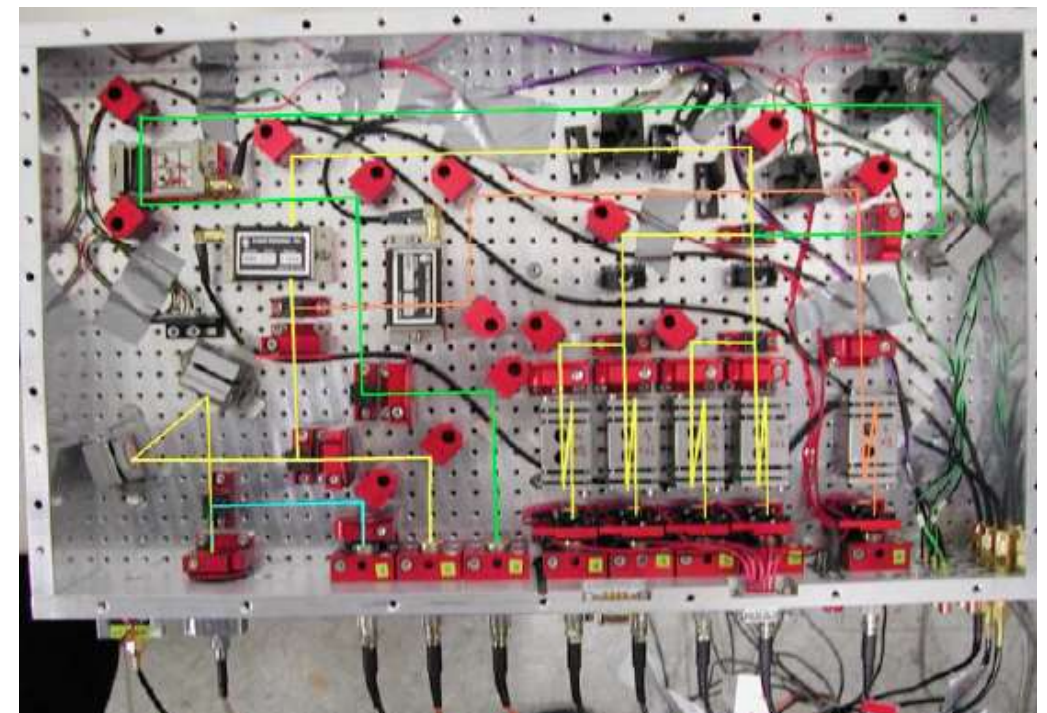
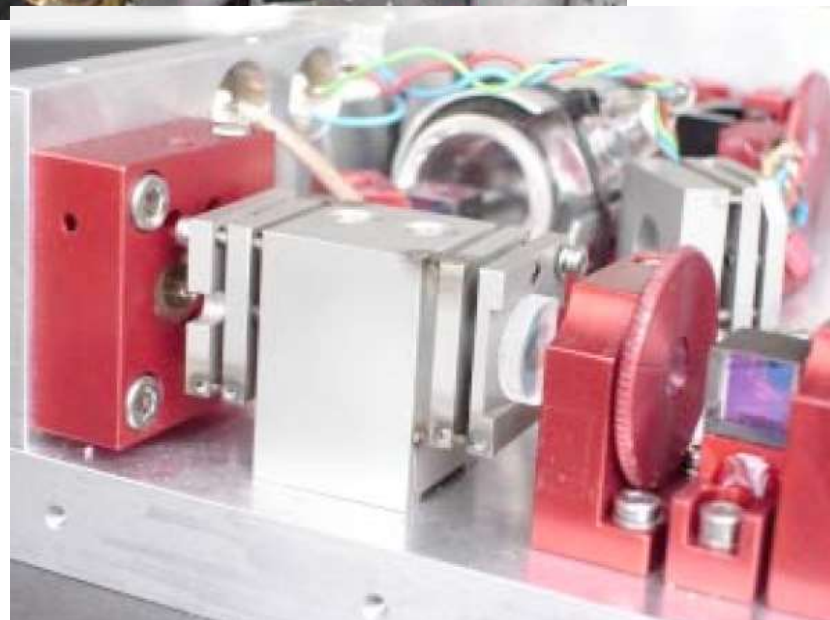
- many frequencies required for laser cooling (780 nm for Rb, 767 nm for K, 850 nm for Cs ...)
- Need precise control of frequency and intensity



- For Raman diffraction, phase noise between lasers is a key

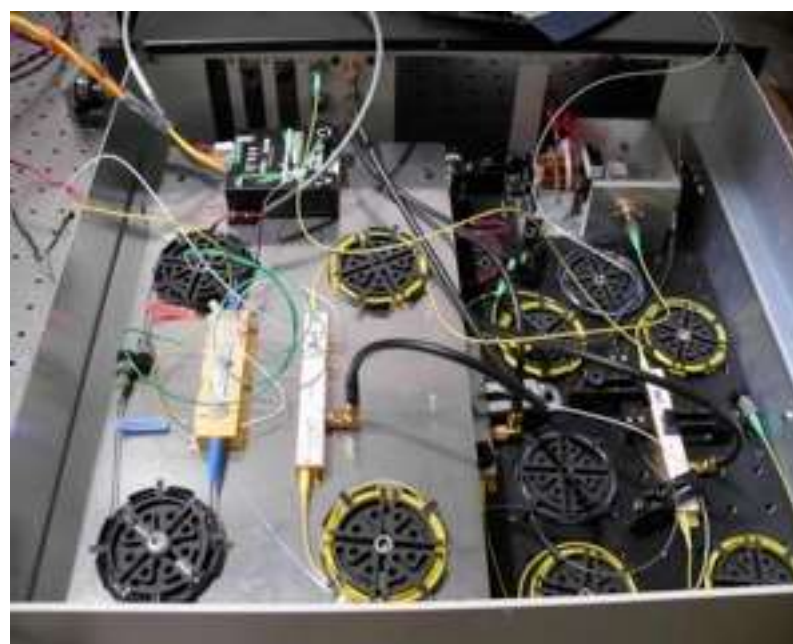


- ✓ Need many optical elements (AOs, mirrors, photodiodes ...)
- ✓ Before : large optical tables
- ✓ Now (Quantus project, AOsense) : compact optical benches



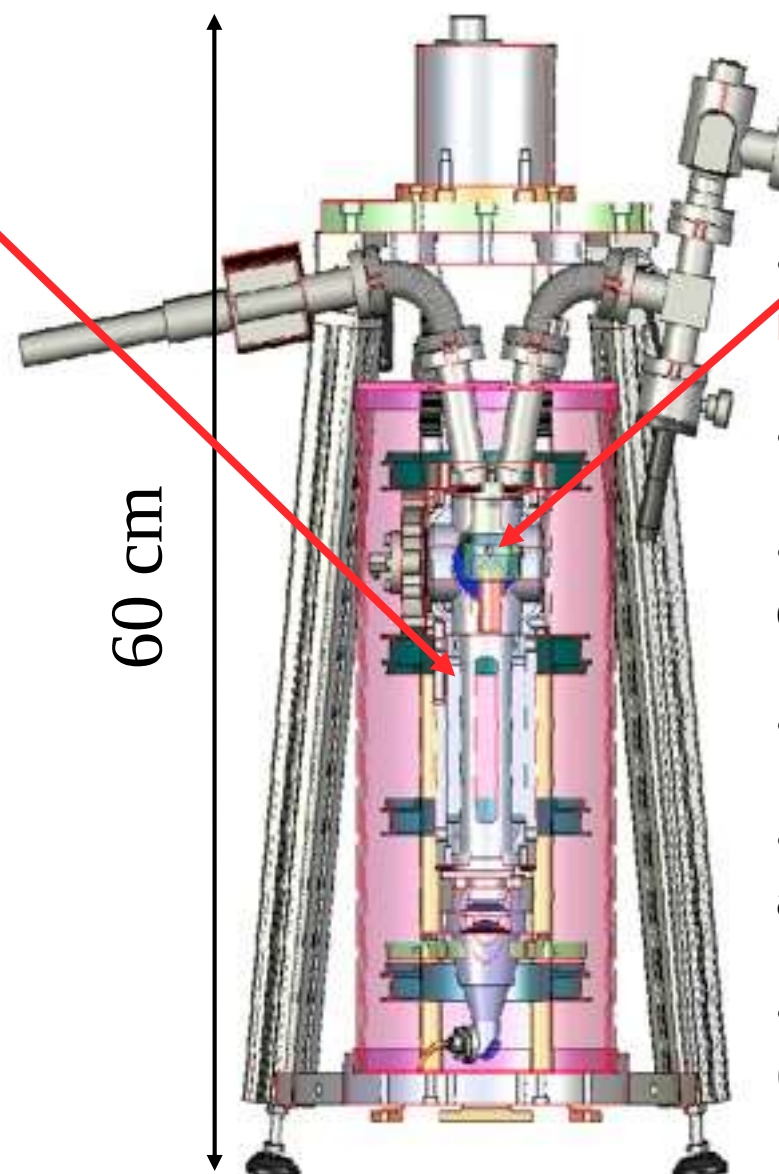
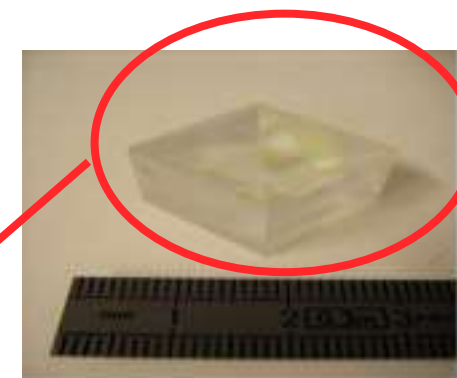


- ✓ One single telecom laser diode, frequency doubled
- ✓ Low noise highly compact HF generation
- ✓ Off-the-shelf fiber coupled telecom & microwave components
- ✓ Drastic reduction of volume (overall packaging $\sim 70 \text{ cm}^3$)

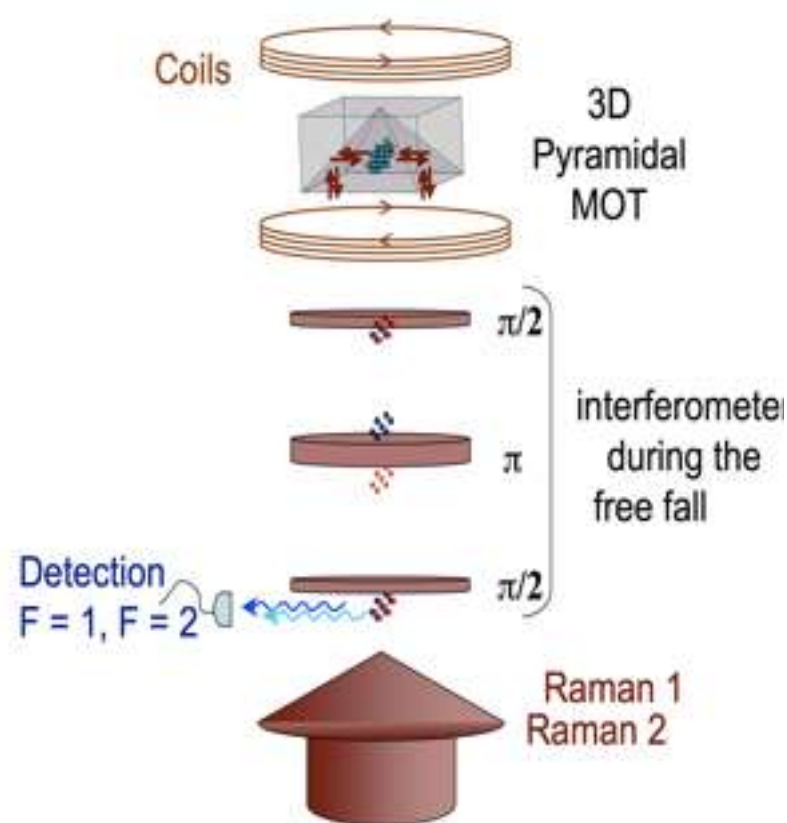




Patent filled (WO 2009/118488 A2)

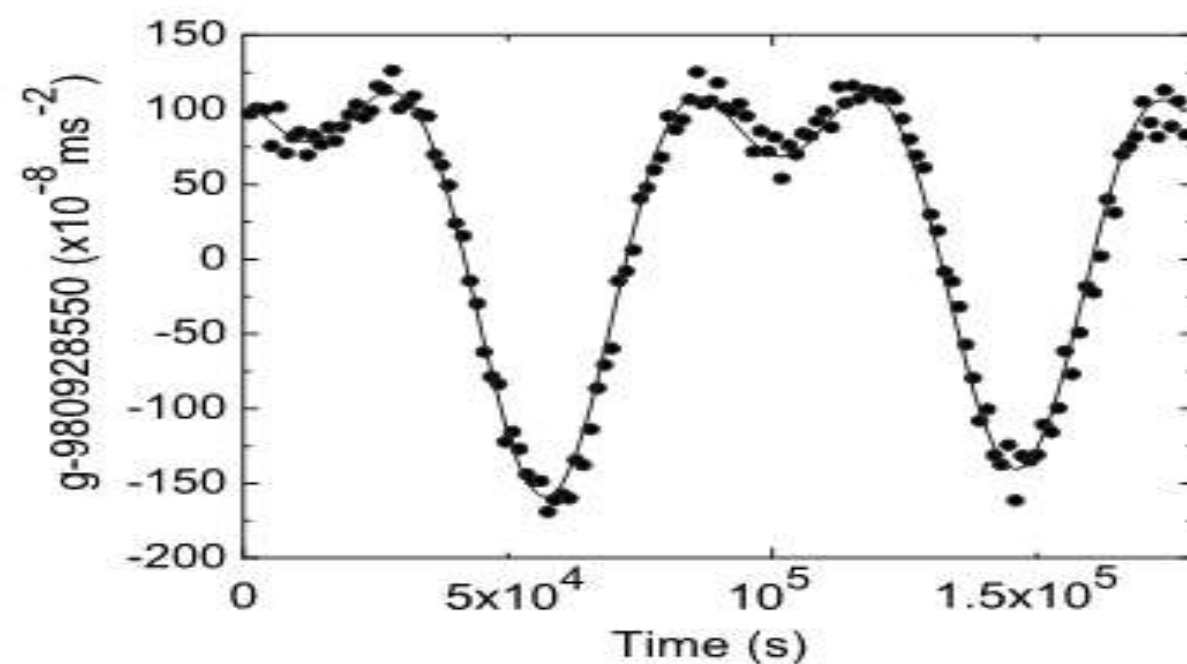
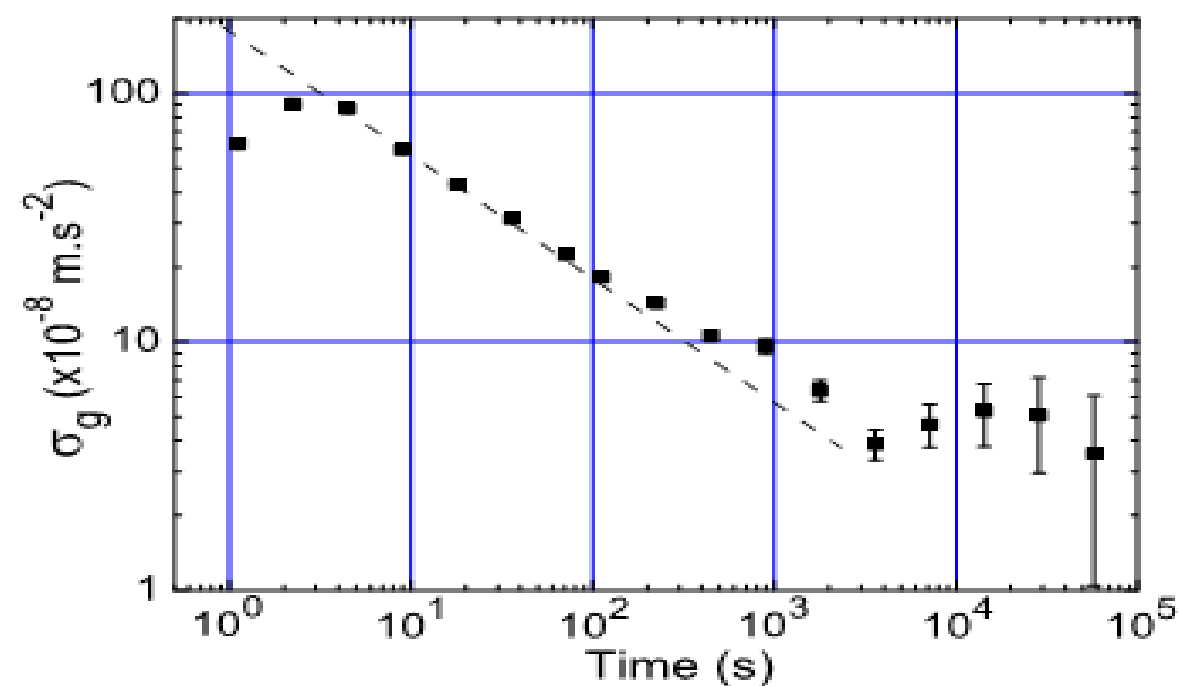
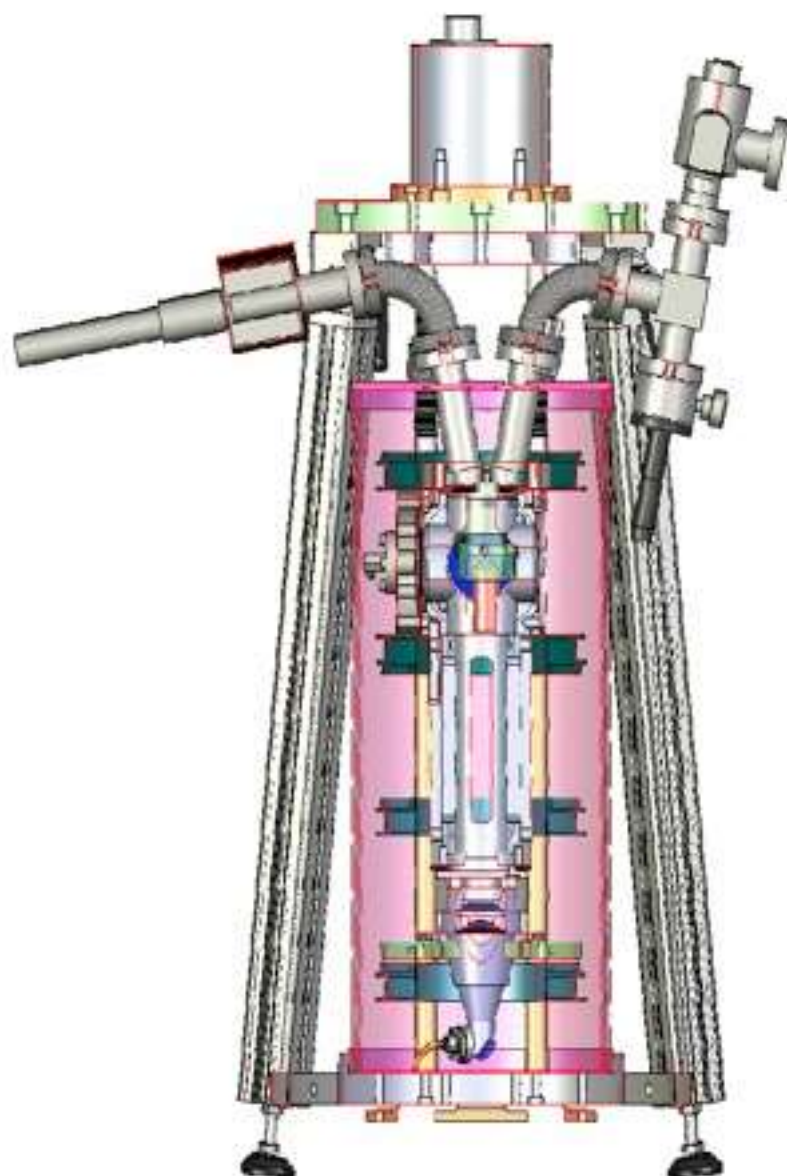


- Innovative hollow pyramidal retro-reflector
- Single beam trap & interferometer
- 2 L Ti magnetically shielded vacuum chamber (Indium sealed)
- Trapping from a vapor ($P \sim 10^{-8}$ hPa)
- Continuous monitoring by a tilt-meter and a seismometer
- Polarization controlled optical injection (through a PMF)
- Development of home-made electrical and HF feedthroughs

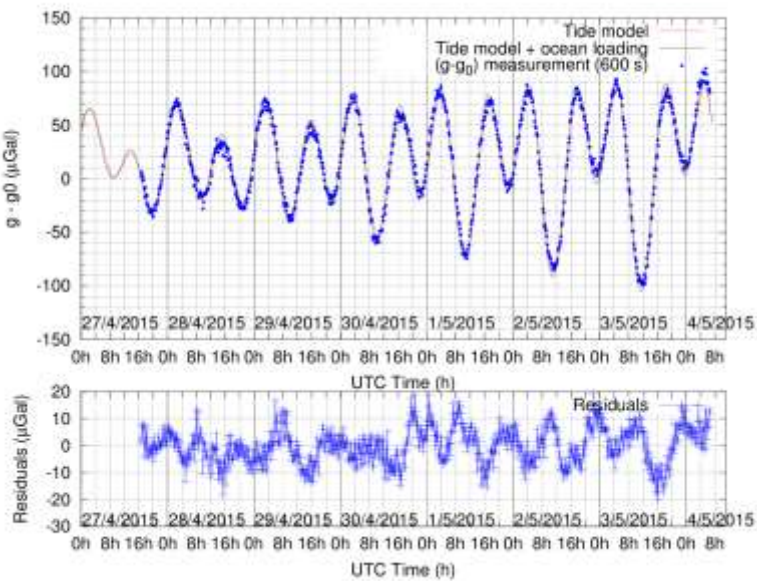


Demonstration of the compact interferometer with a hollow pyramid:
[Bodard et al., Applied Physics Letters 96, 134101, 2010]

Patent filled (WO 2009/118488 A2)

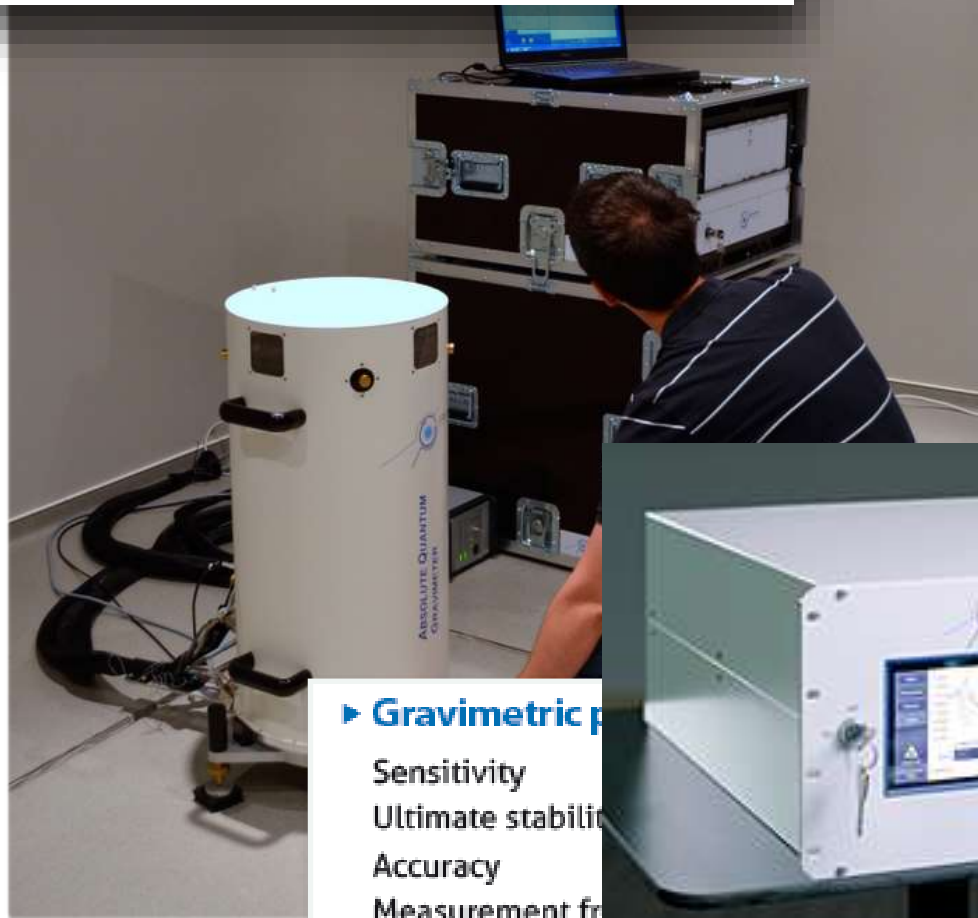


Demonstration of the compact interferometer with a hollow pyramid:
[Bodard et al., Applied Physics Letters 96, 134101, 2010]



μQUANS

PRECISION QUANTUM SENSORS



► Gravimetric performance

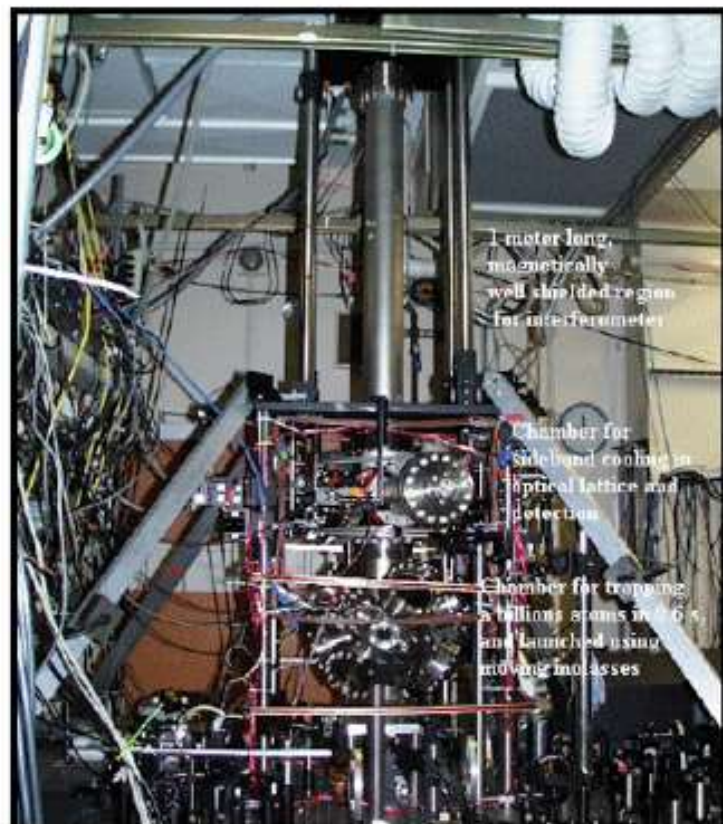
Sensitivity
Ultimate stability
Accuracy
Measurement frequency

► Power

Operating power 300W
Peak power (warm-up) 400W

► Frequency stability

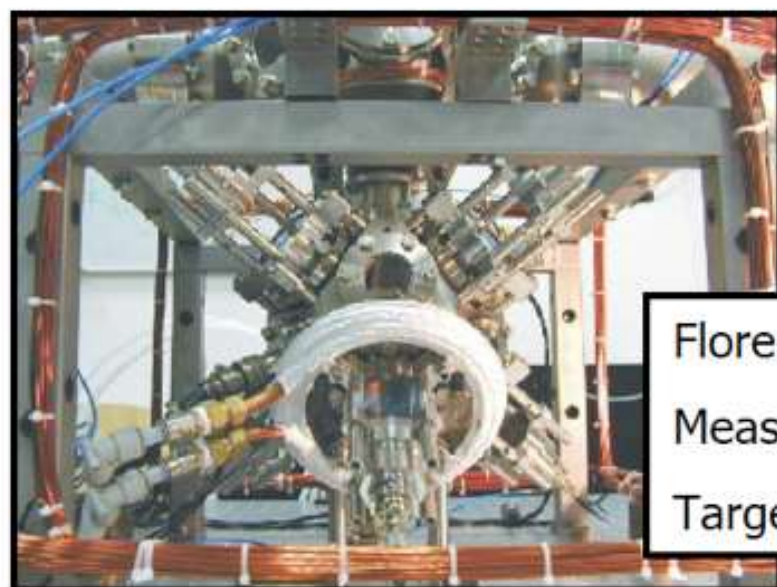
1 s	$\leq 3.0 \cdot 10^{-13}$
10 s	$\leq 9.5 \cdot 10^{-14}$
100 s	$\leq 3.0 \cdot 10^{-14}$
1000 s	$\leq 9.5 \cdot 10^{-15}$
10000 s	$\leq 3.0 \cdot 10^{-15}$
1 day	$\leq 2.0 \cdot 10^{-15}$
Flicker floor	$\leq 2.0 \cdot 10^{-15}$ (@ 10 days)



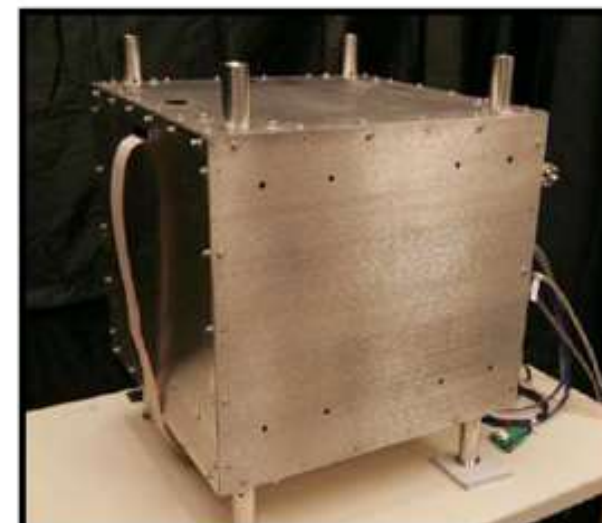
Stanford Gravimeter (non-mobile)
Achieved Accuracy: $4 \cdot 10^{-9} \text{ g}$ (?)



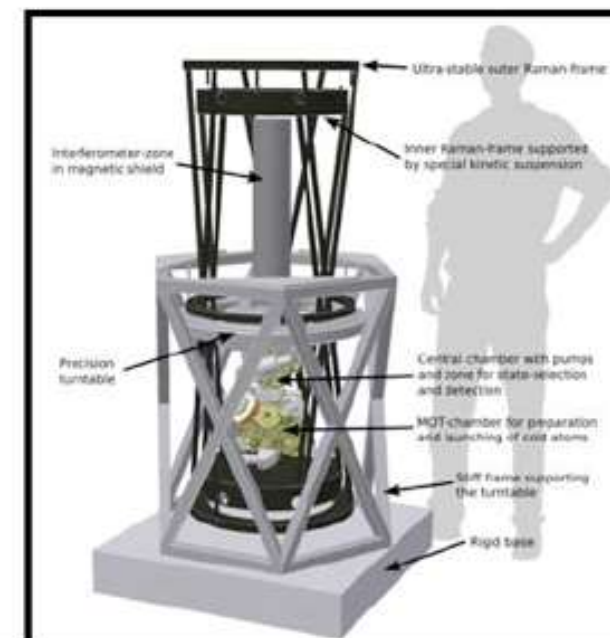
Paris Gravimeter („mobile“)
Achieved Accuracy: $1.4 \cdot 10^{-8} \text{ g}$



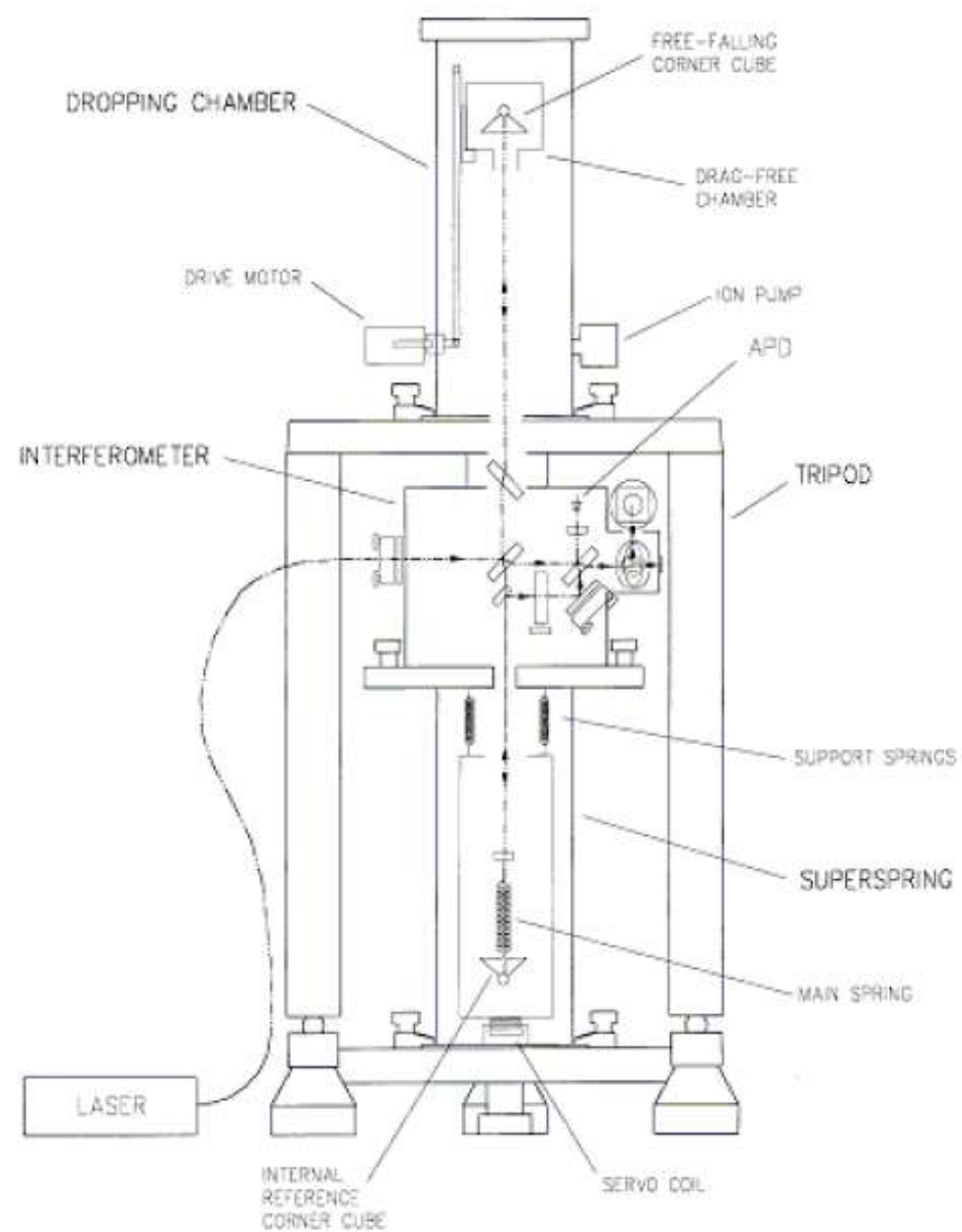
Florenz INFN Gravity Gradiometer MAGIA
Measurement of the gravitational constant G
Targeted Accuracy: $\Delta G/G = 1 \cdot 10^{-4}$

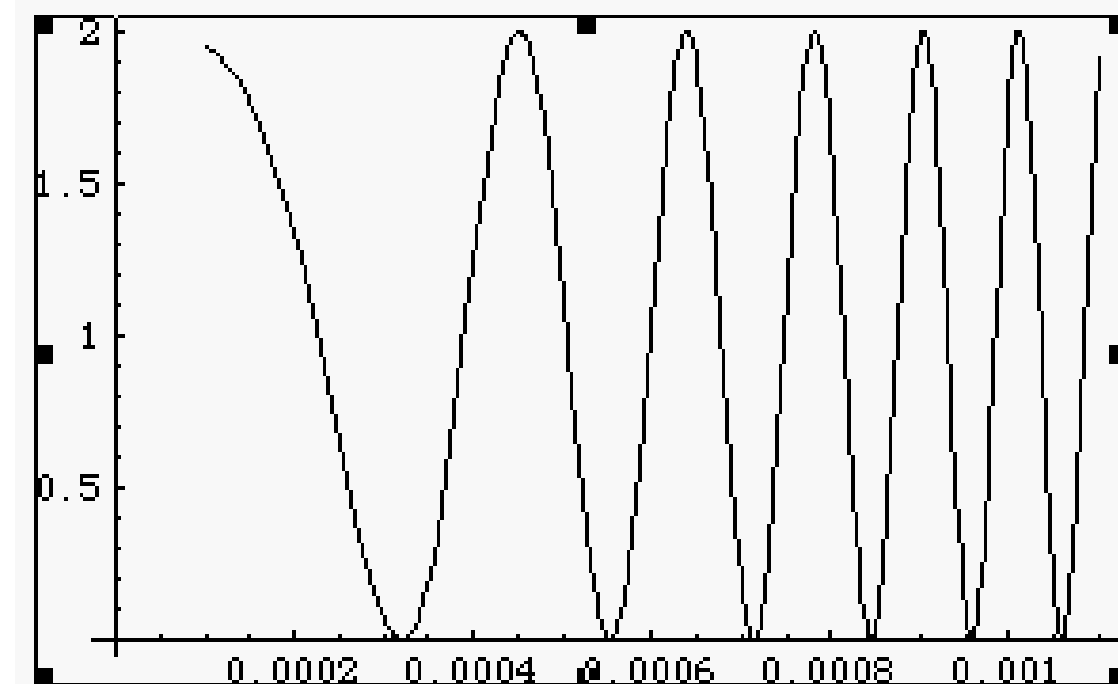
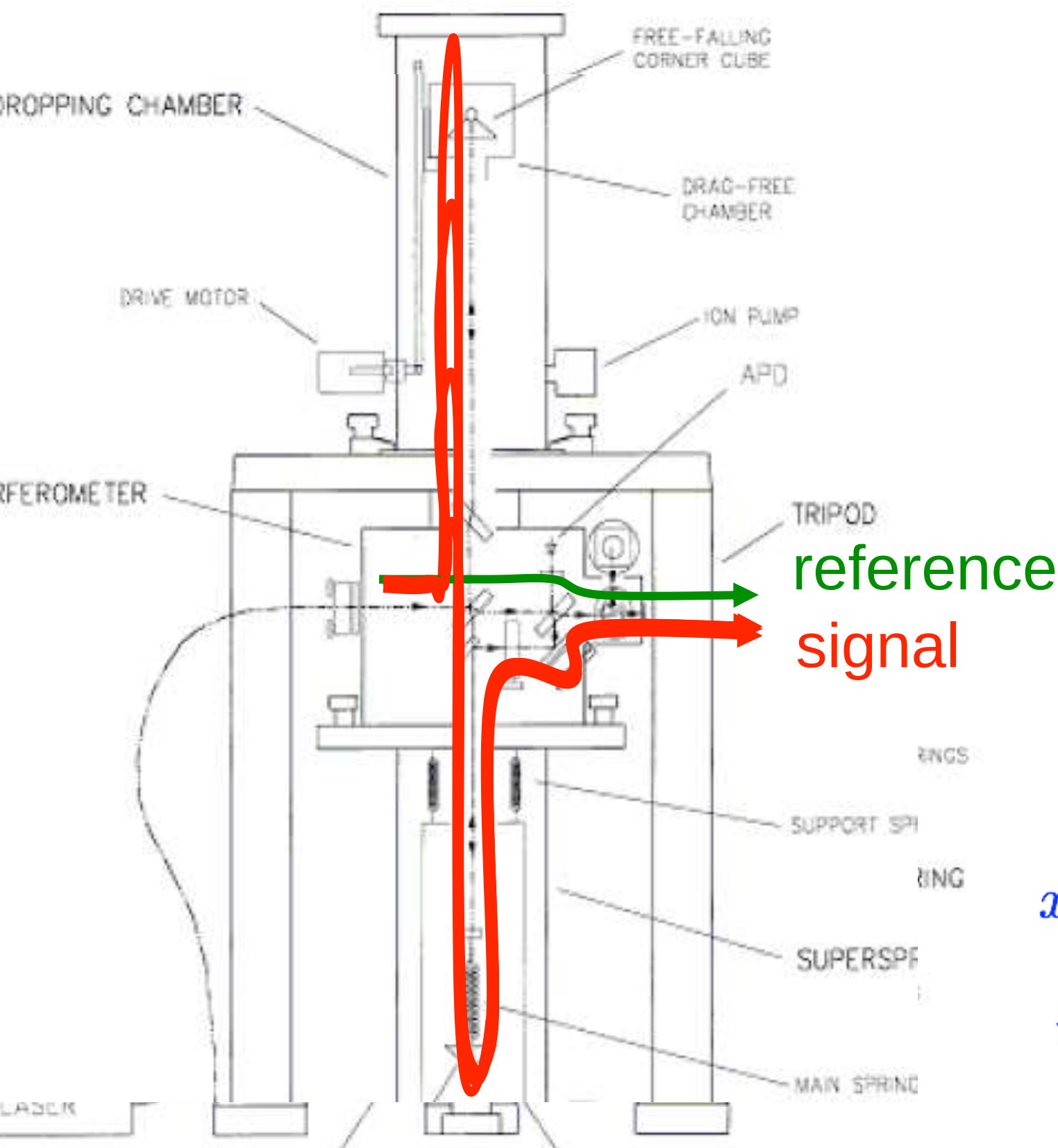


Kasevich Gravimeter (mobile)
Bias Stability: $< 10^{-10} \text{ g}$



Berlin Gravimeter GAIN
(mobile, under construction)
Targeted Accuracy: $5 \cdot 10^{-10} \text{ g}$

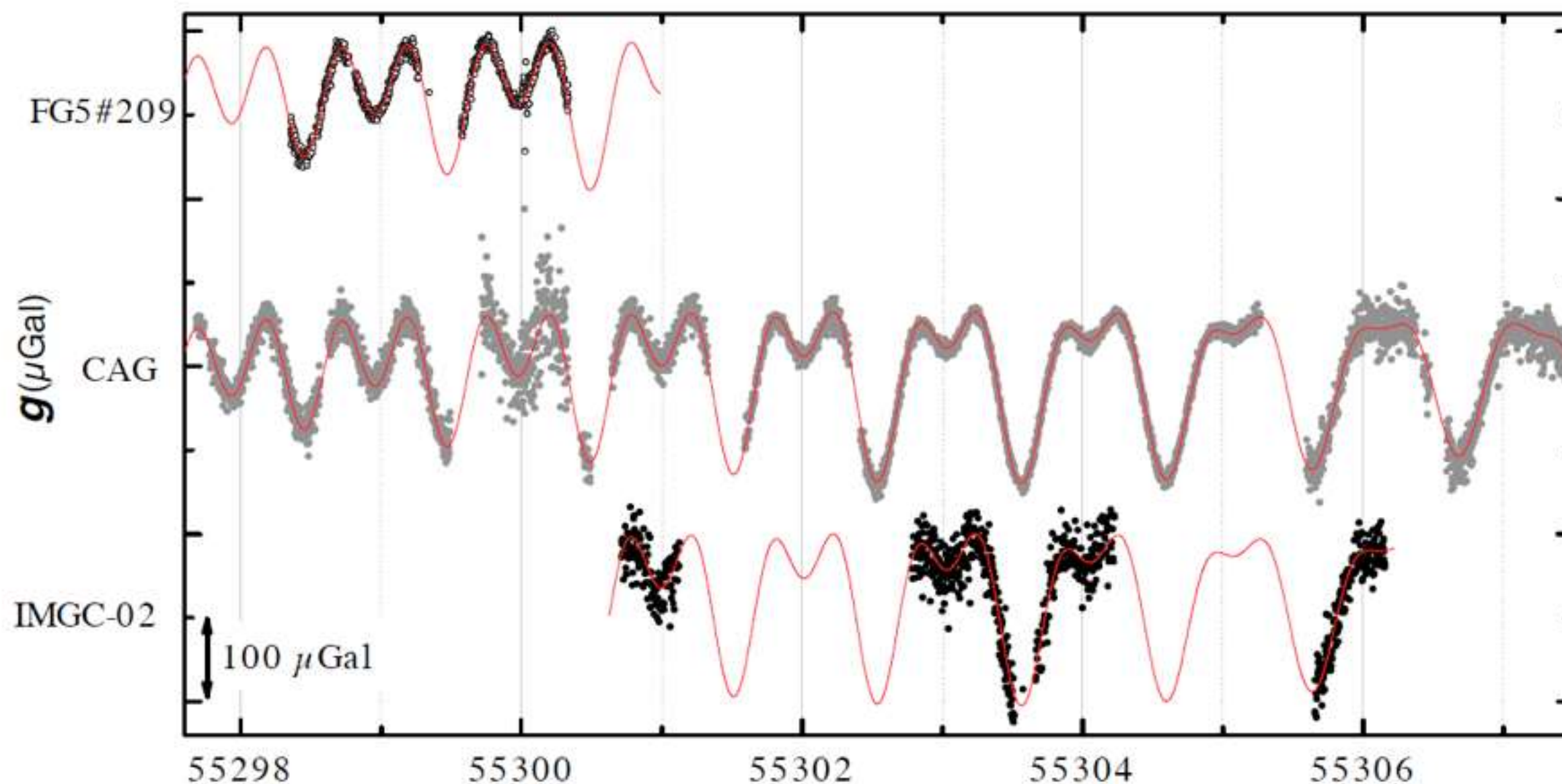




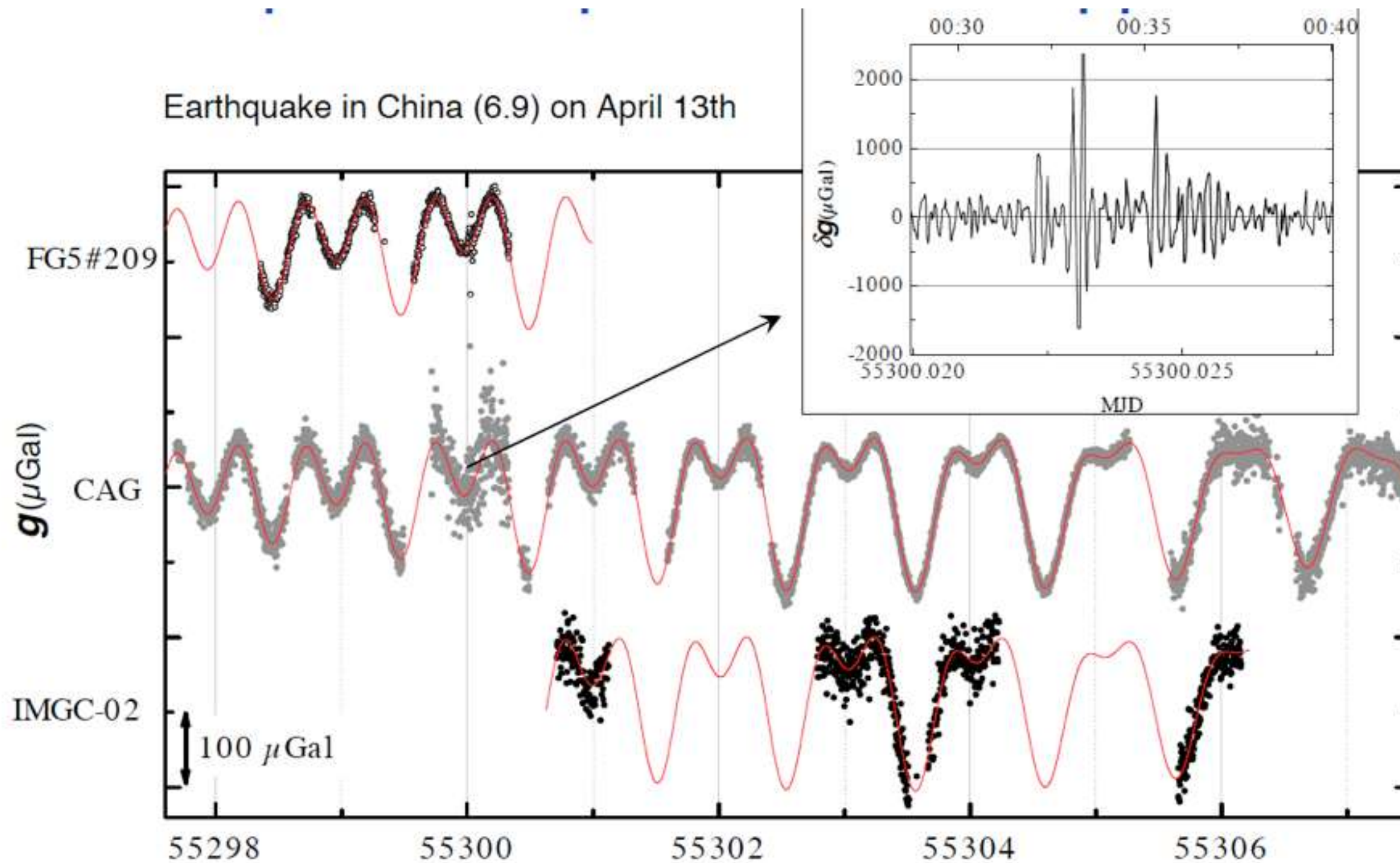
$$x_i = x_0 + v_0 \tilde{t}_i + \frac{g_0}{2} \left(\tilde{t}_i^2 + \frac{\gamma \tilde{t}_i^4}{12} \right)$$

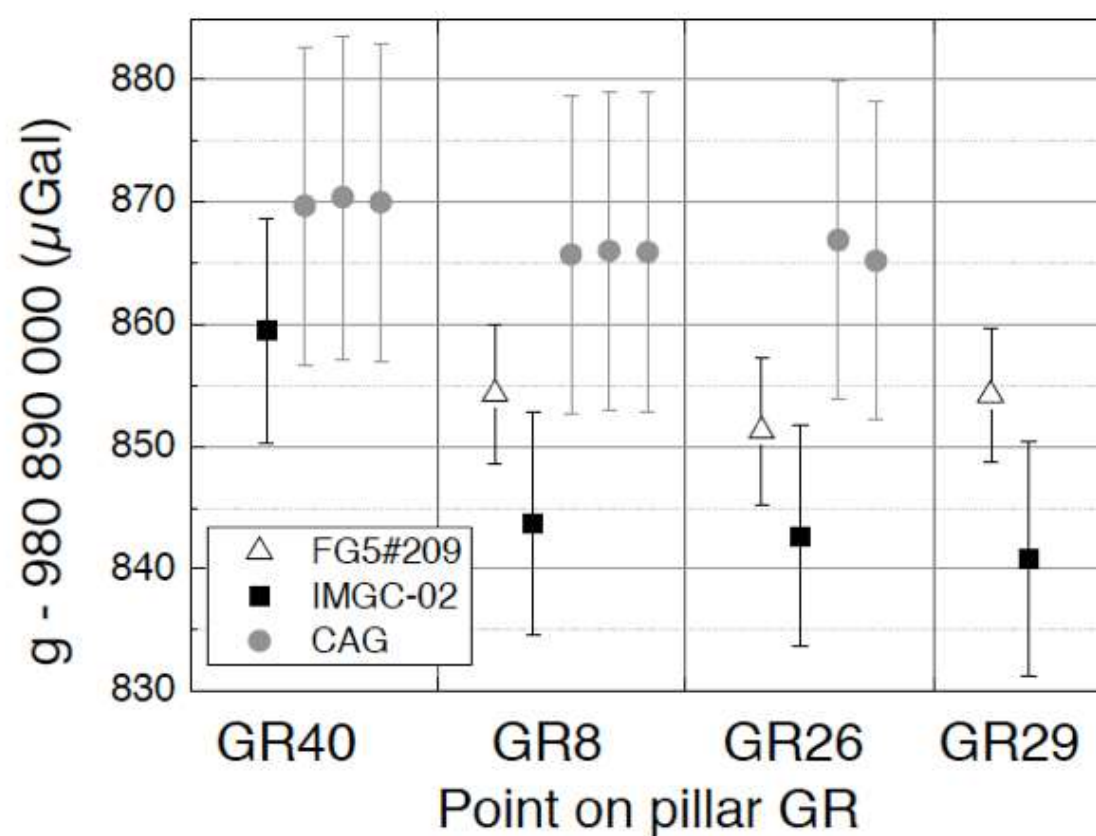
$$\tilde{t}_i = t_i - \frac{x_i - x_0}{c}$$

Done in Trappe, near Paris, with the gravimeter developed at Observatoire de Paris



Earthquake in China (6.9) on April 13th



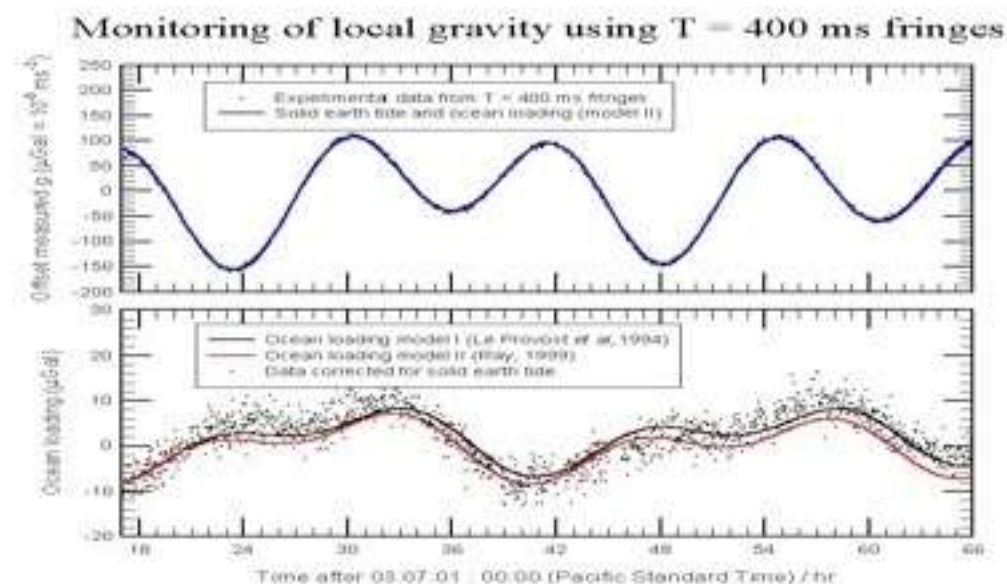


→ Reproducible differences

→ Limit of agreement

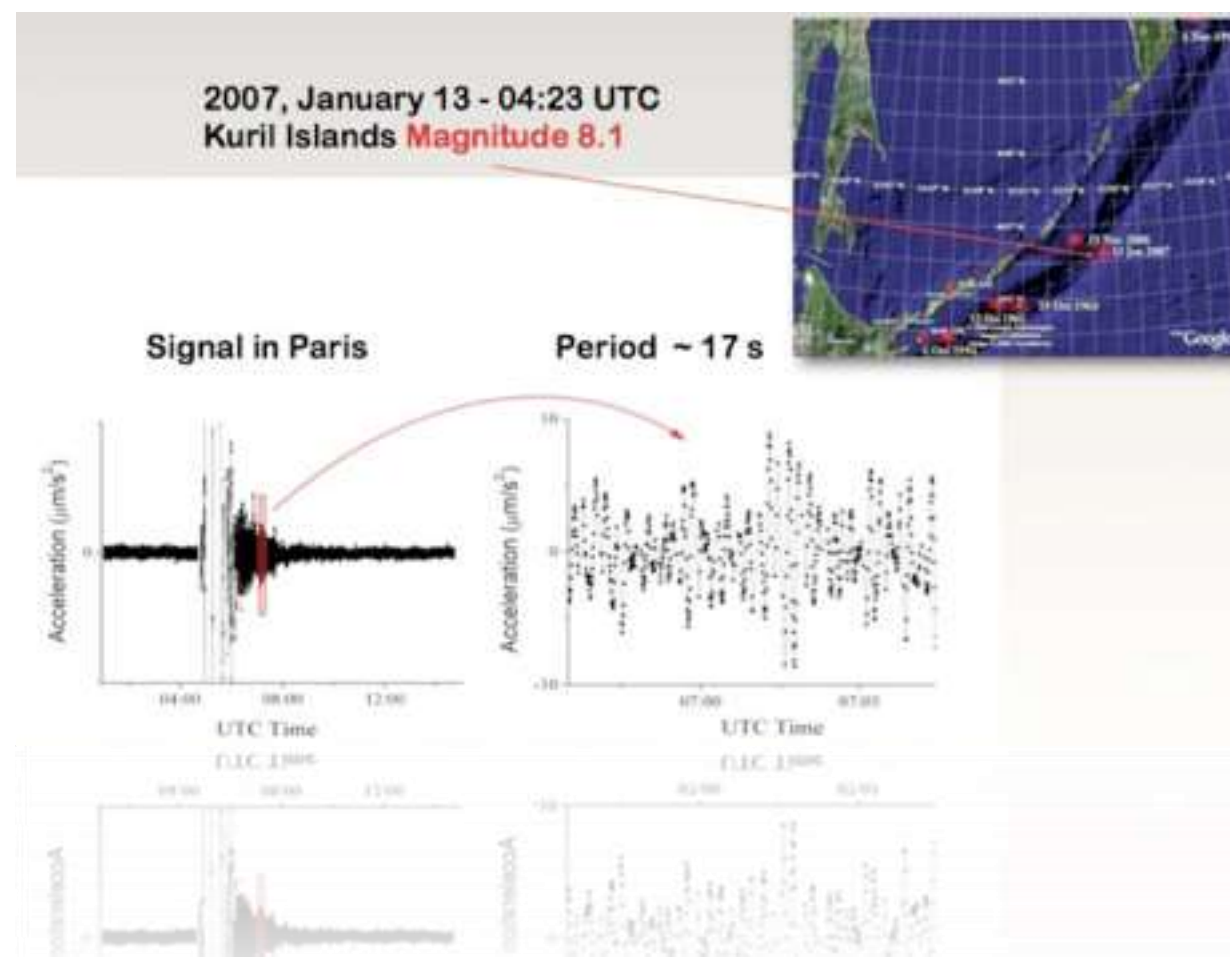
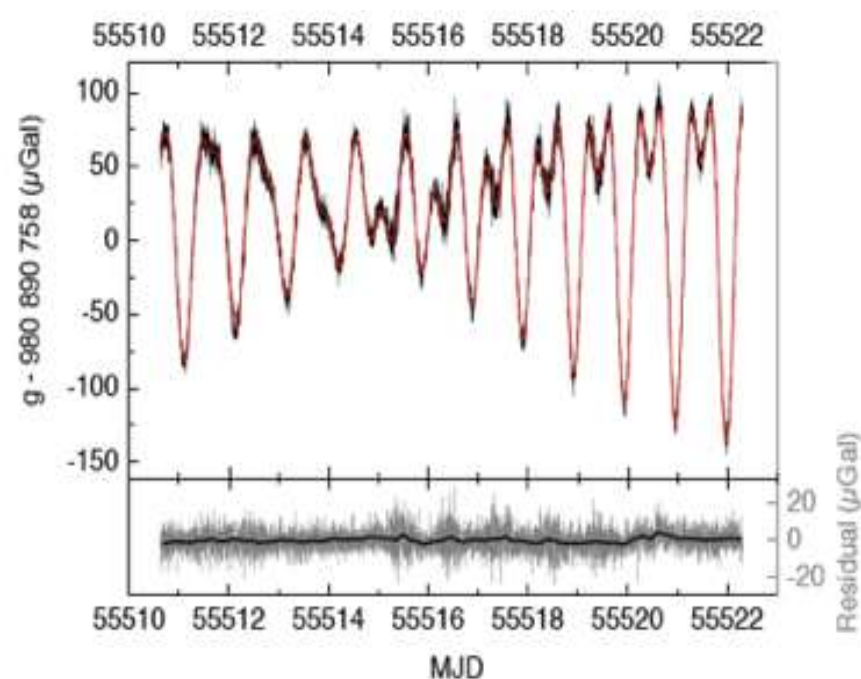
→ Some systematics not well controlled

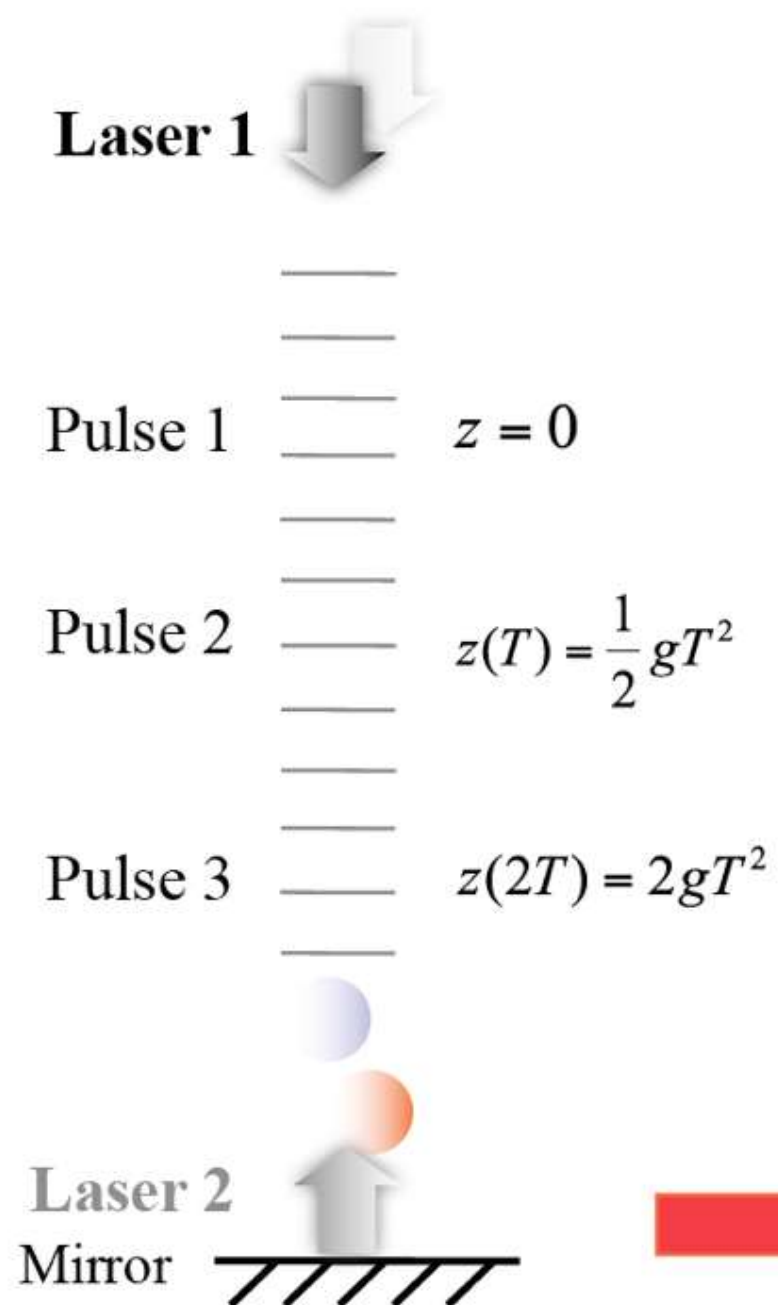
	Accuracy (μGal)	s_{gm} (μGal)	U (k=2) (μGal)
FG5#209	2.0	1.1 – 1.7	5.4 – 5.9
IMGC-02	4.1	1.2 – 2.0	9.0 – 9.5
CAG	6.5	0.2 – 1.2	13.0 – 13.2



✓ In laboratories, atom gravimeters beat the best commercial gravimeters

- ⇒ Seismic wave detections
- ⇒ Gravity monitoring
- ⇒ Watt balance : quantum definition of the kilogram

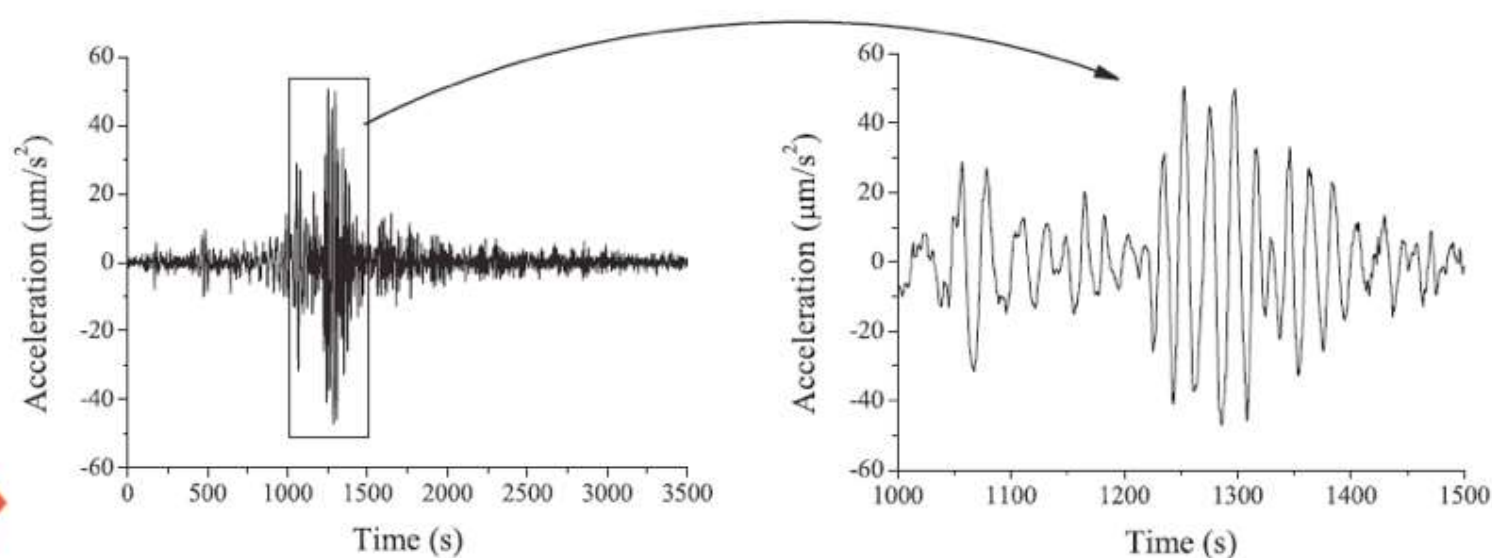




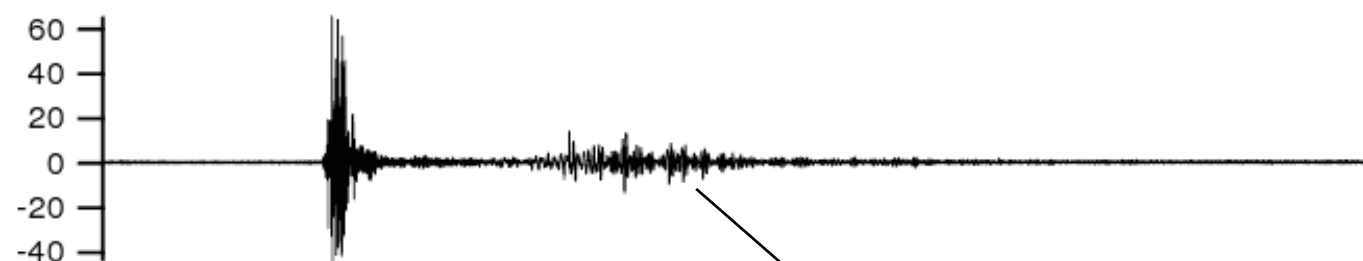
Vertical interferometer
 $\Rightarrow$ Free fall along the equiphase planes

Sensitivity to gravity acceleration :

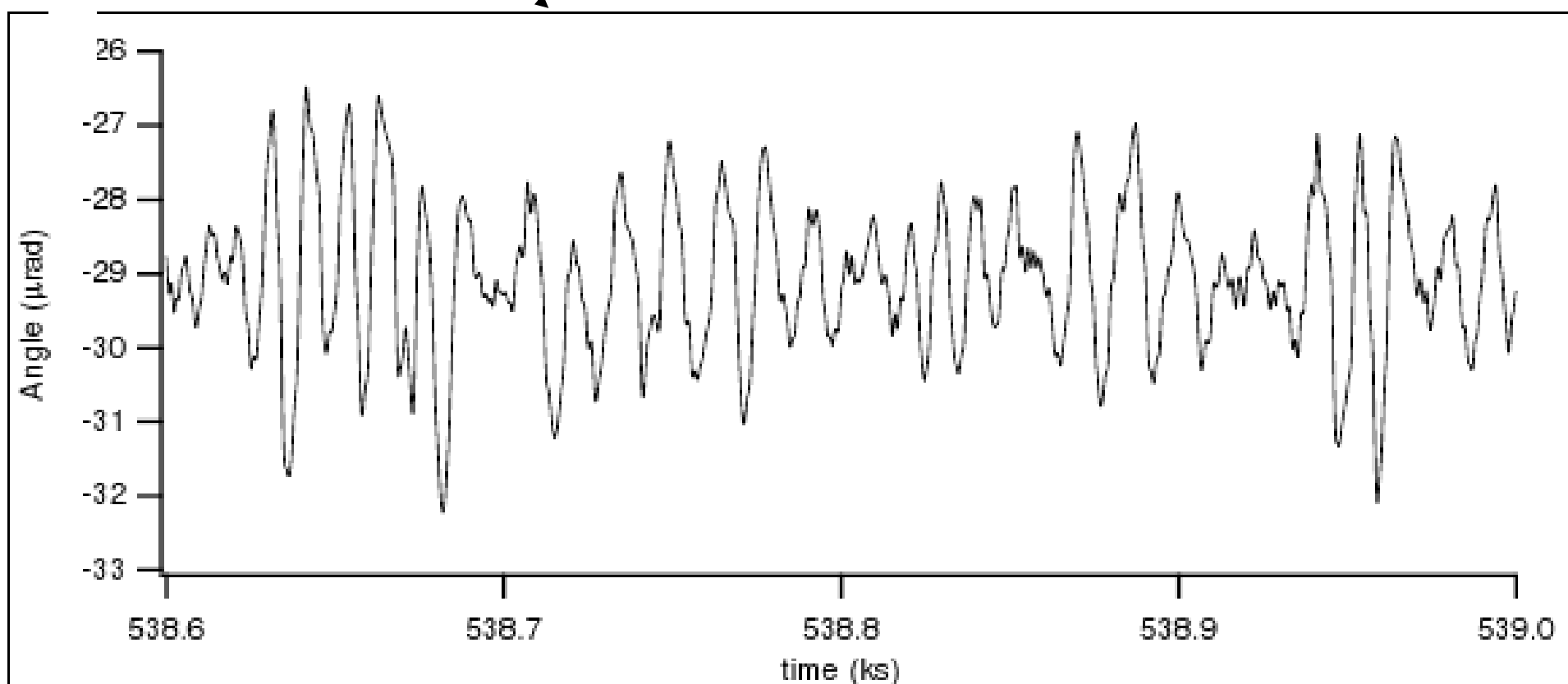
$$\Delta\Phi_{\text{int}} = -\vec{k}_{\text{eff}} \vec{g} T^2 + \delta\Phi_{\text{noise}} + \delta\Phi_{\text{sys}}$$



Earthquake in China 2 Mars 20th 2008

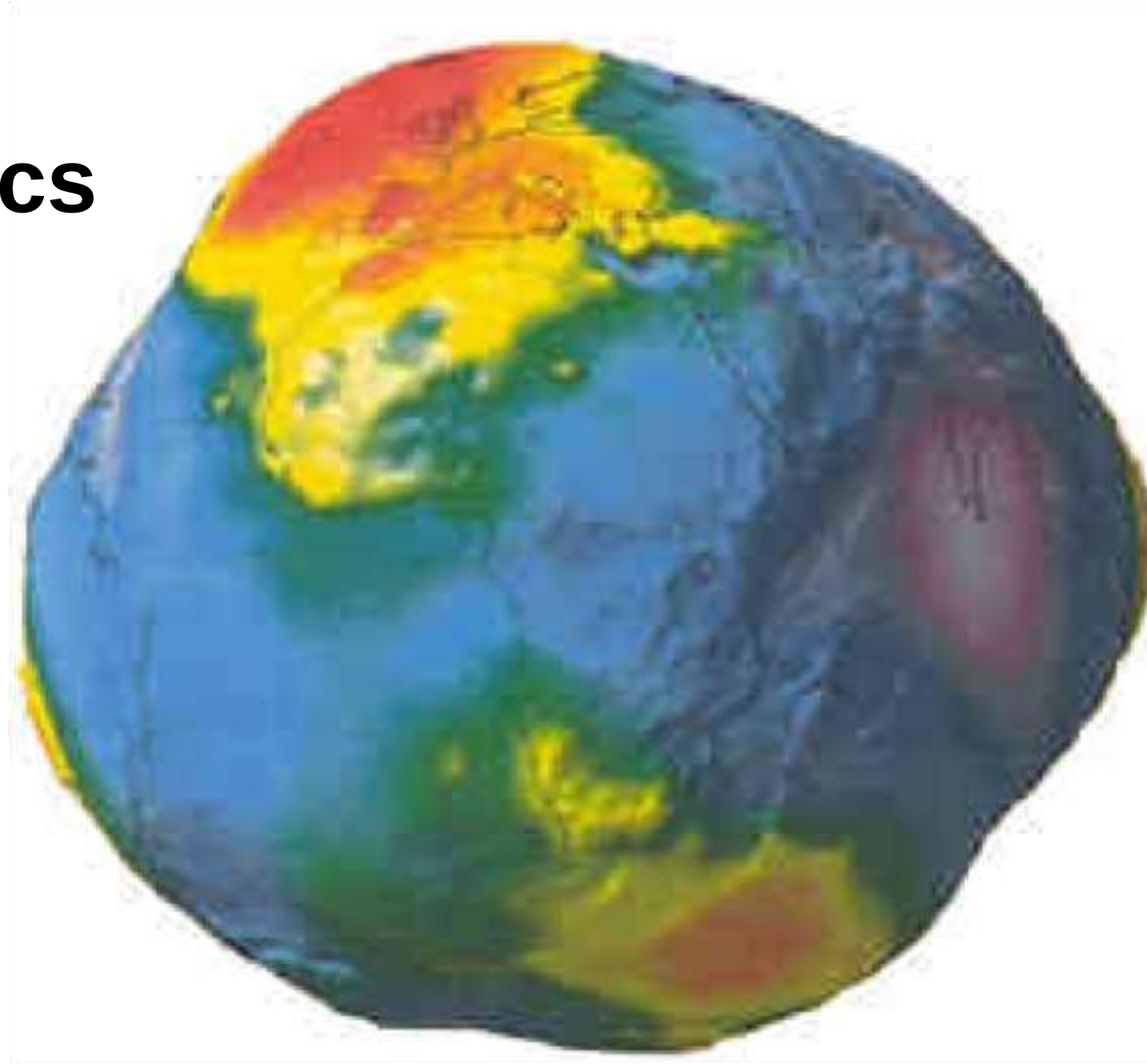


Honduras/offshore 7.3
+30 min



Gyroscope output necessary to disambiguate tilt from horizontal motion (navigation problem).

Geophysics



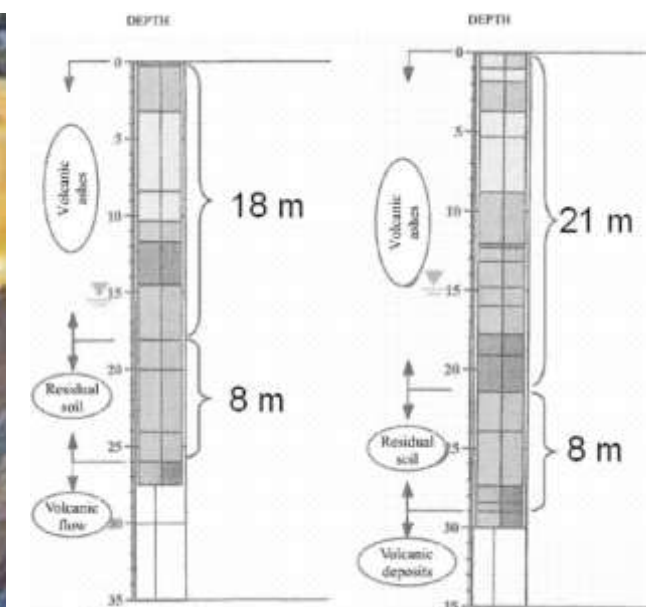
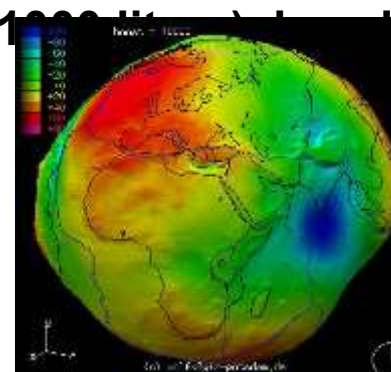
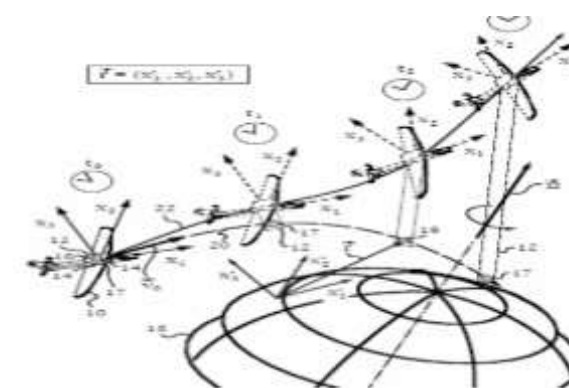
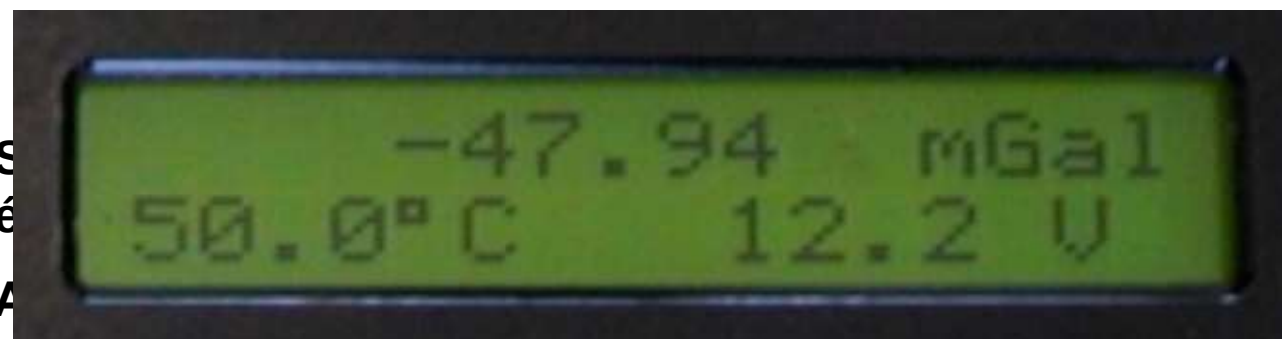
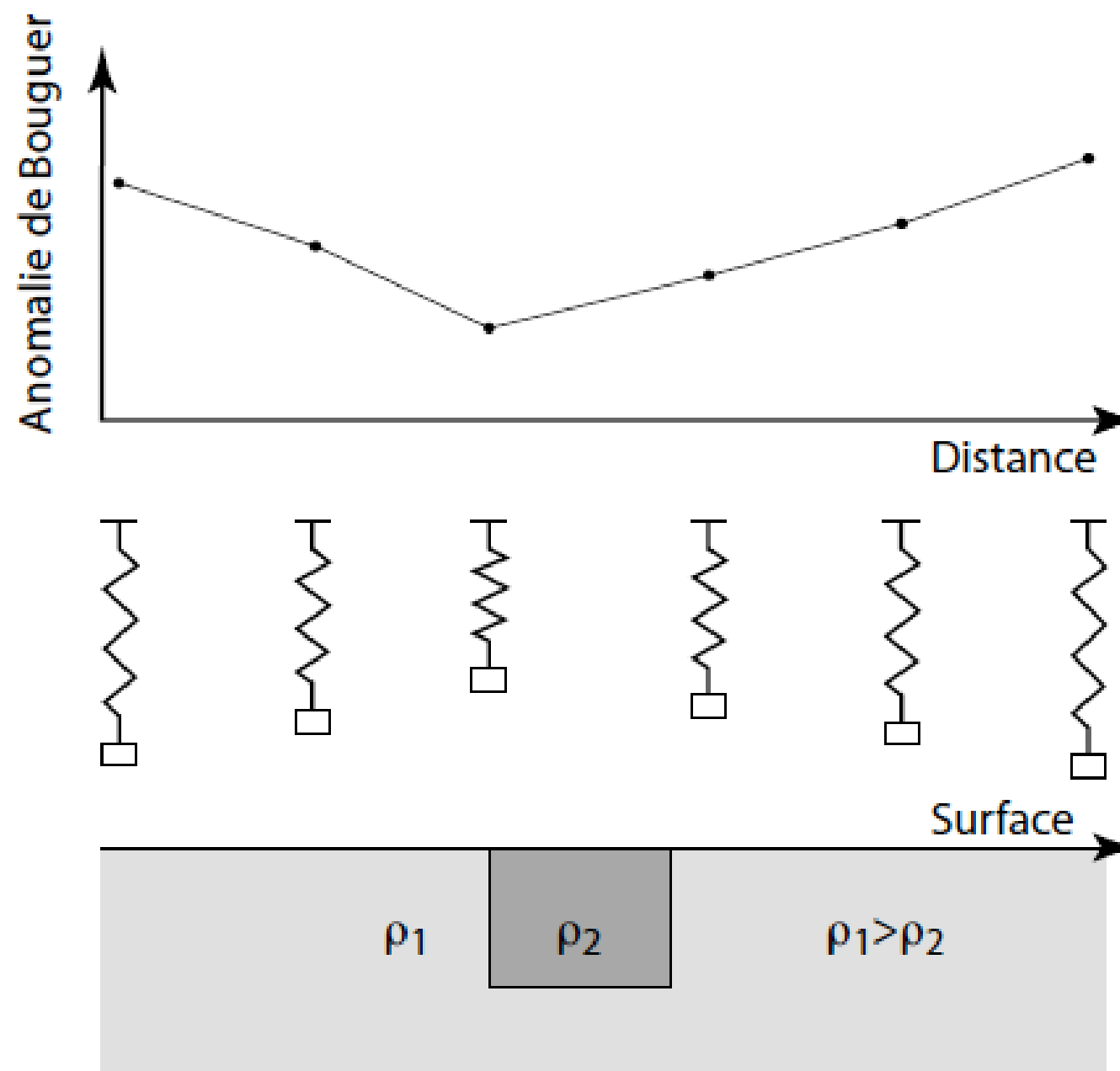
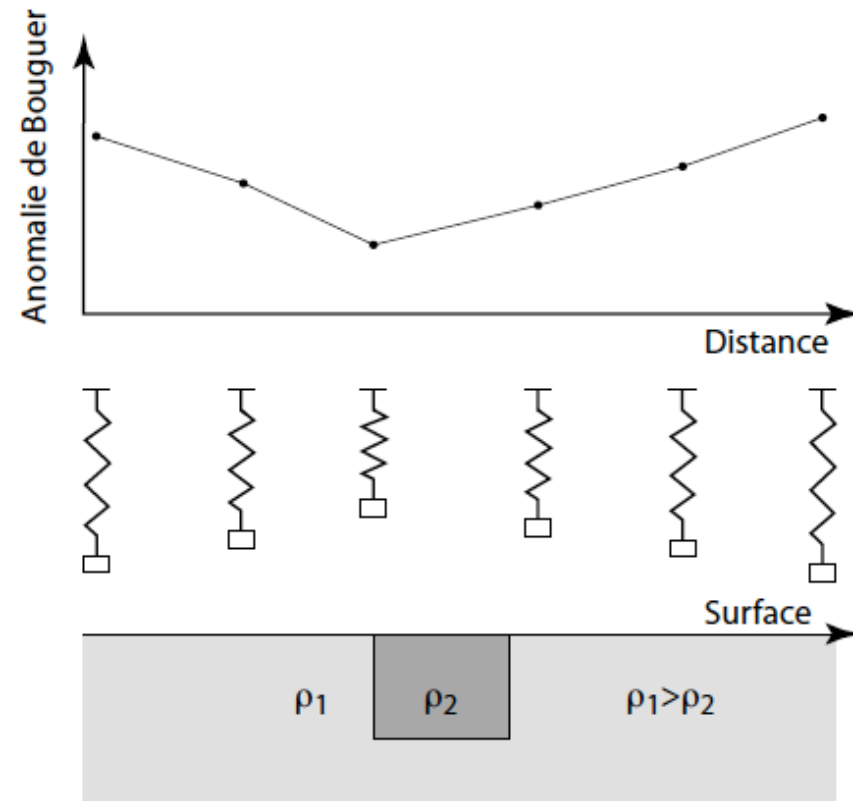


Figure 2. Boreholes P4 and P6 corresponding to Puerto Espejo and University of Qandio.

- ✓ Deduce from the variations of gravity locally the density variations the ground





✓ Small variations

✓ nanogravi : 10^{-9} g with $g \sim 9.87 \text{ m/s}^2 = 987 \text{ Gal}$

✓ microgravi : 10^{-6} g, $1 \mu\text{Gal} = 10^{-6} \text{ Gal}$

✓ Looking for small changes 10 - 100 μGal

✓ Variation of g with height : 3 $\mu\text{Gal/cm}$

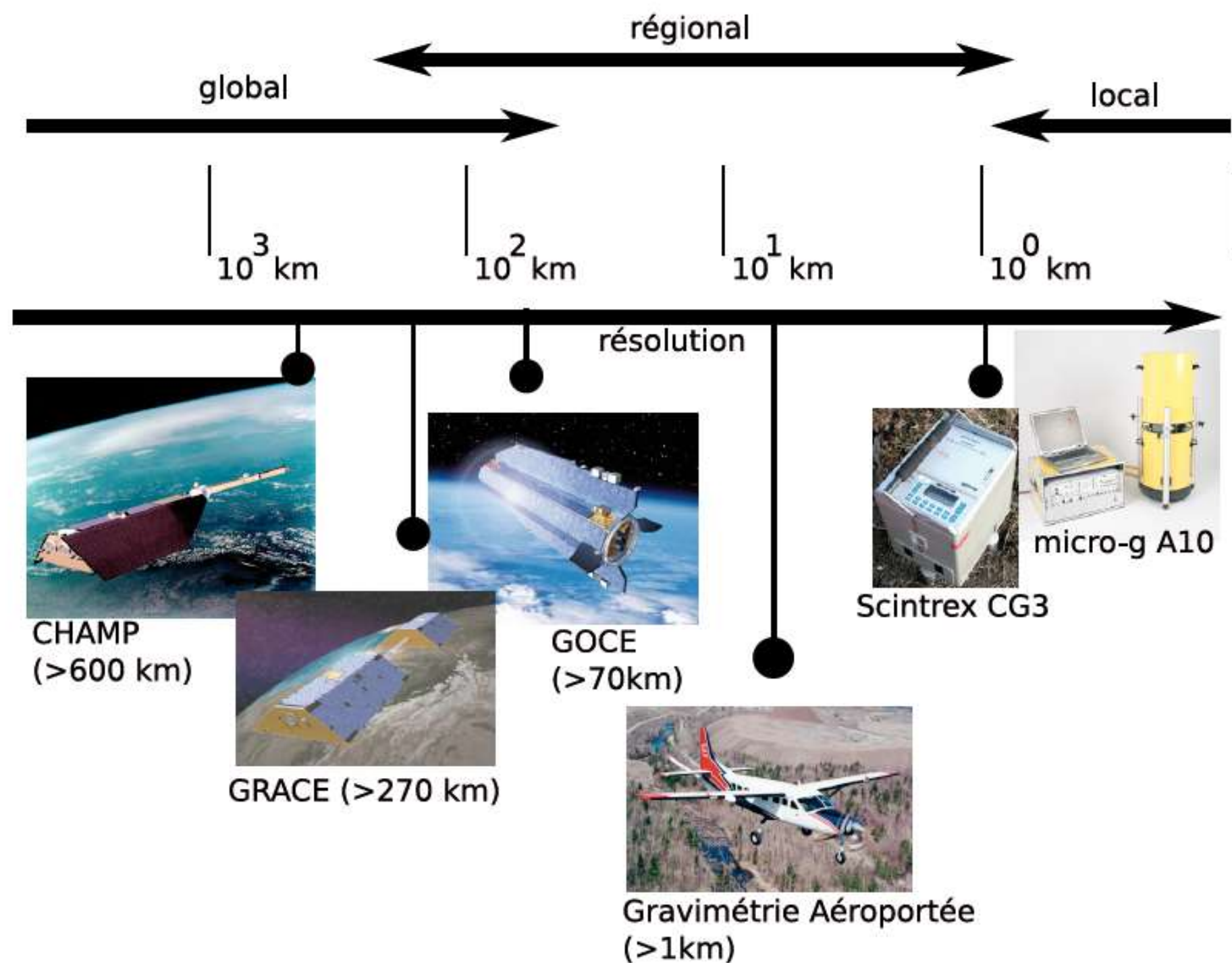
TABLE 1: Field of application, accuracy target, type of instruments and their availability.

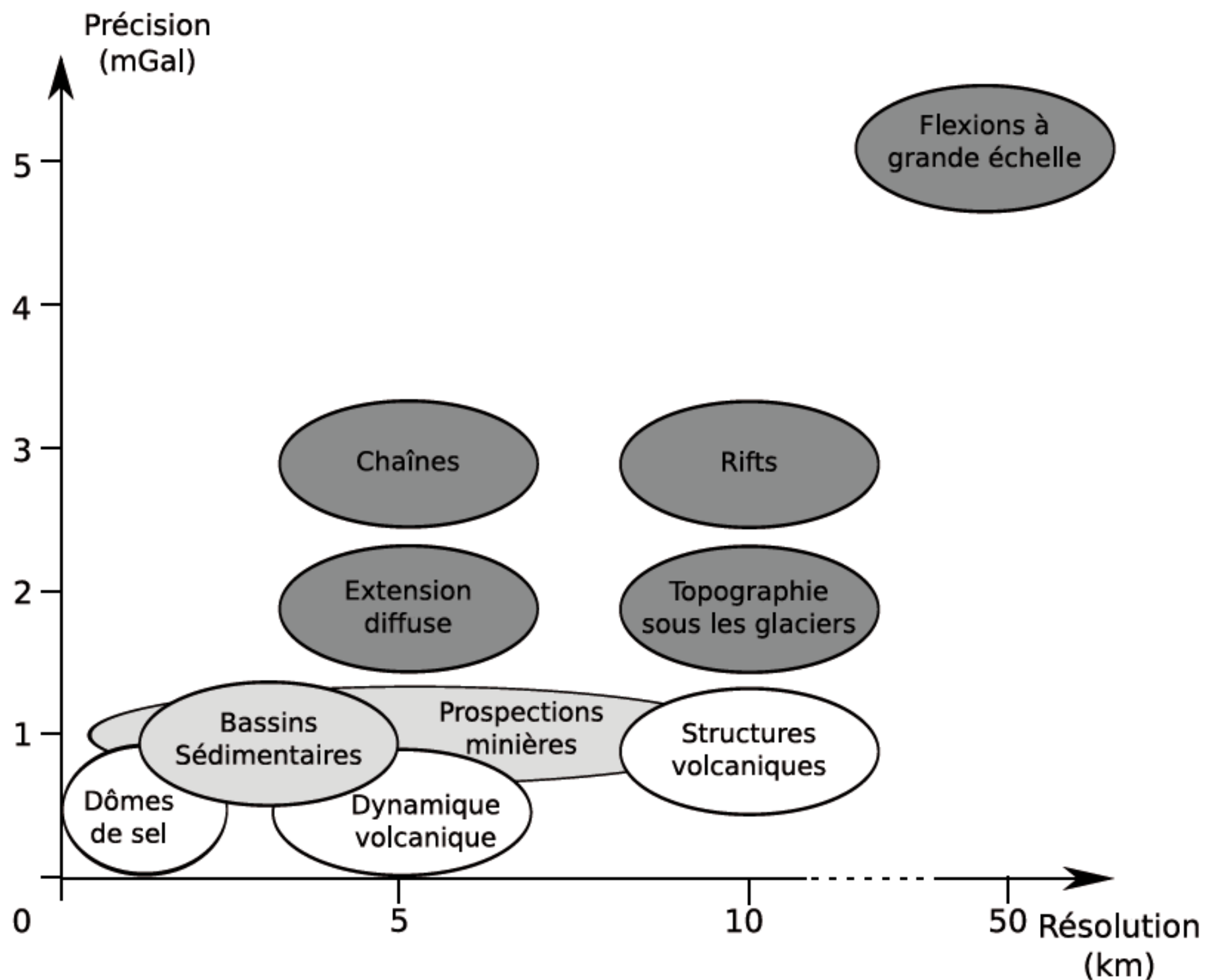
Application	Accuracy target	Instruments/availability
Isostasy	10^{-5}	Relative gravimeters (pendulum)/1817
Prospection for oil and gas	10^{-6} — 10^{-7}	Relative gravimeters/1950
Structural geology	10^{-6}	Relative gravimeters/1950
Microgravity prospection	10^{-8}	Relative gravimeters/1980
Metrology (forces)	10^{-6} — 10^{-7}	Absolute gravimeters/1980
Metrology (kilogram)	10^{-8} — 10^{-9}	Absolute gravimeters/1990
Geodetic metrology	10^{-8}	Absolute gravimeters/1980
Geodesy	10^{-7} — 10^{-8}	Rel + Abs gravimeters/1980
Earth tide and earth dynamic	10^{-10} — 10^{-11}	Superconducting gravimeters/1970

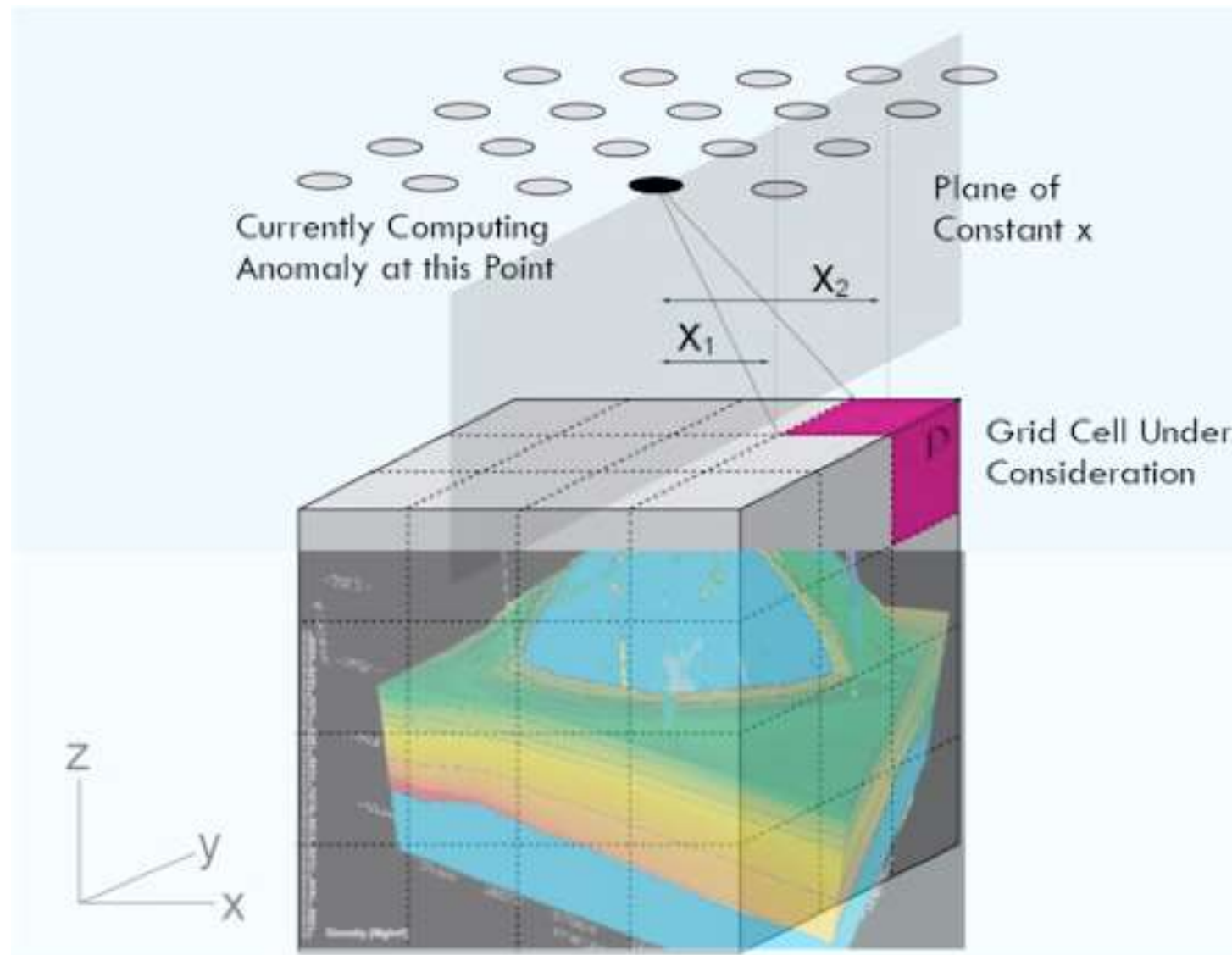
TABLE 2: Progress in measuring g (partially compiled after Torge 1989, page 14, figure 1.1 [1])

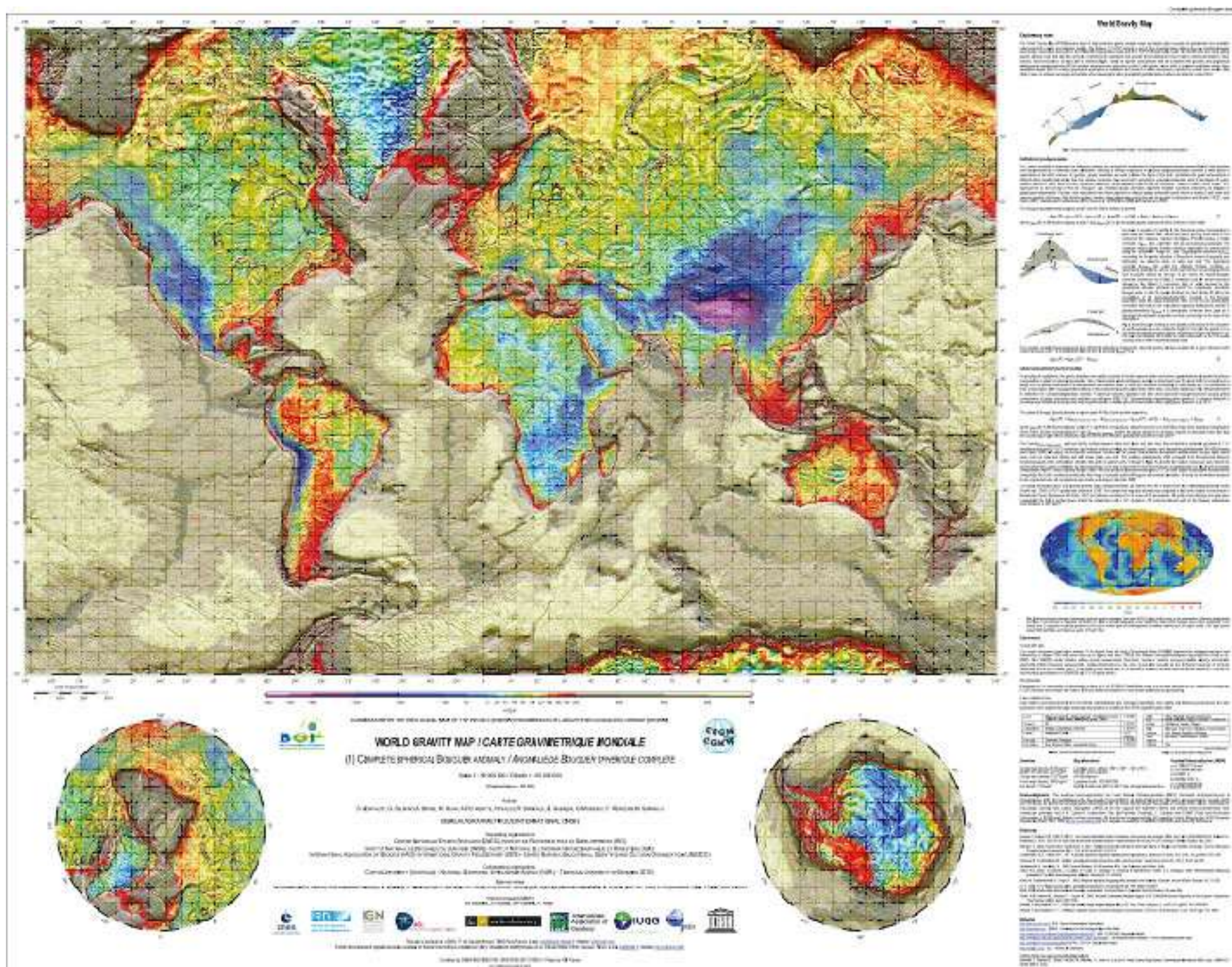
Type of instrument	Time span	Uncertainty range	Rate
Reversible pendulum	1817 to 1945	4×10^{-5} to 2×10^{-6}	One order of magnitude/130 years
Spring type gravimeters	1935 to 2010	4×10^{-7} to 3×10^{-9}	Two orders of magnitude/75 years
Superconducting gravimeters	1965 to 2010	5×10^{-10} to 5×10^{-12}	Two orders of magnitude/45 years
Absolute gravimeters	1970 to 2010	8×10^{-7} to 2×10^{-9}	Two orders of magnitude/40 years

RESOLUTION VS HEIGHT OF MEASUREMENT

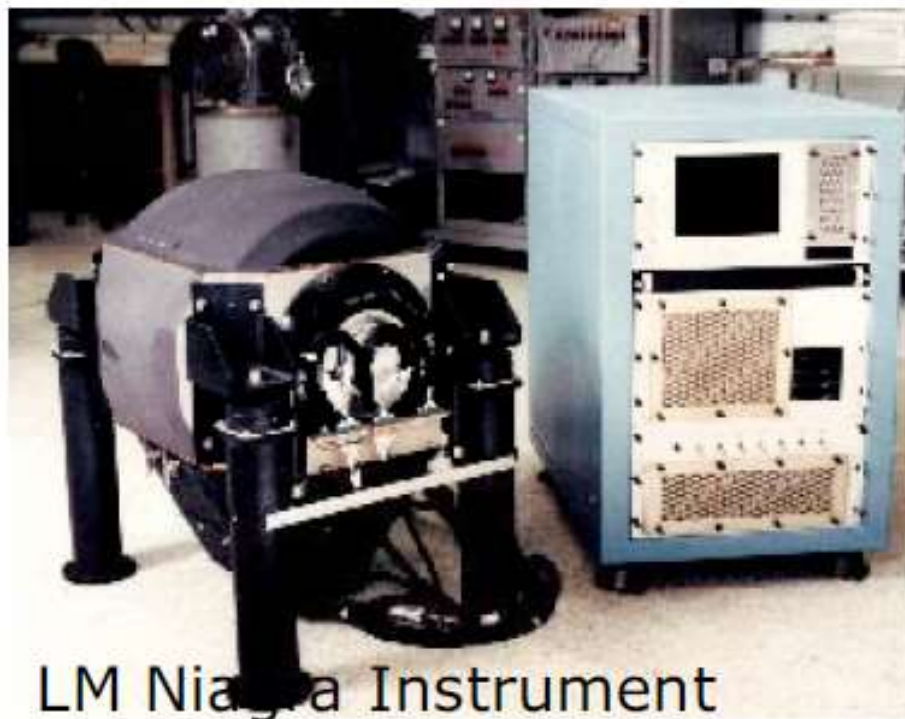




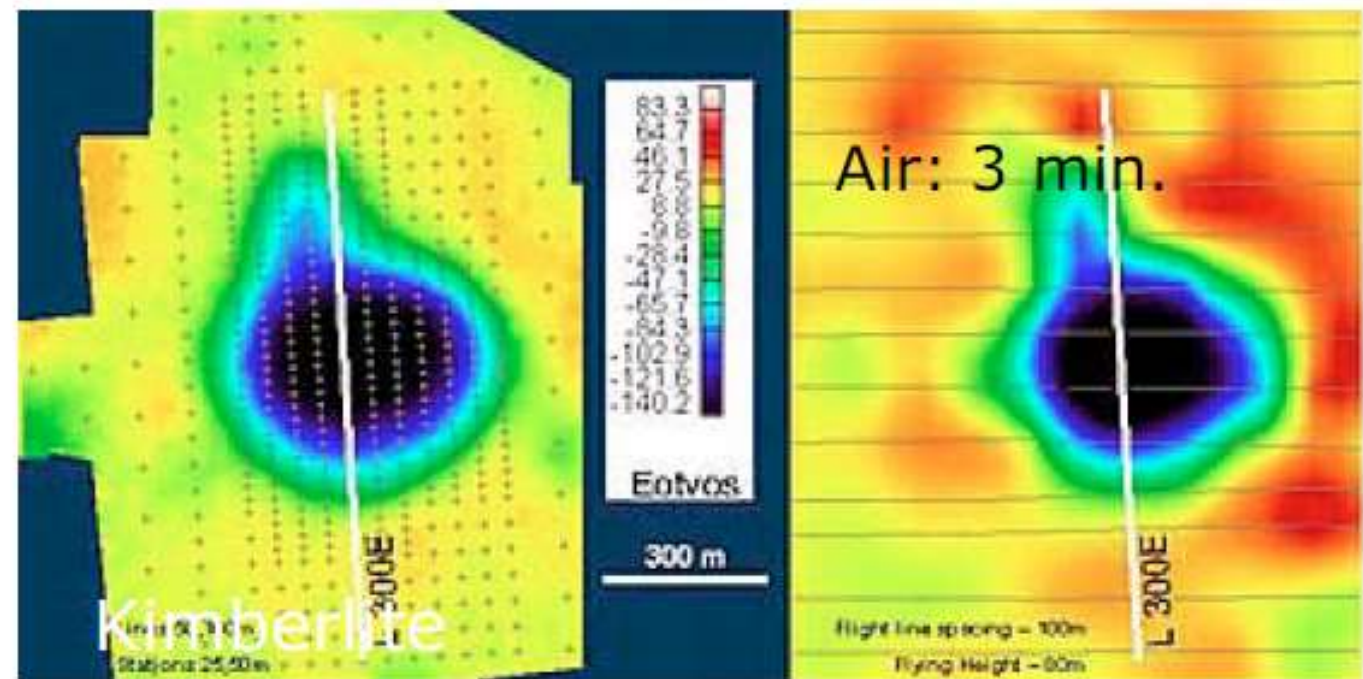




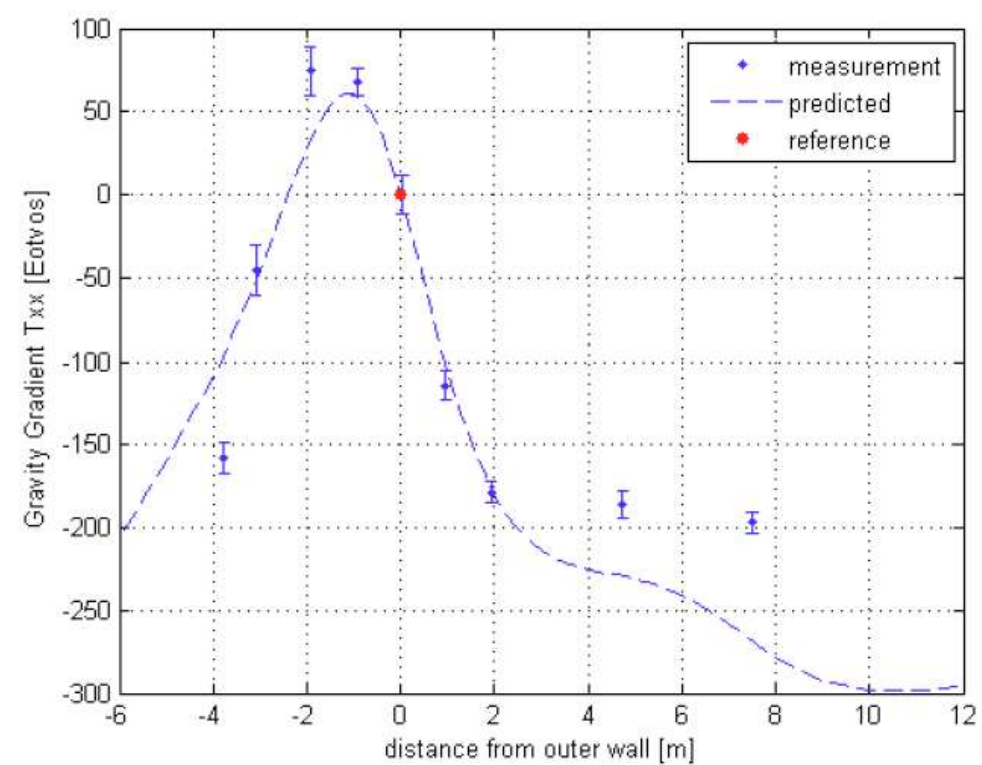
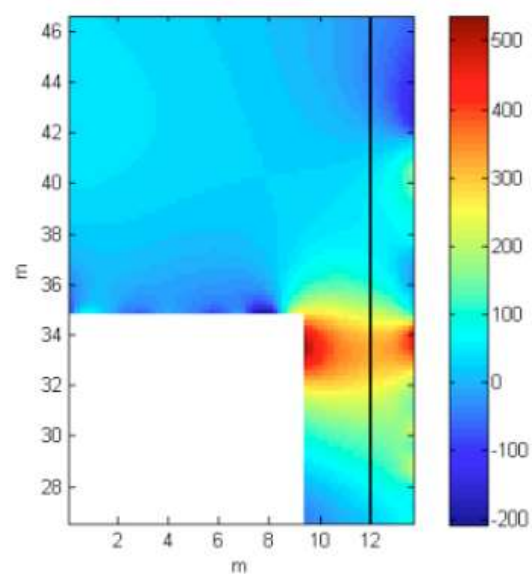
Existing technology

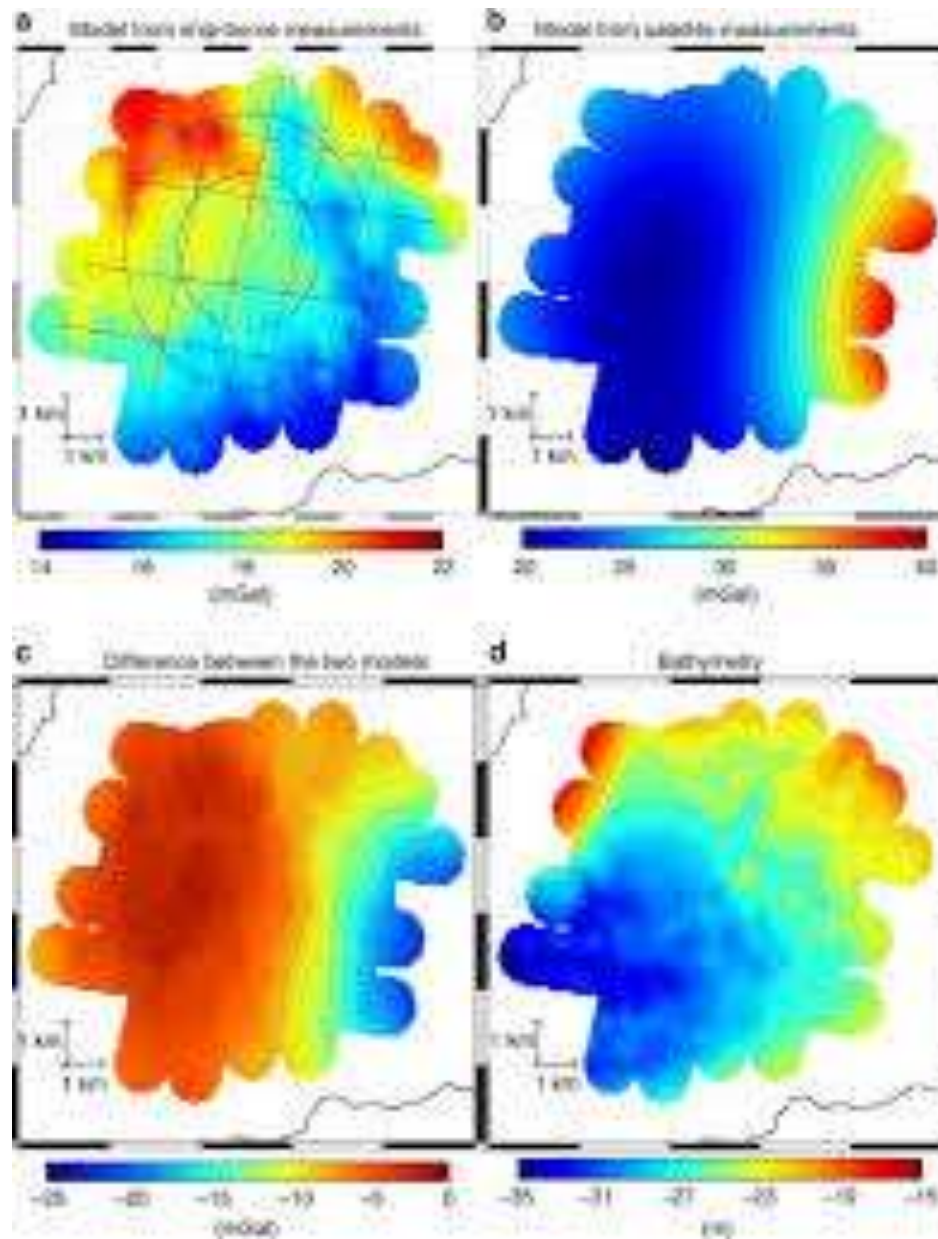


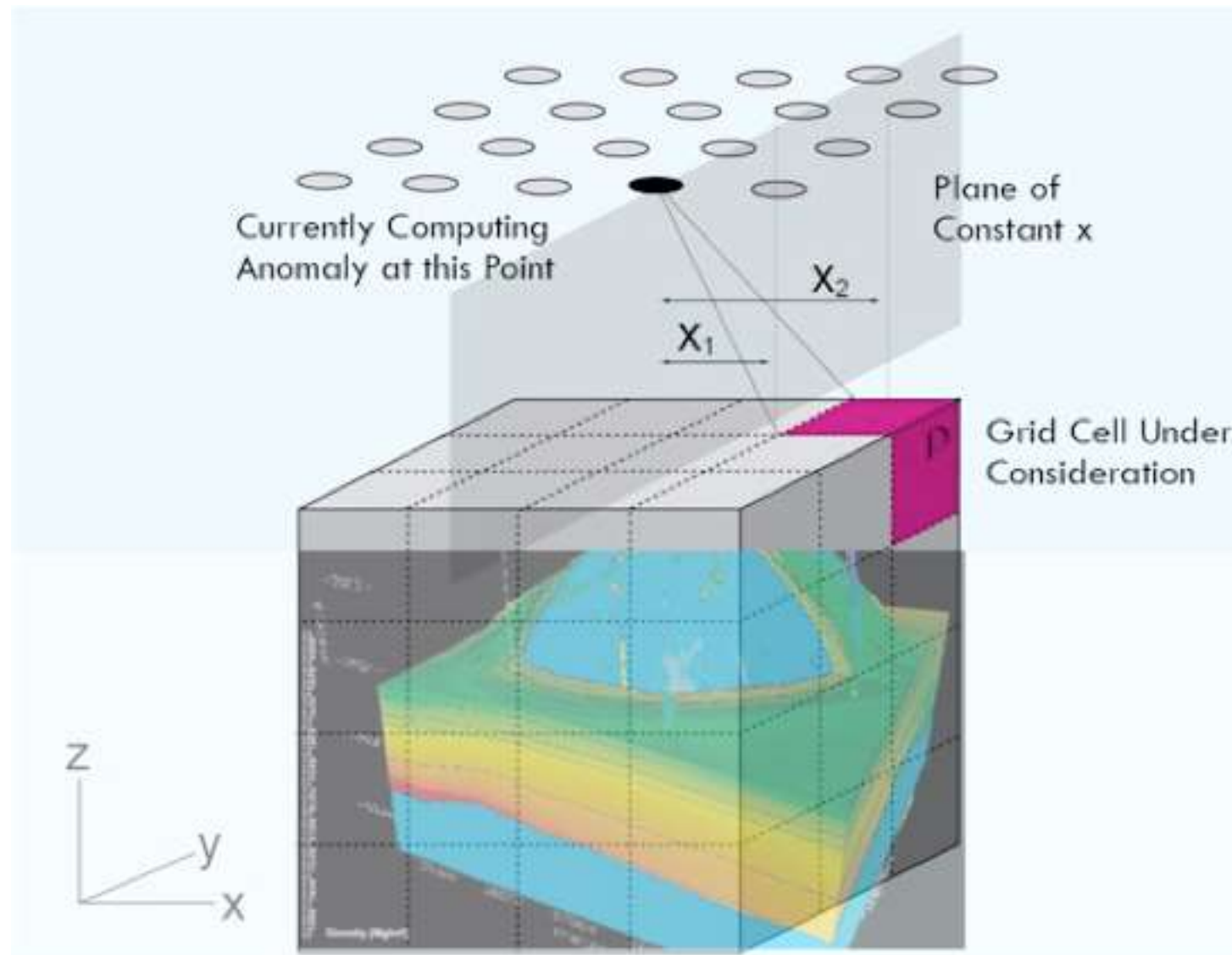
Land: 3 wks.

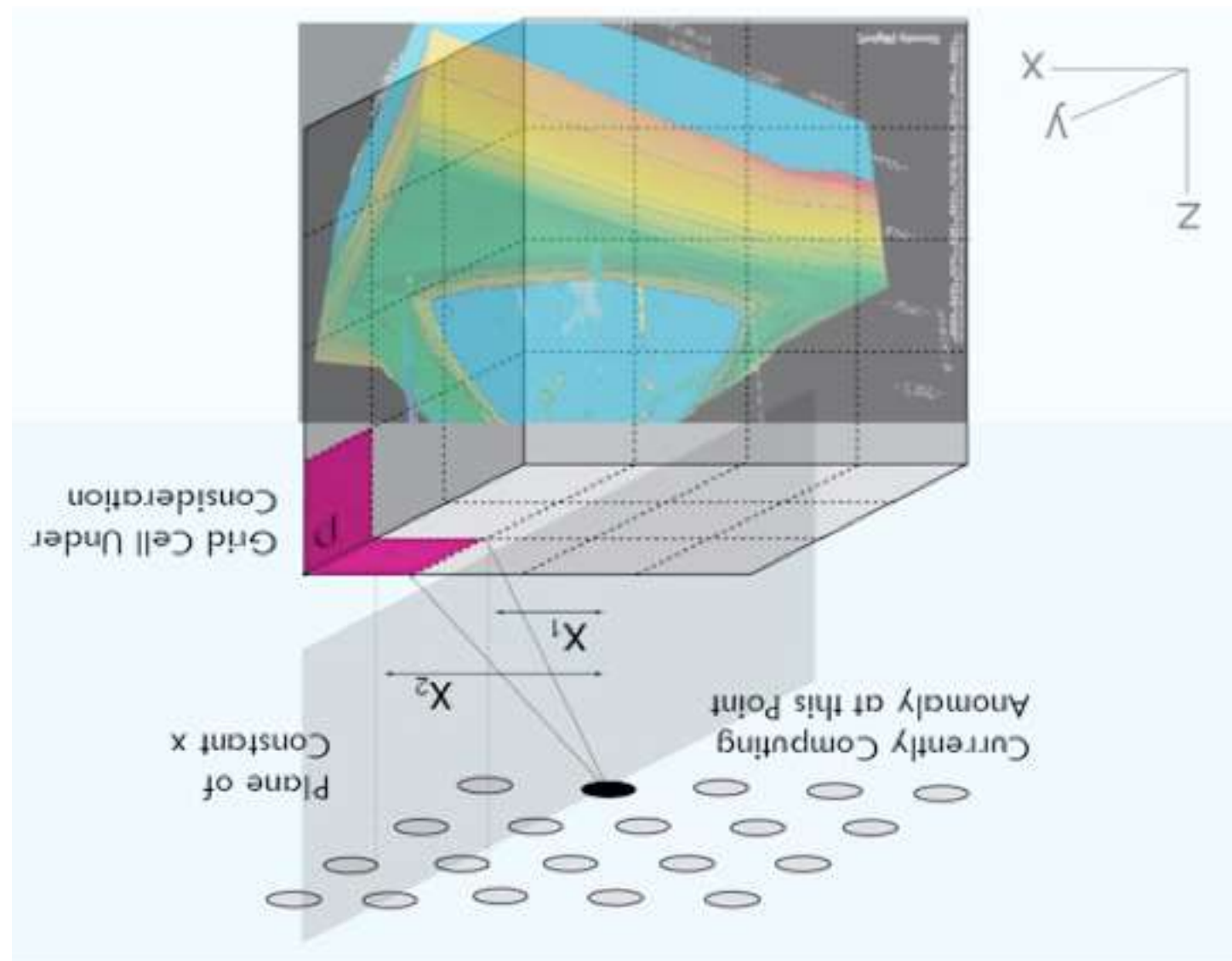


AI sensors potentially offer 10 x – 100 x improvement in detection sensitivity at reduced instrument costs.



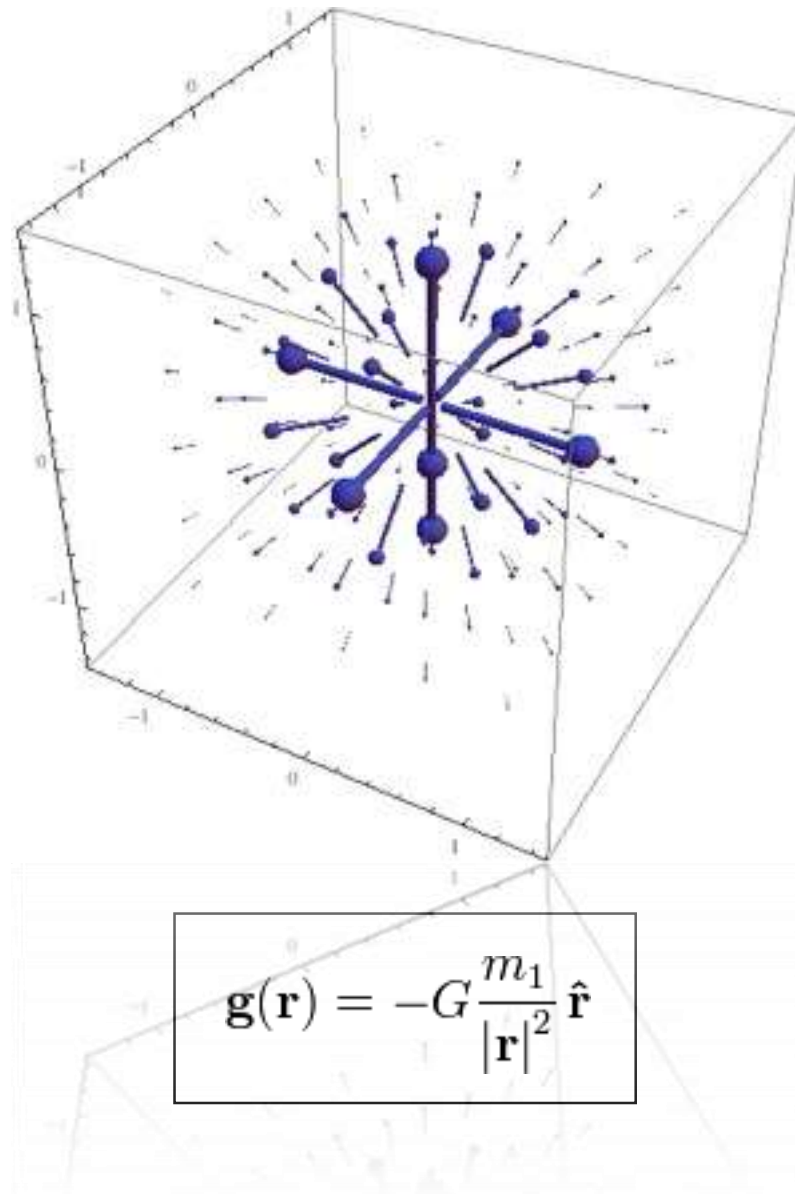




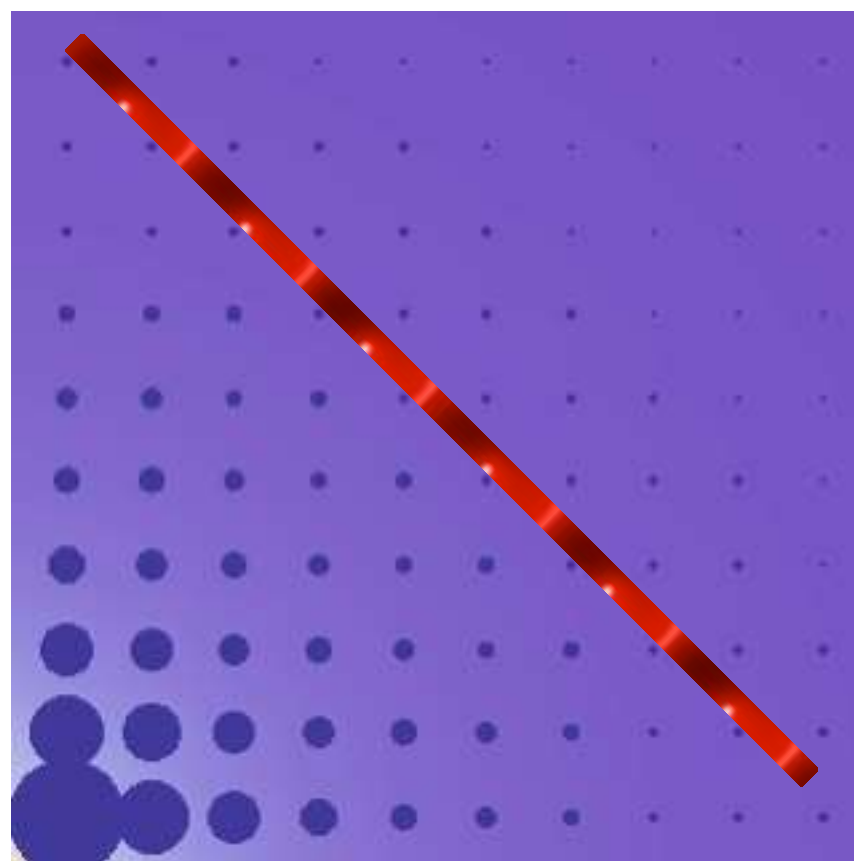
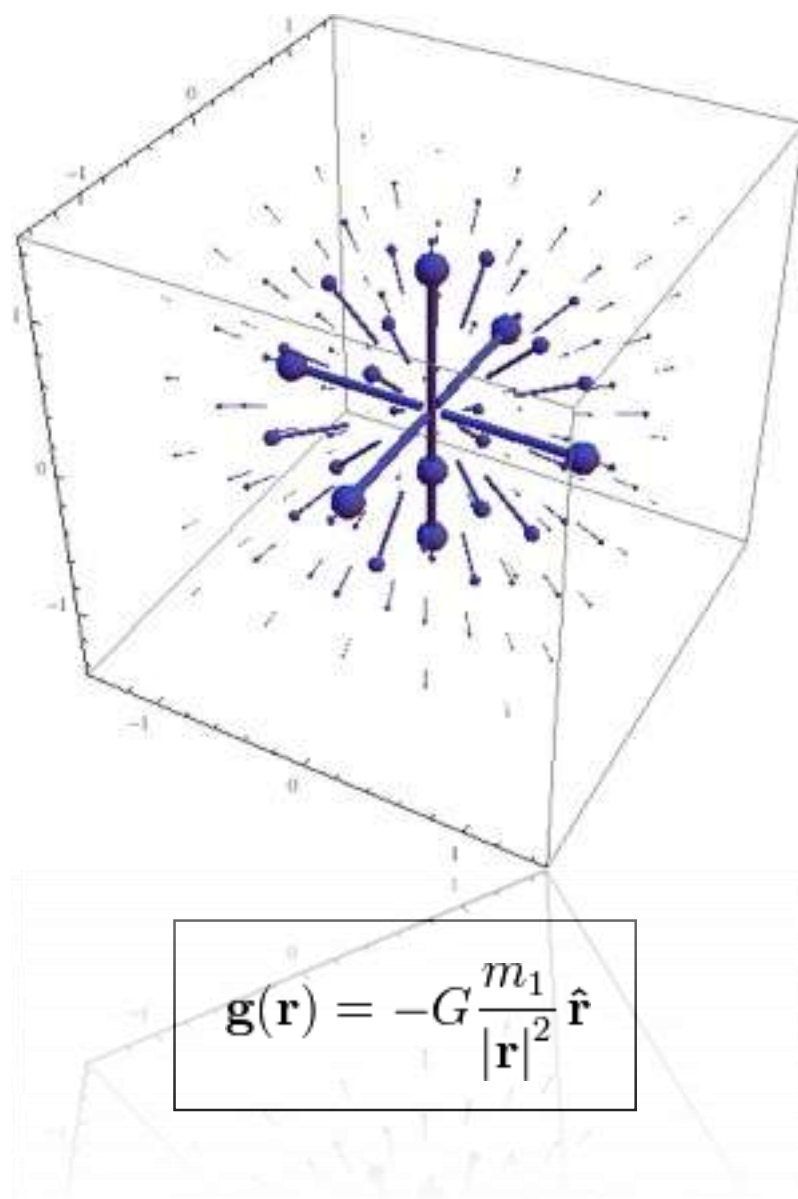


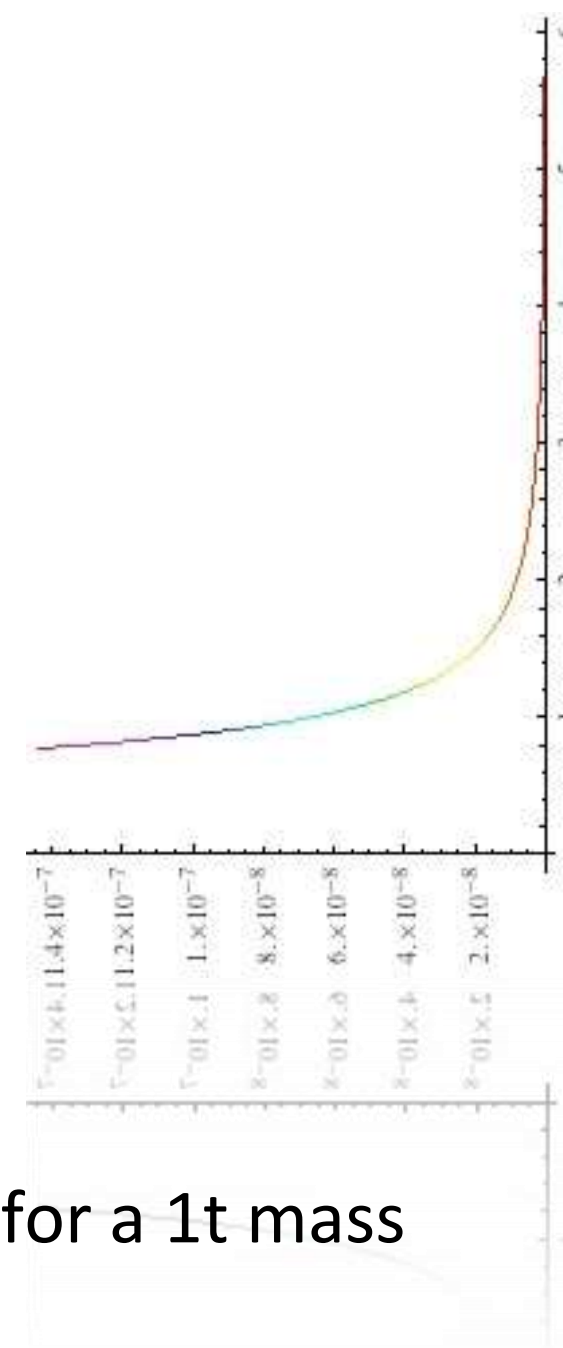
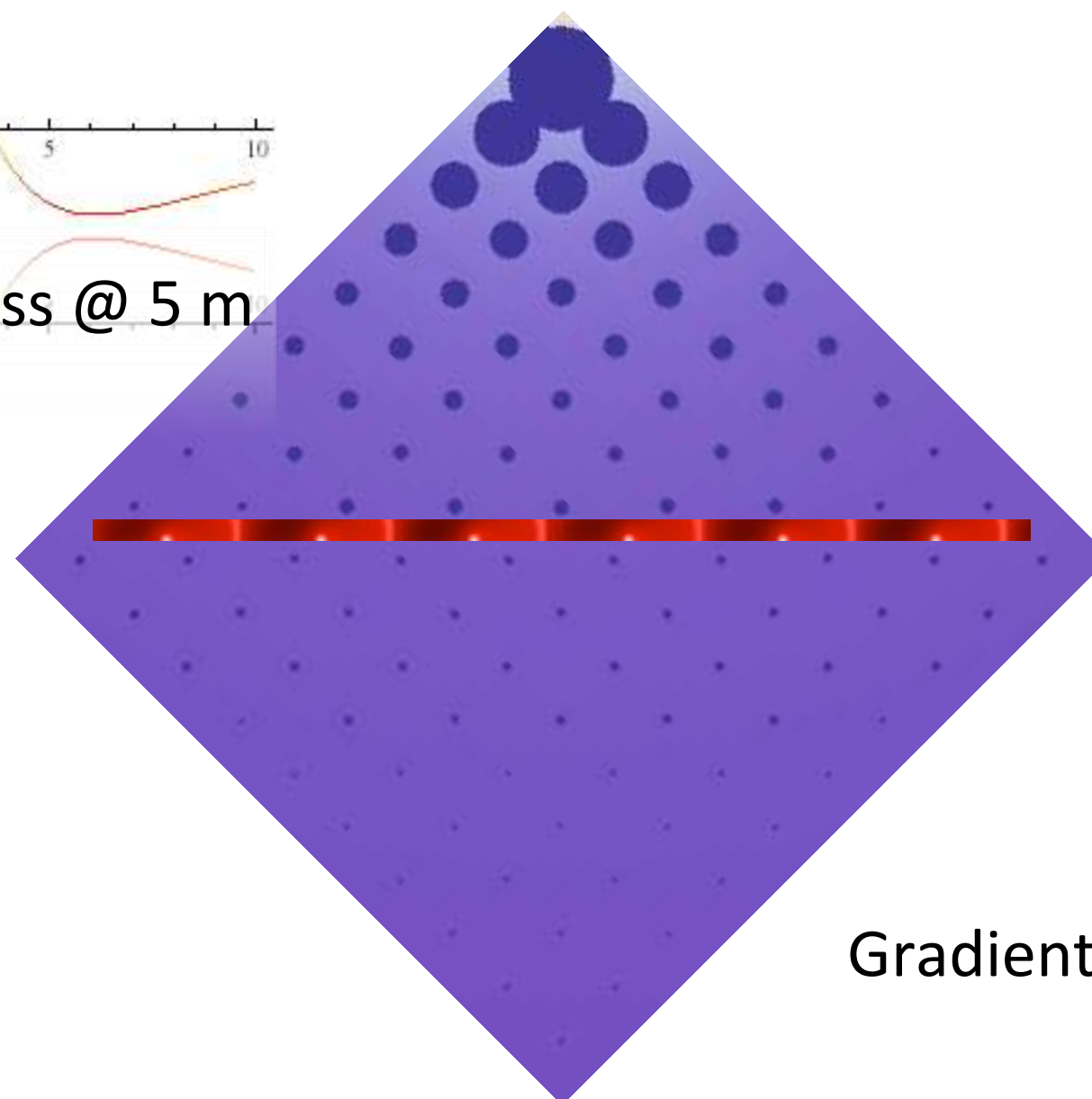
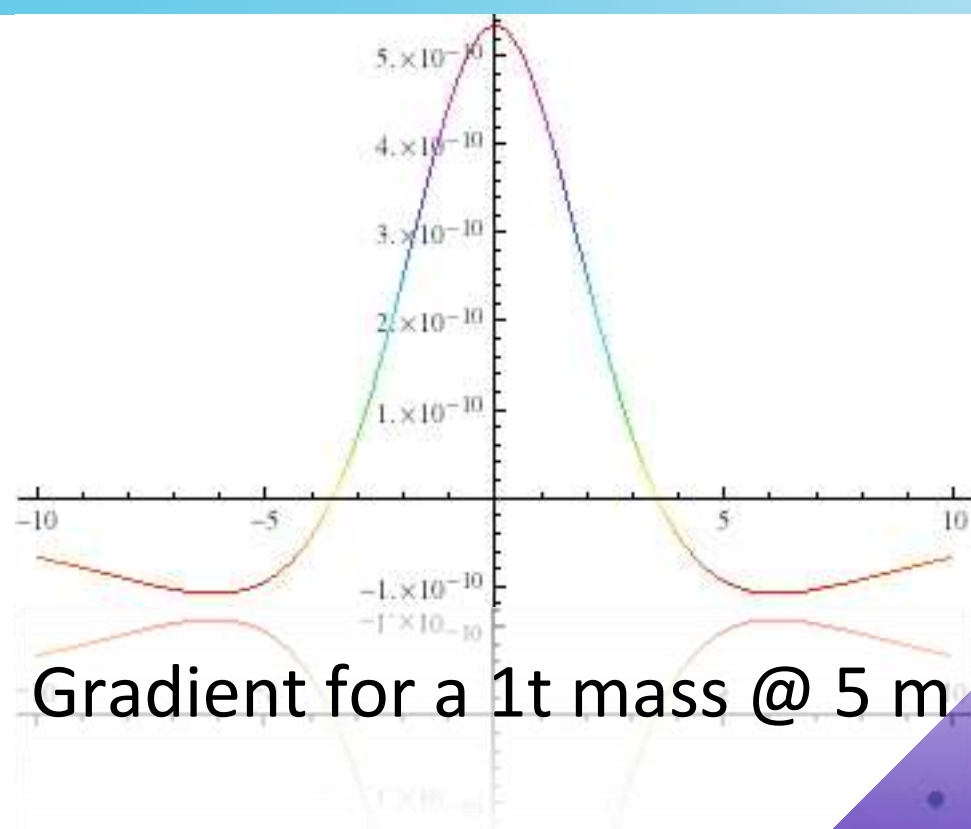
- Underground gravity monitoring can provide informations about seisms, water transfer in rocks, gradient noise ...
- Underground laboratories are very quiet : improved performance limits on AI sensors.

point source of gravity



point source of gravity





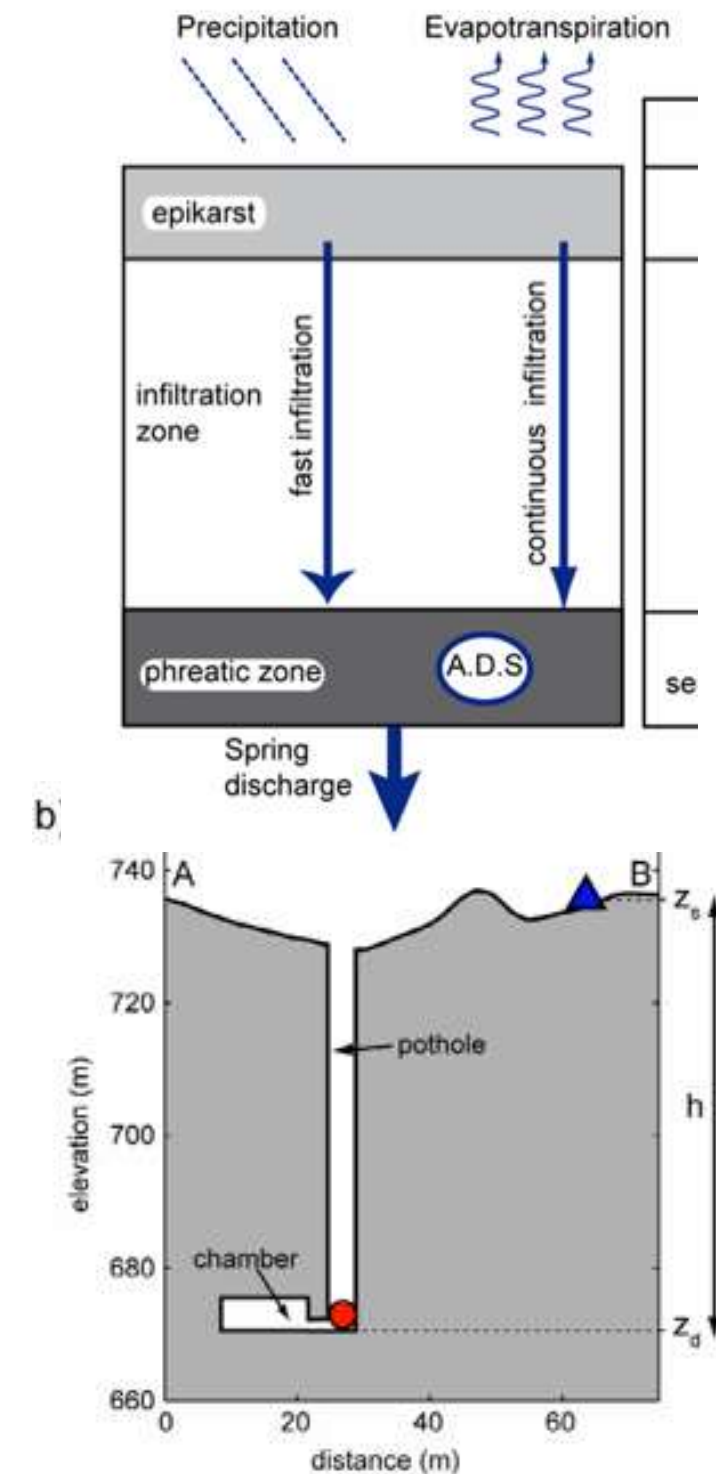
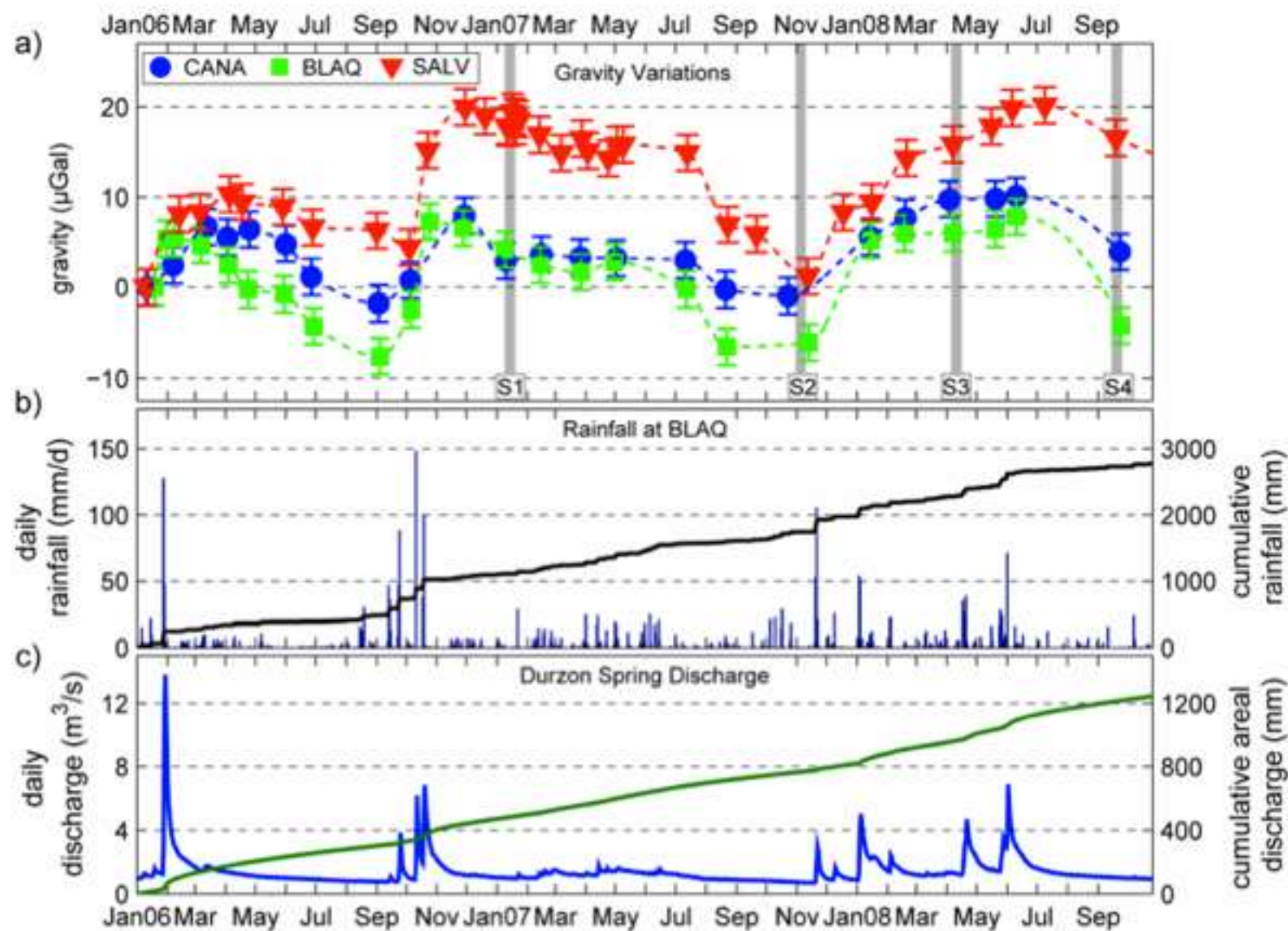
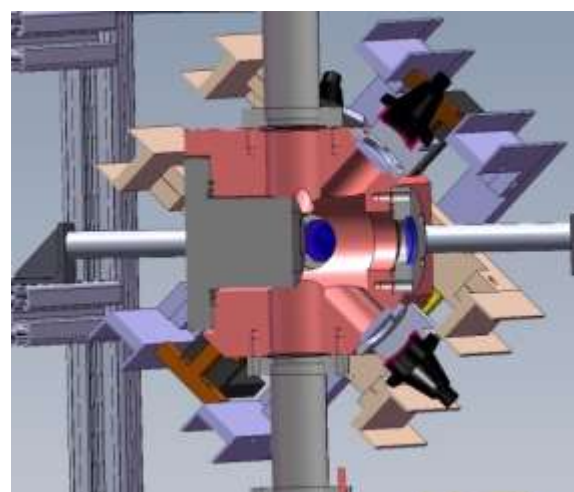
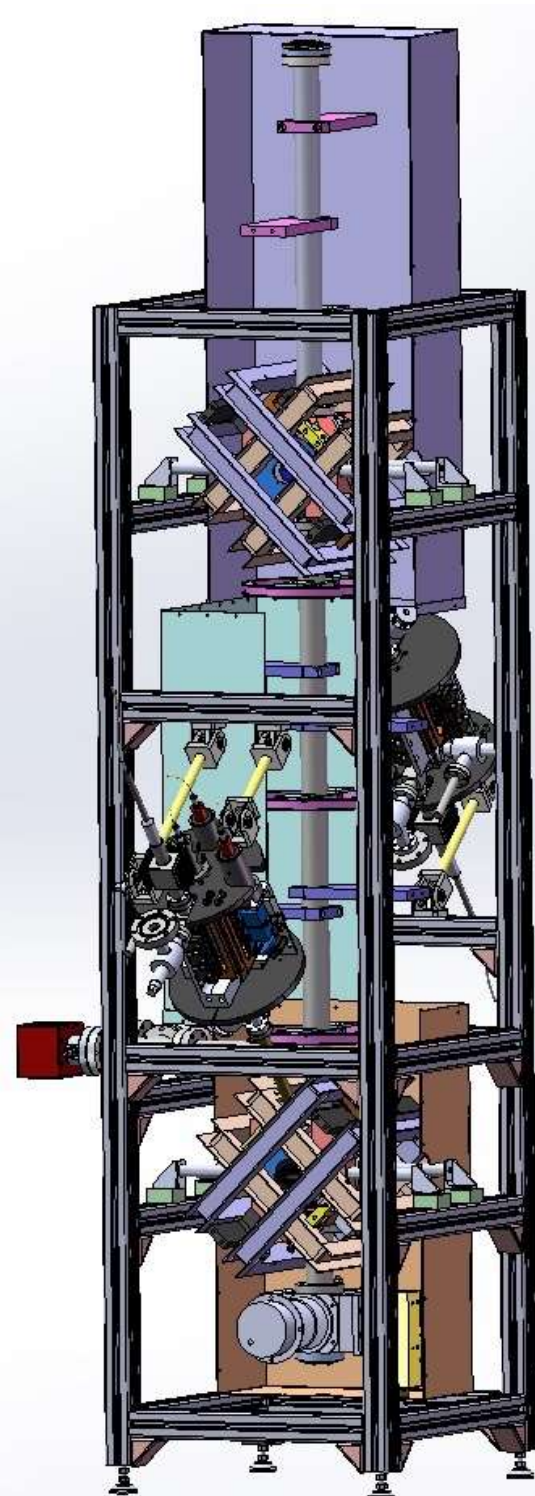
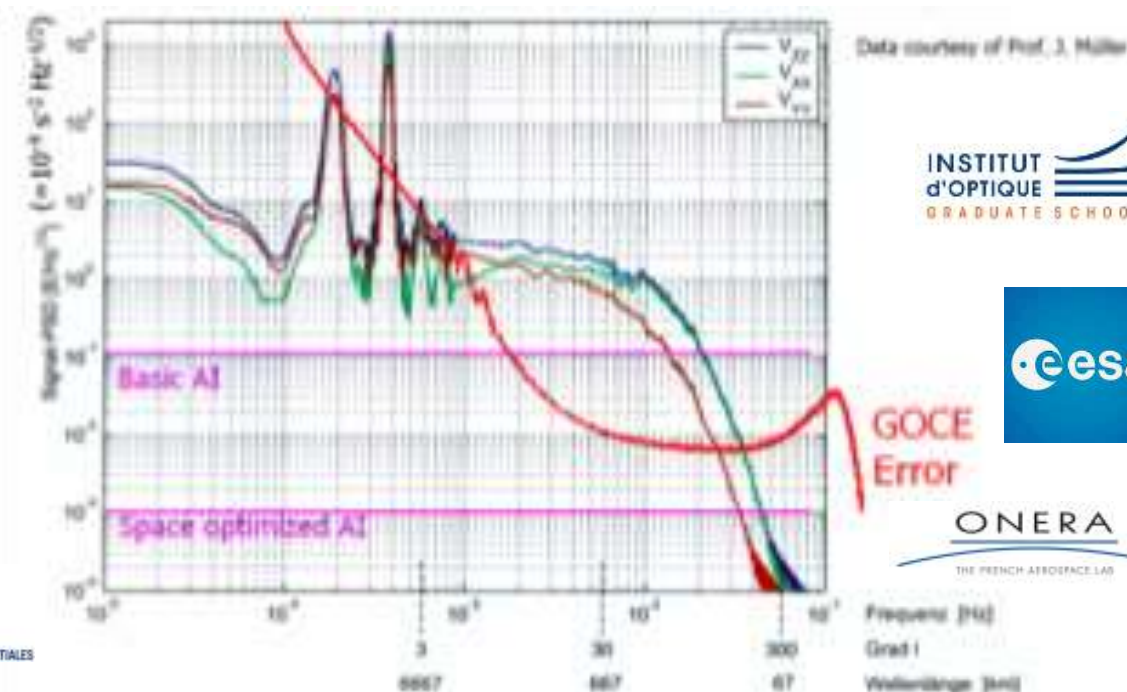


Figure 7.2 : a) Absolute gravity variations at site CANA, BLAQ and SALV, corrected for regional hydrology (see text for explanations). Survey periods S1 to S4 are indicated by vertical grey bars. b) Daily and cumulative rainfall measured at BLAQ, c) Daily Durzon spring discharge (blue line) and cumulative areal discharge (green line).



l'Observatoire de Paris SYRTE

cnes
CENTRE NATIONAL D'ÉTUDES SPATIALES



Concept 1

Single satellite (GOCE-like) using Mach-Zehnder interferometry, 3-axis

- Conceivable sensitivity: 10^{-12} ms^{-2} @ $T=10 \text{ s}$, $\text{SNR}=1000:1$ (improved sensitivity possible...)

100 to 1 $\text{mE Hz}^{-1/2}$ level @ baseline=1m

Technology already demonstrated outside of lab environment, possible to measure complete gravity gradient tensor plus absolute value

- No calibration problems, no drag free needed

Concept 2

Multiple ultra-compact satellites, size of approximately one soda bottle each

- Conceivable sensitivity: 10^{-12} ms^{-2} @ $T=10 \text{ s}$, $\text{SNR}=1000:1$ (improved sensitivity possible...)

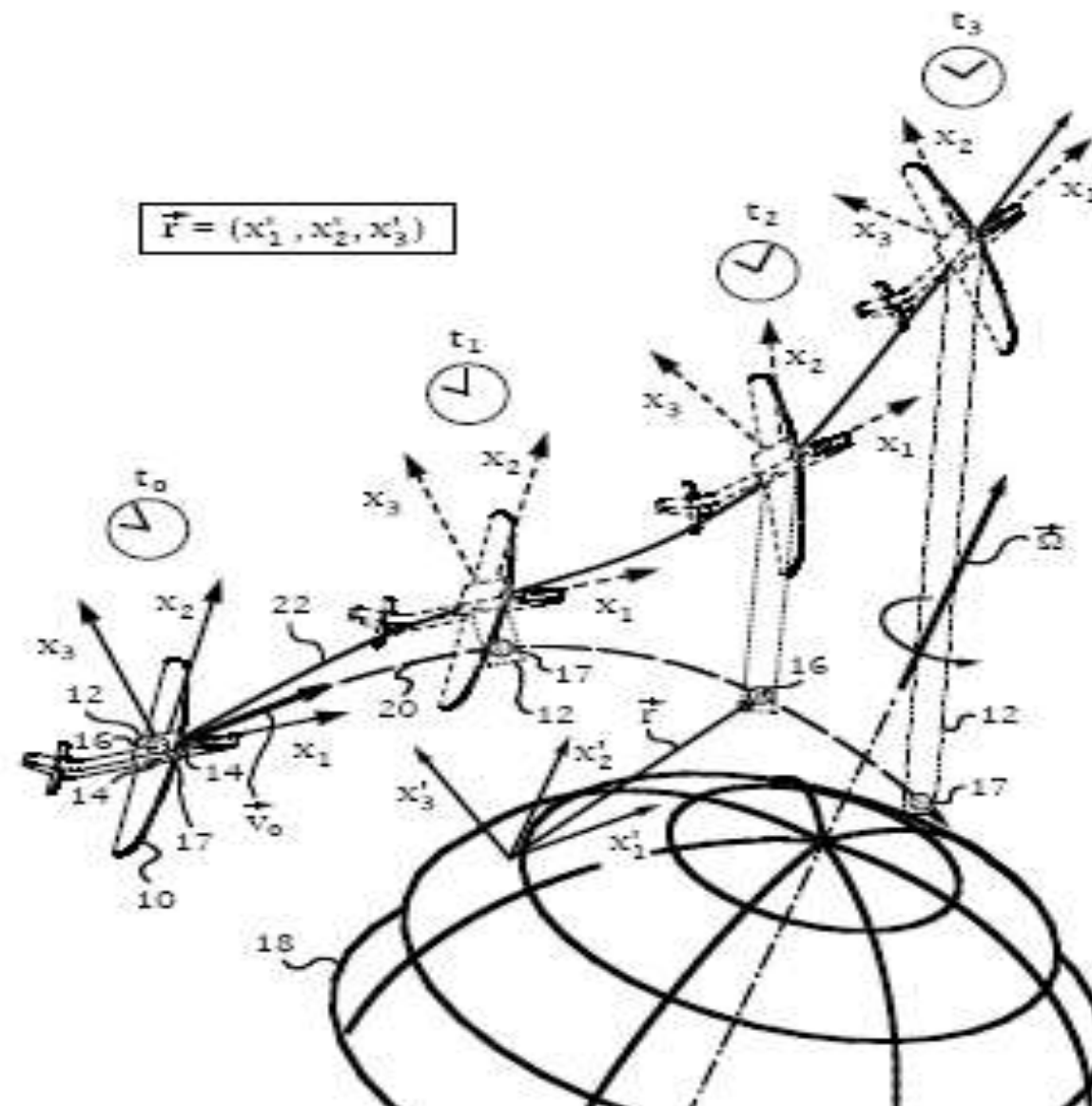
1 $\mu\text{E Hz}^{-1/2}$ level @ baseline=1km

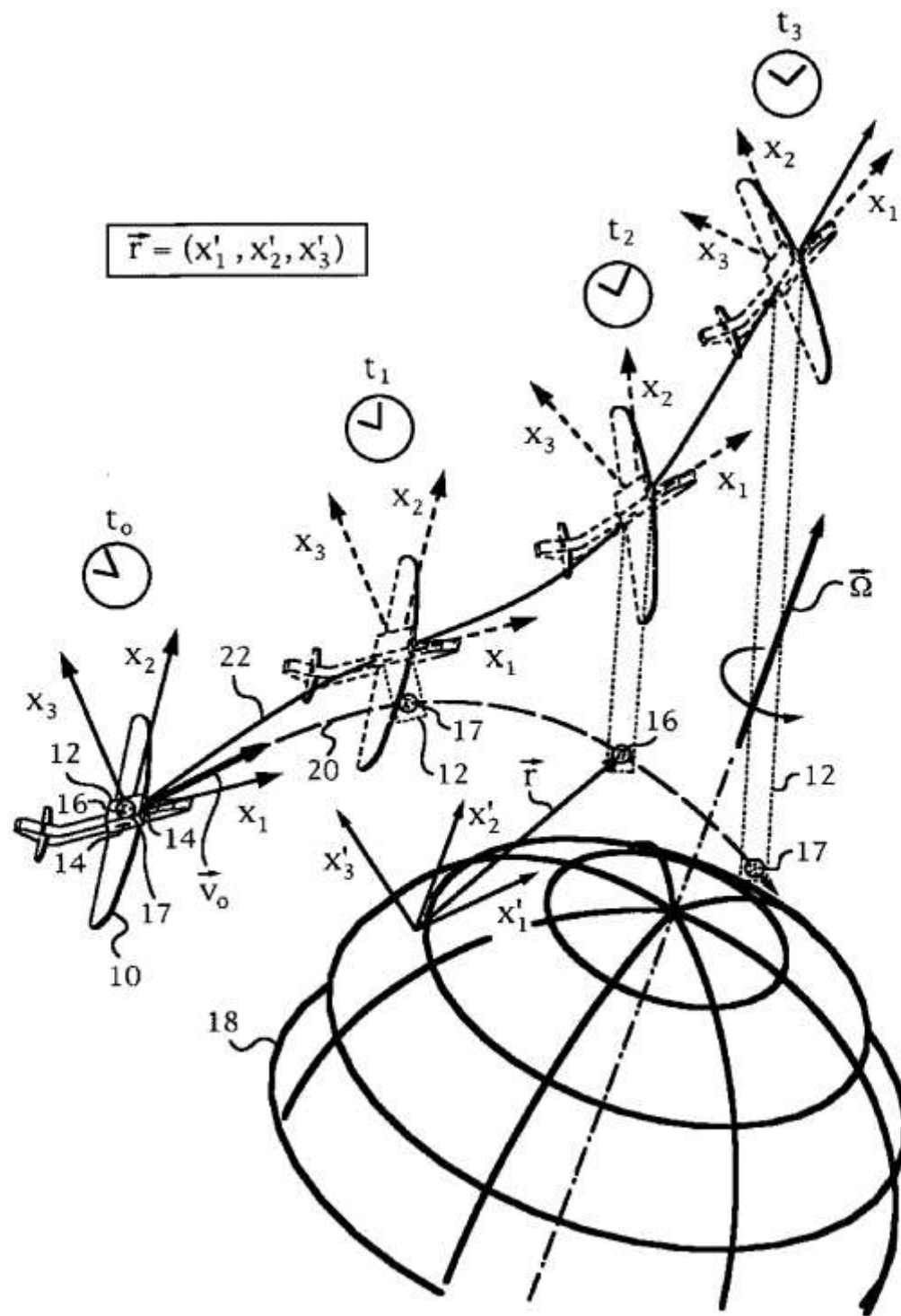
probably limited by mission complexity

Possible to measure complete gravity gradient tensor plus absolute value at very high sensitivity

- No calibration problems, drag free or high sensitivity standard accelerometers

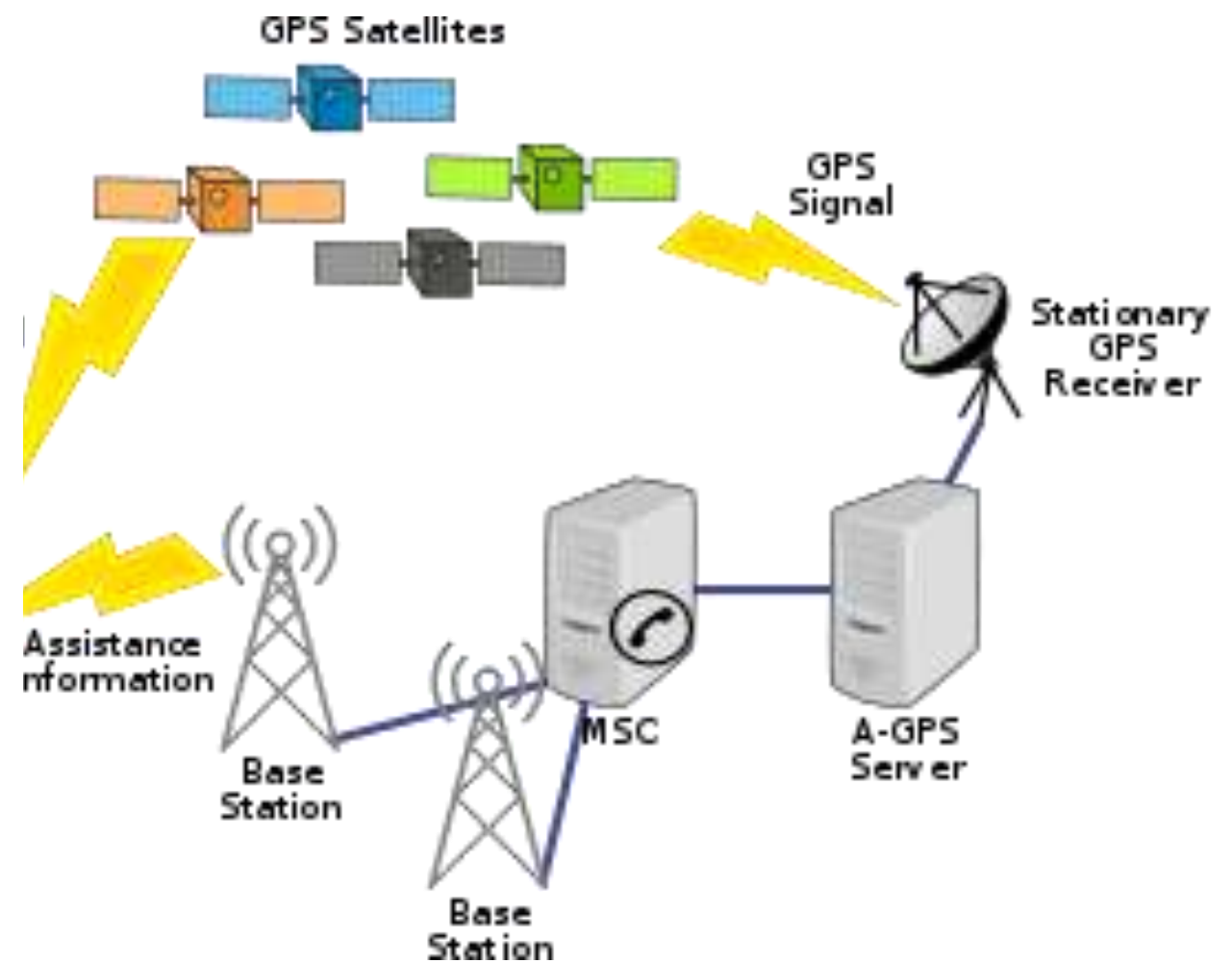
Navigation

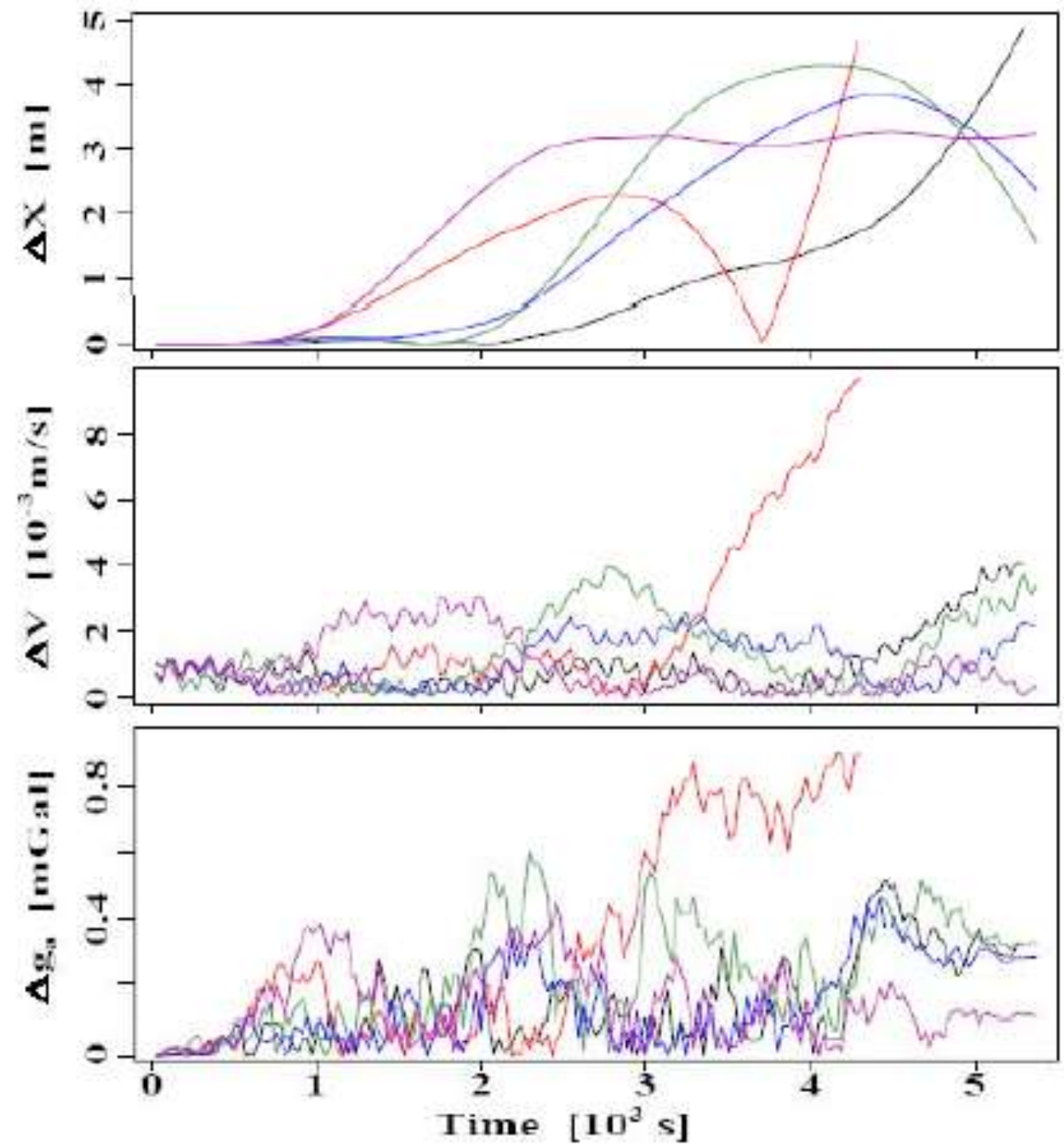
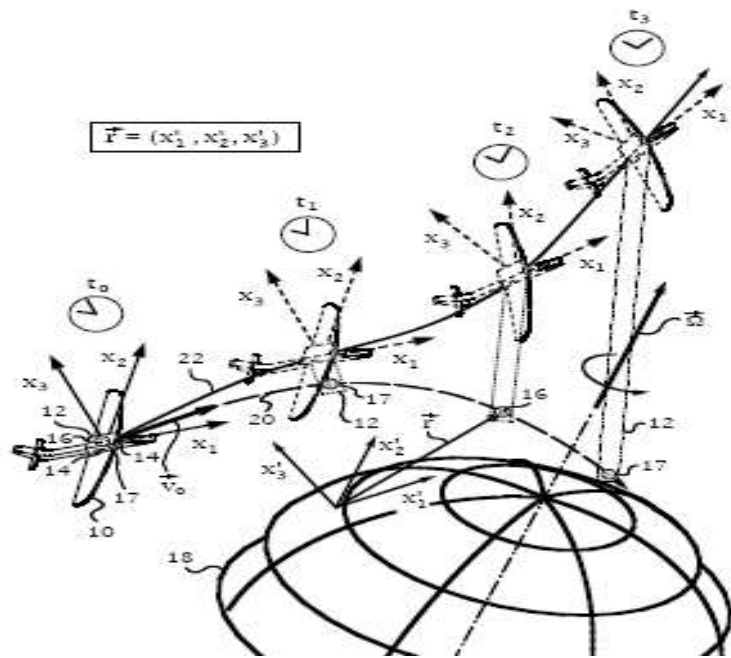




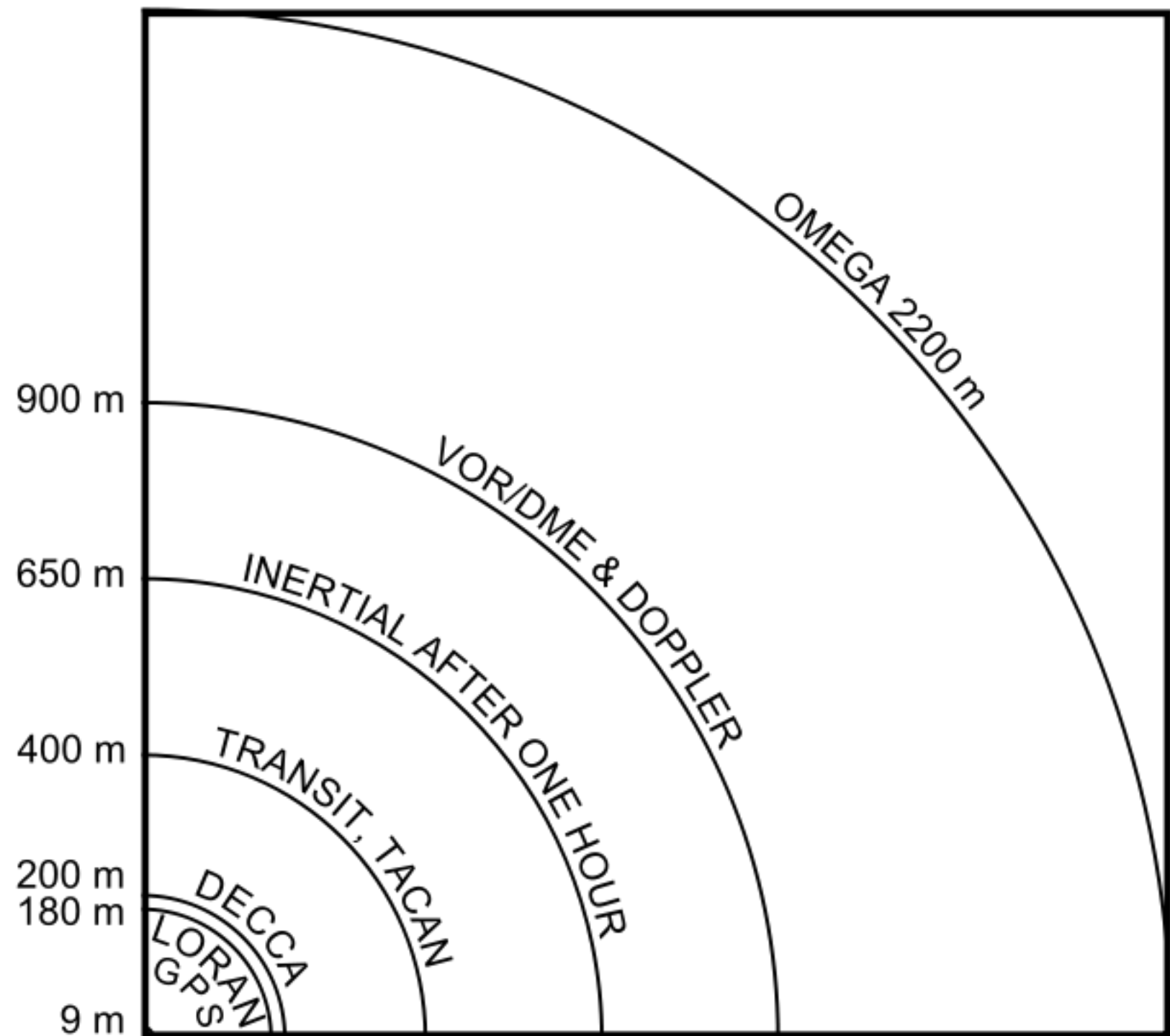
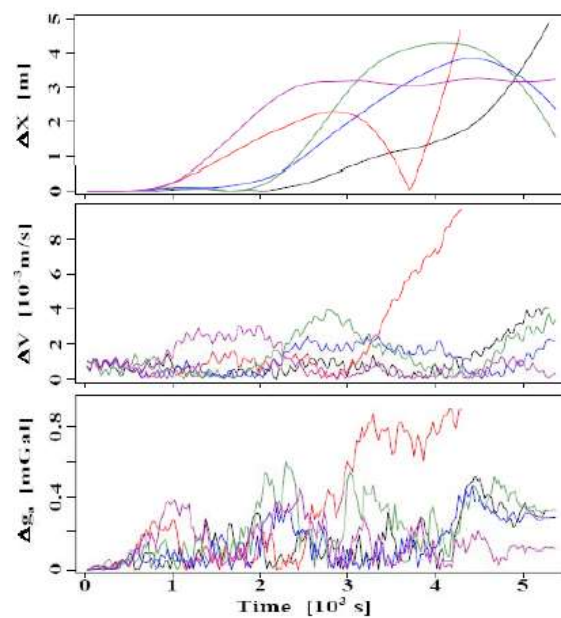
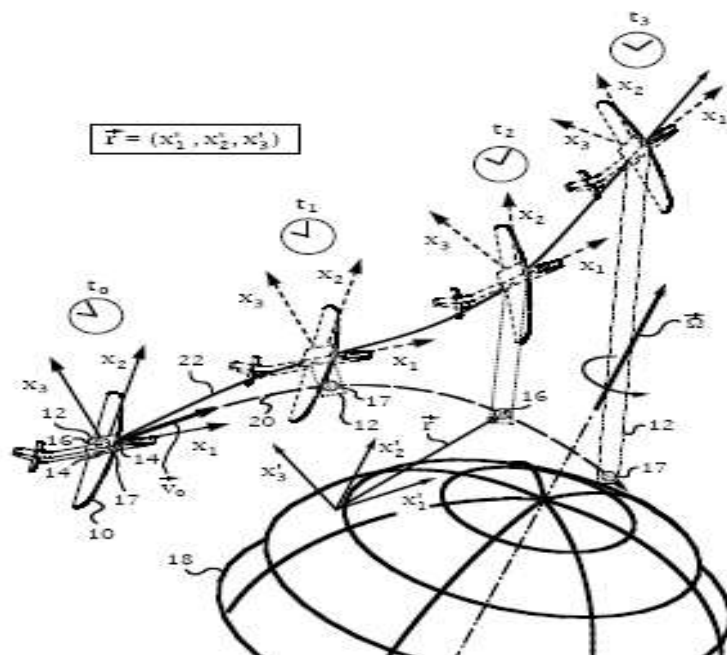
Navigation system

- ✦ Radionavigation
- ✦ On sight radio beacons (GPS, VOR, ...)





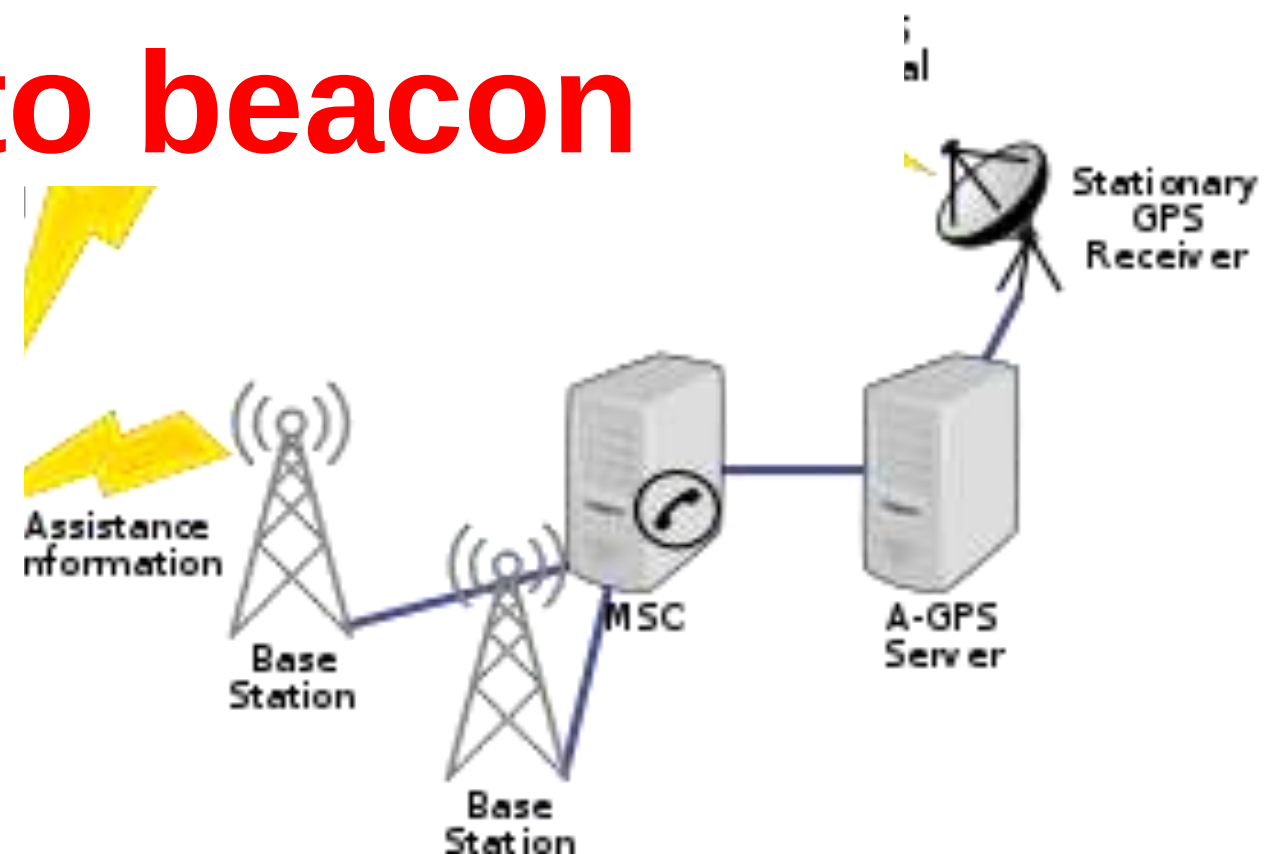
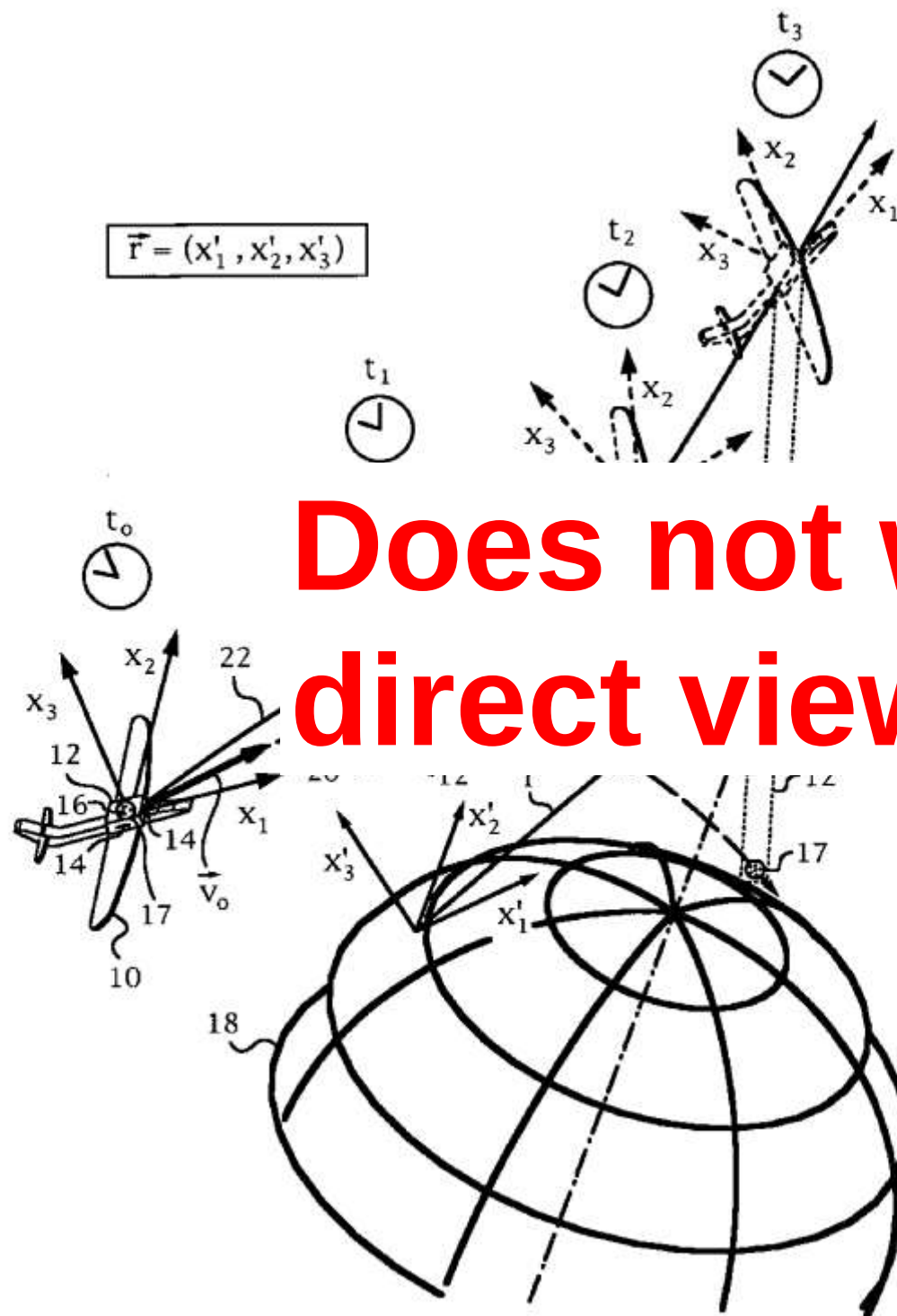
ACCURACY OF NAVIGATION SYSTEMS (2-dimensional)



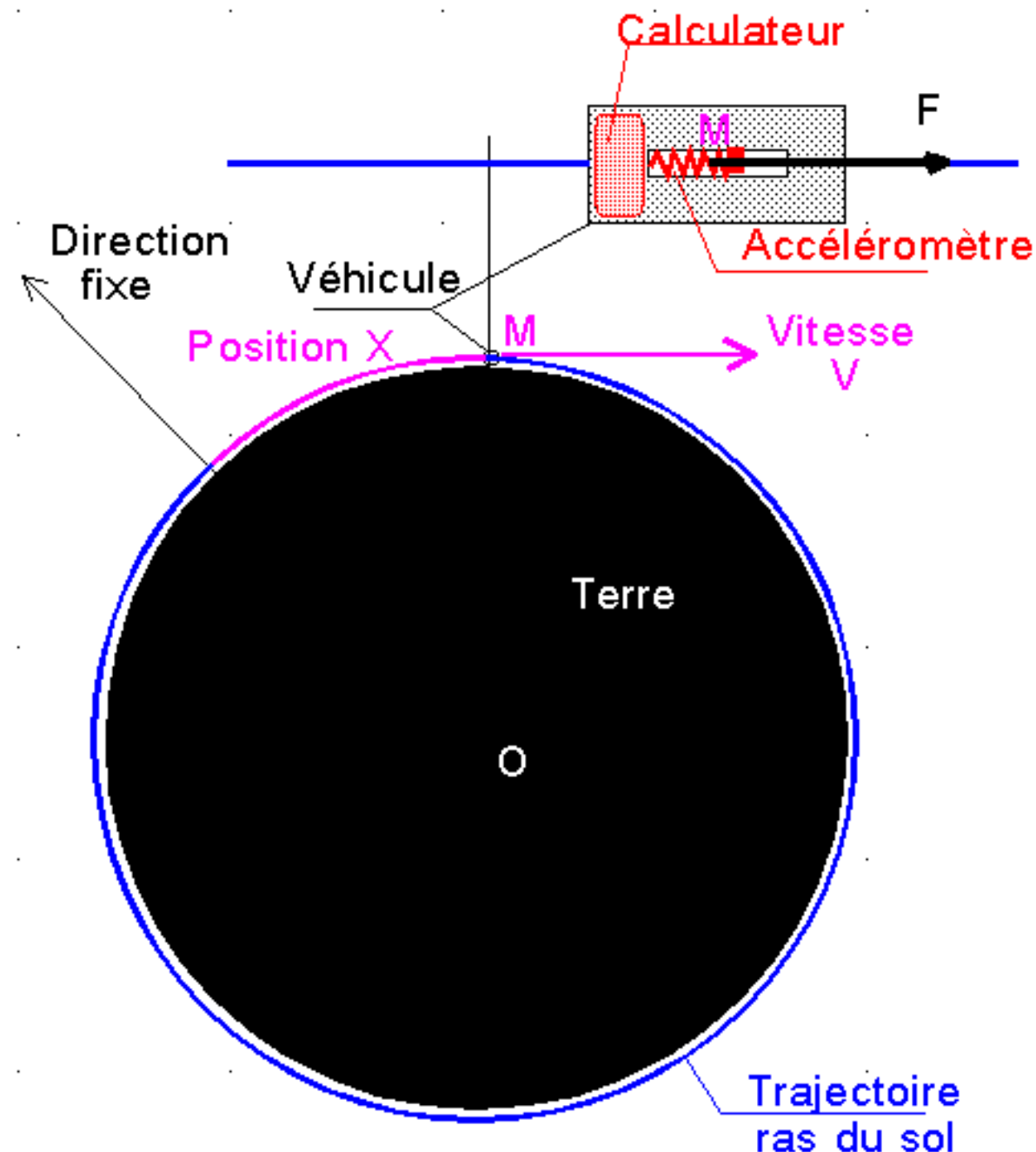
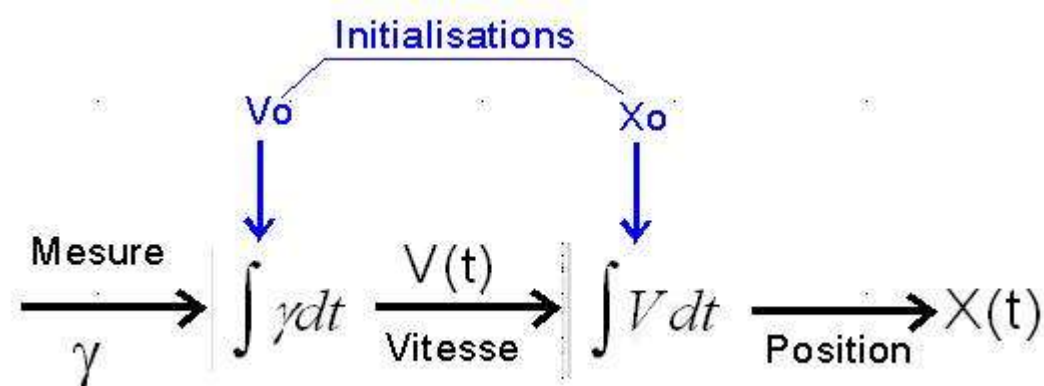
Navigation system

- ✦ Radionavigation
- ✦ On sight radio beacons (GPS, VOR, ...)

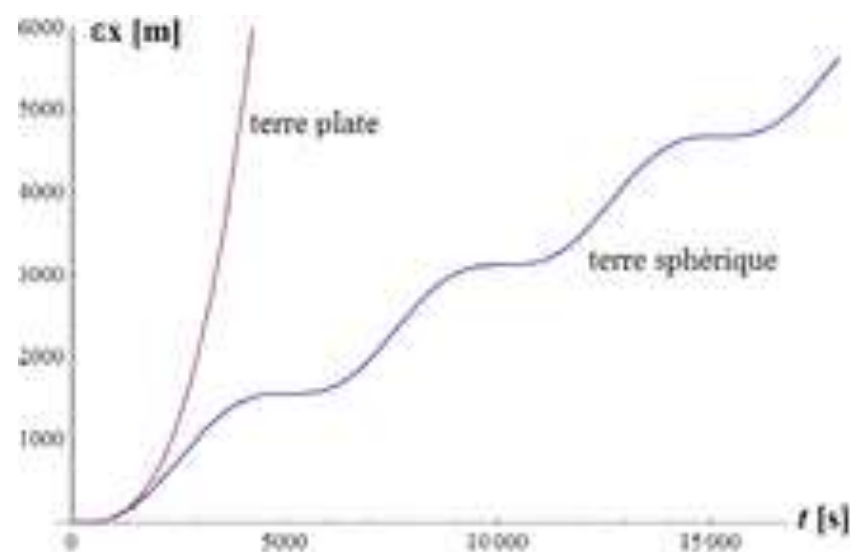
**Does not work without
direct view to beacon**



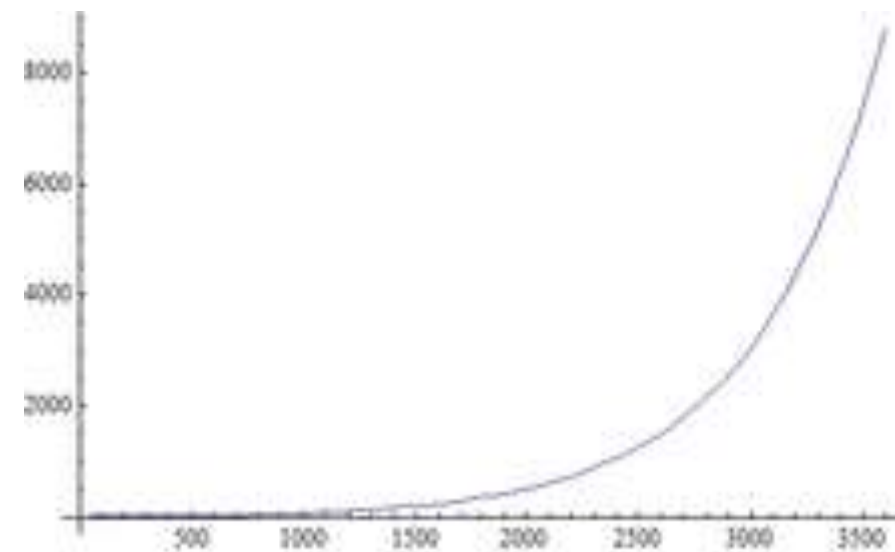
- ✦ Uses the onboard measurement of the specific force as well as the instantaneous rotation vector.
- ✦ Measurement of acceleration (other than gravity)
- ✦ Embedded calculation integration
- ✦ Knowledge of initial conditions
- ✦ Error handling and resetting



- ✦ The bias of the standard inertial sensors is non-zero and varies over time
- ✦ After integration, it leads to a position error that exponentially believes
- ✦ For a few NM/h of precision
- ✦ Knowledge of rotation bias at a few °/h (degrees per hour = $\mu\text{radian/s}$)
- ✦ Knowledge of acceleration bias of a few tens of μG



Gyromètre affecté d'un biais de 0,01 °/h

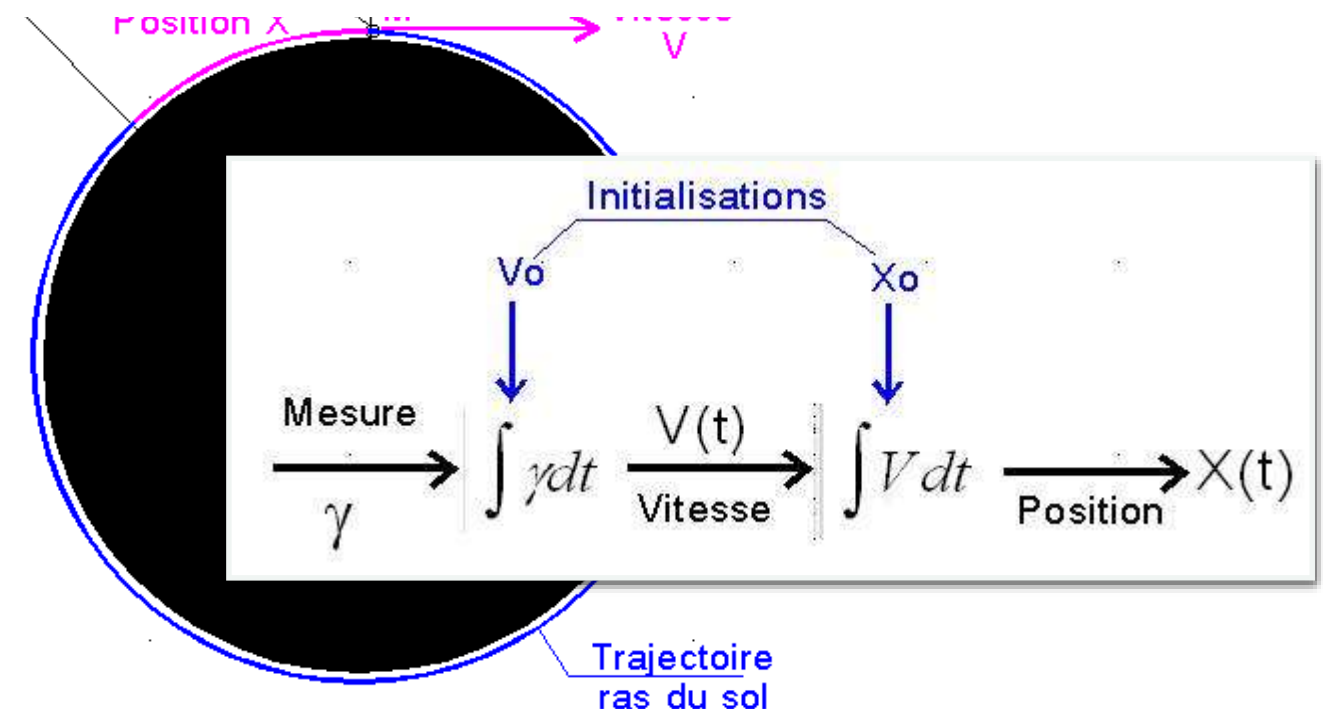
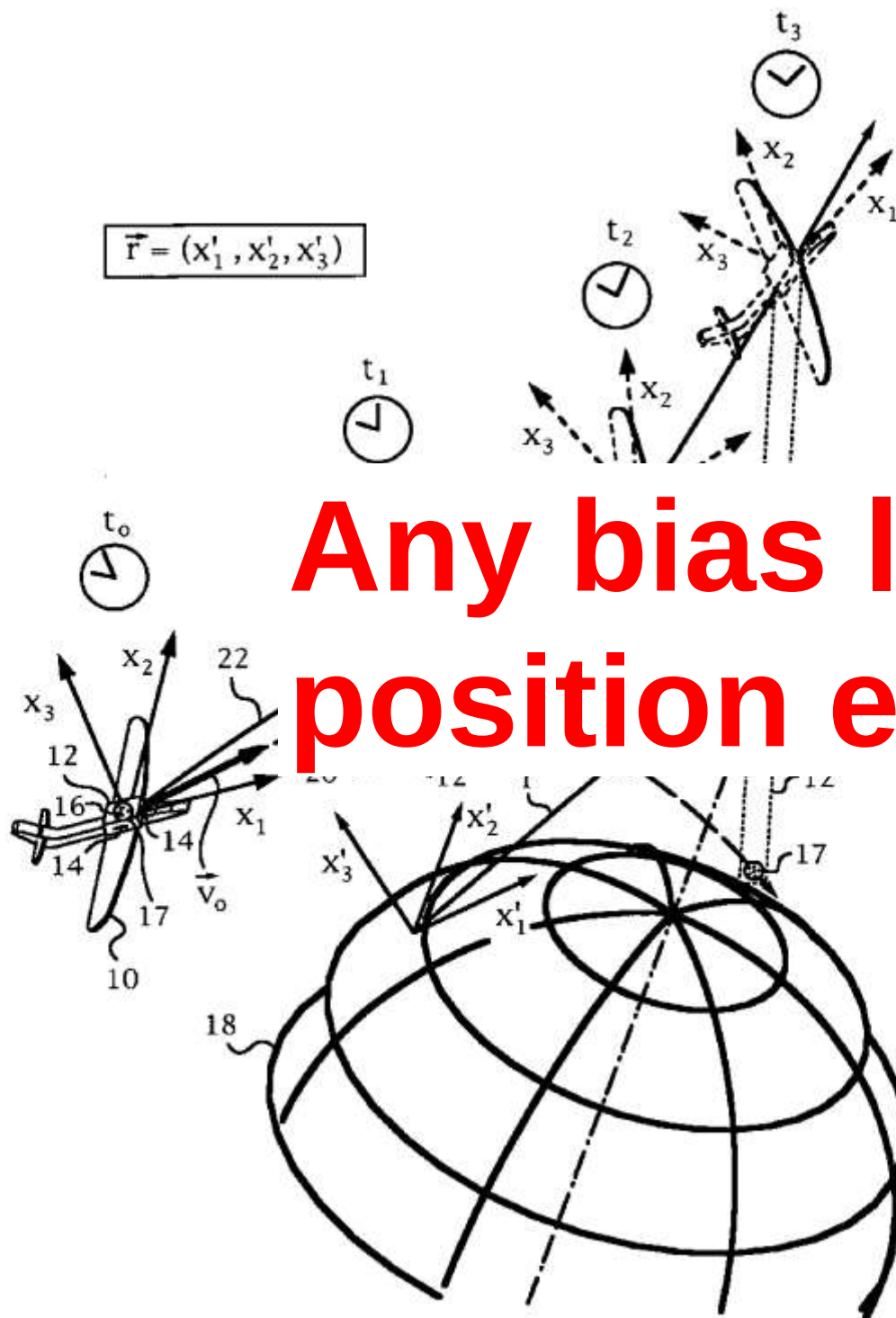


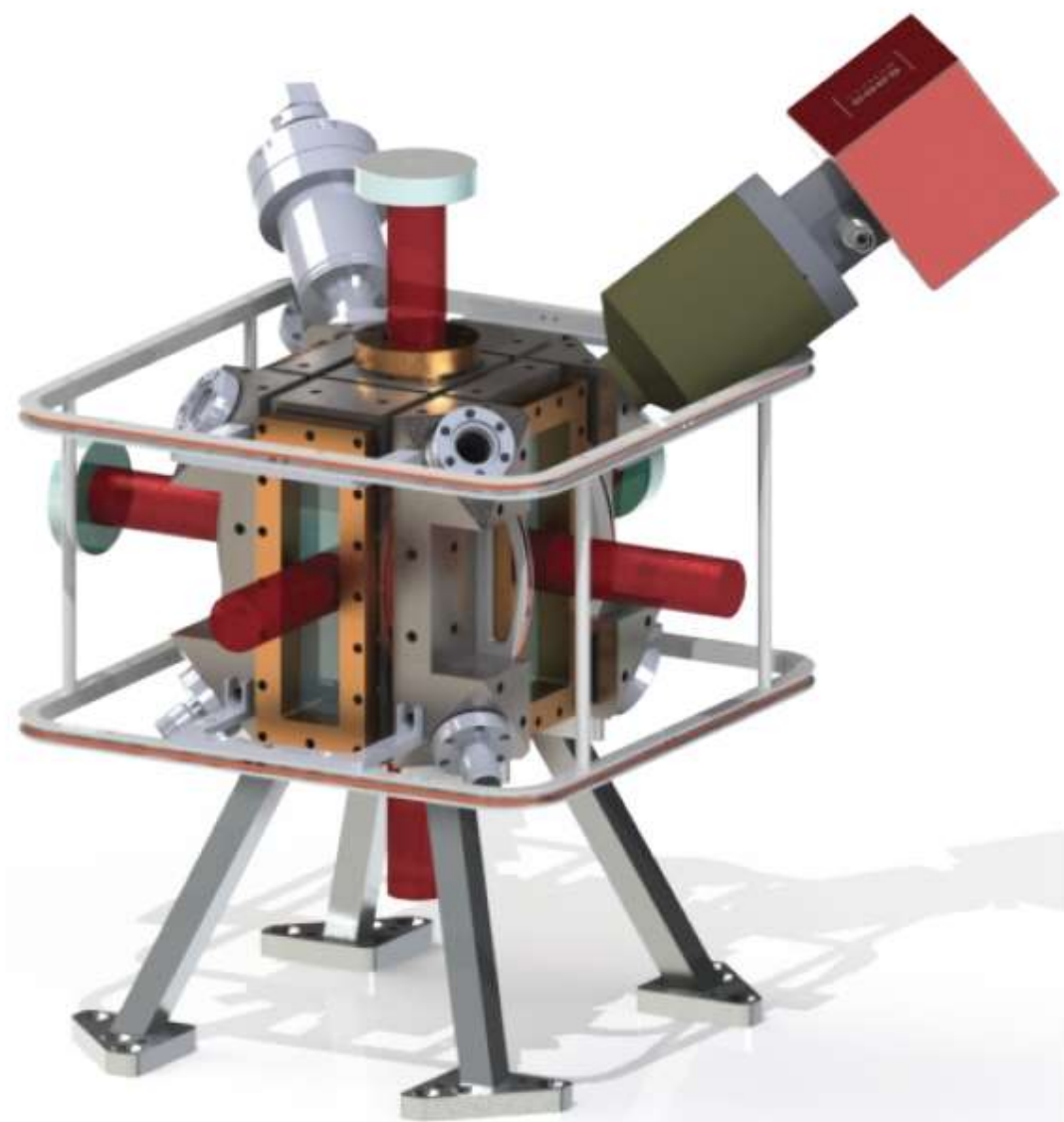
Accéléromètre affecté d'un biais de 10 μg

Navigation system

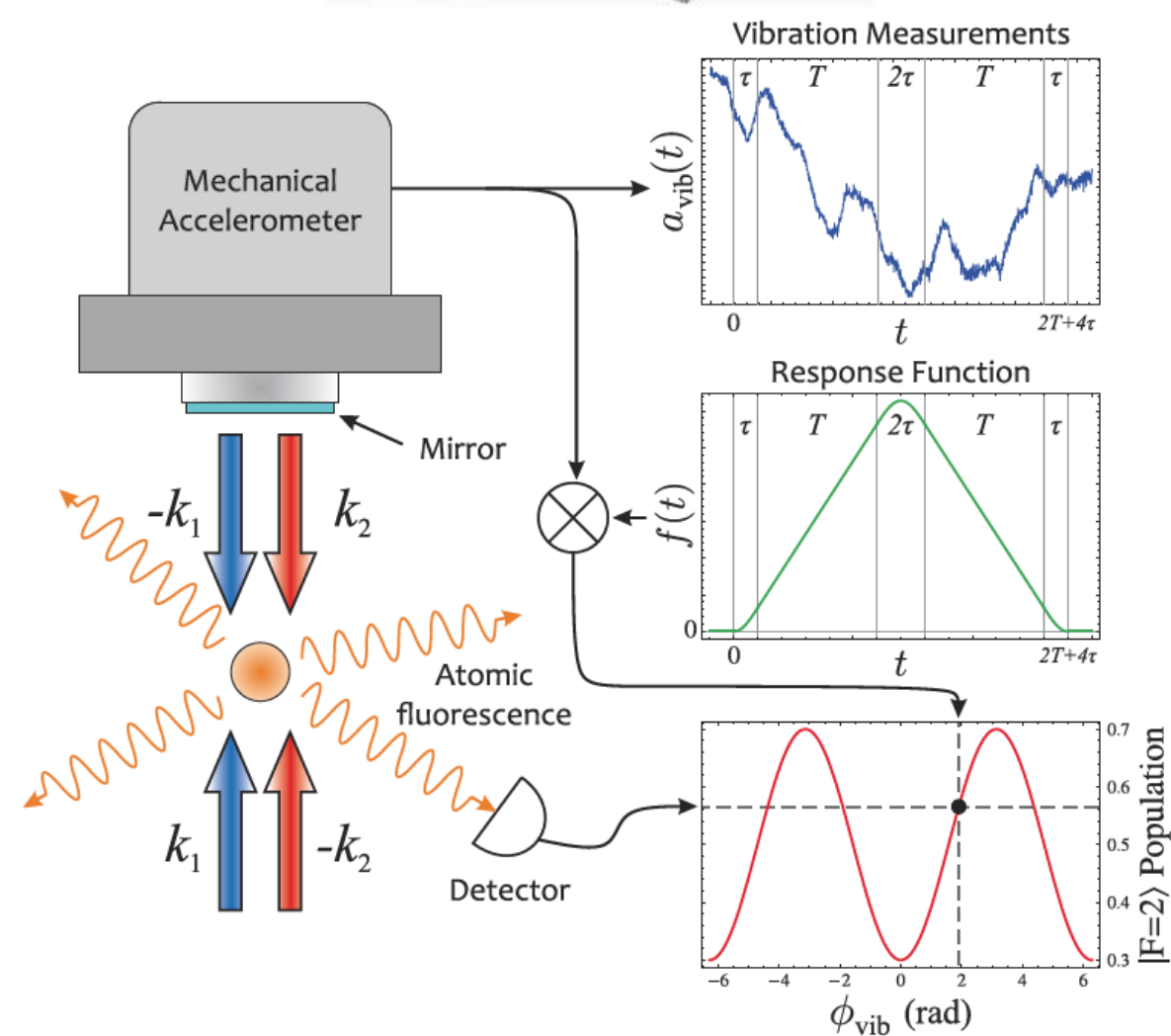
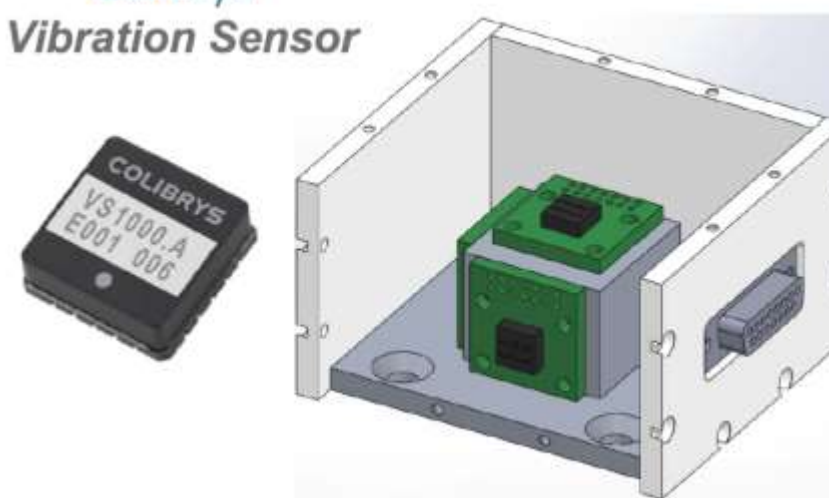
- ✦ Inertial
- ✦ Integrate acceleration and angle measurement
- ✦ Block box

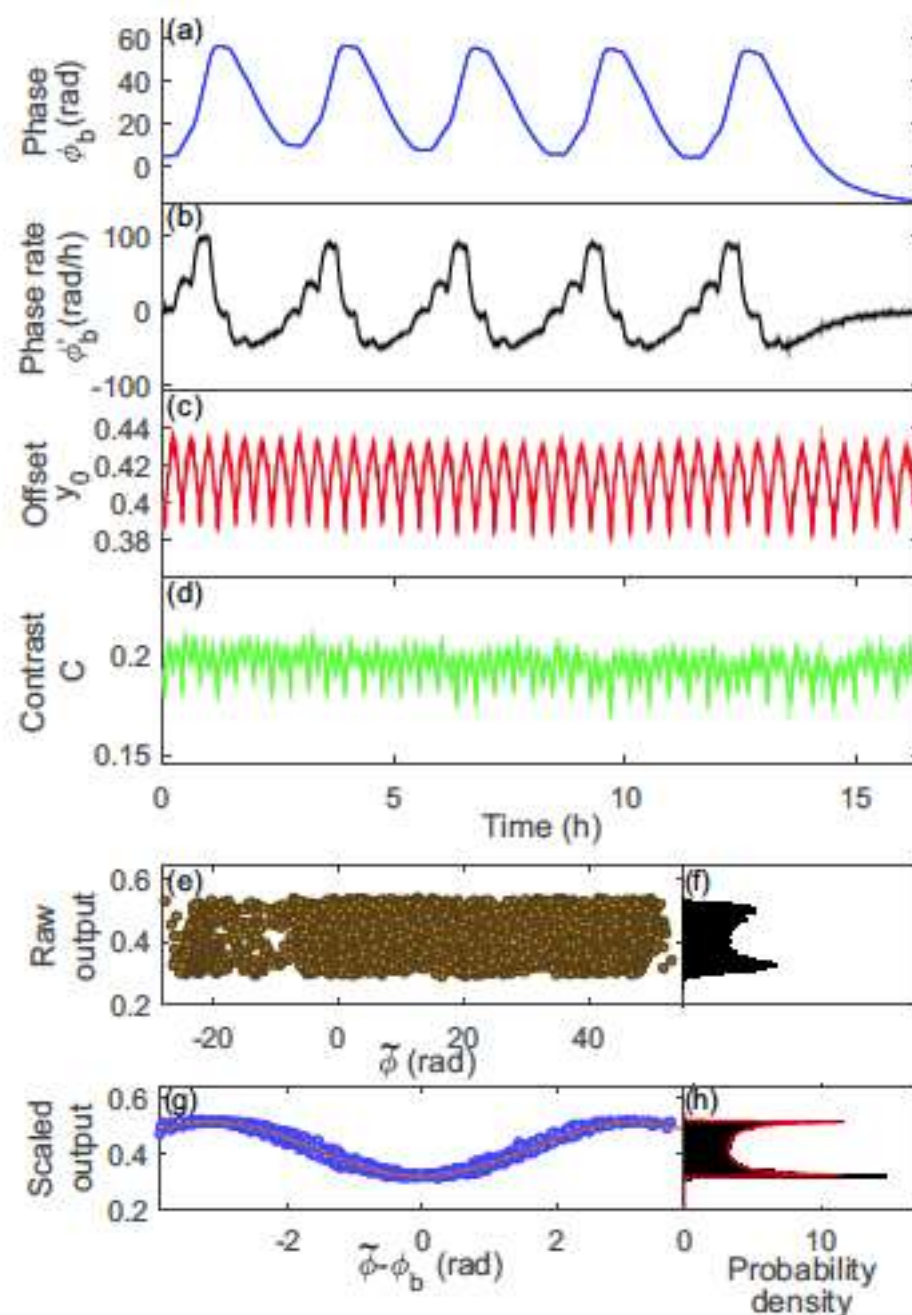
Any bias leads to large position error



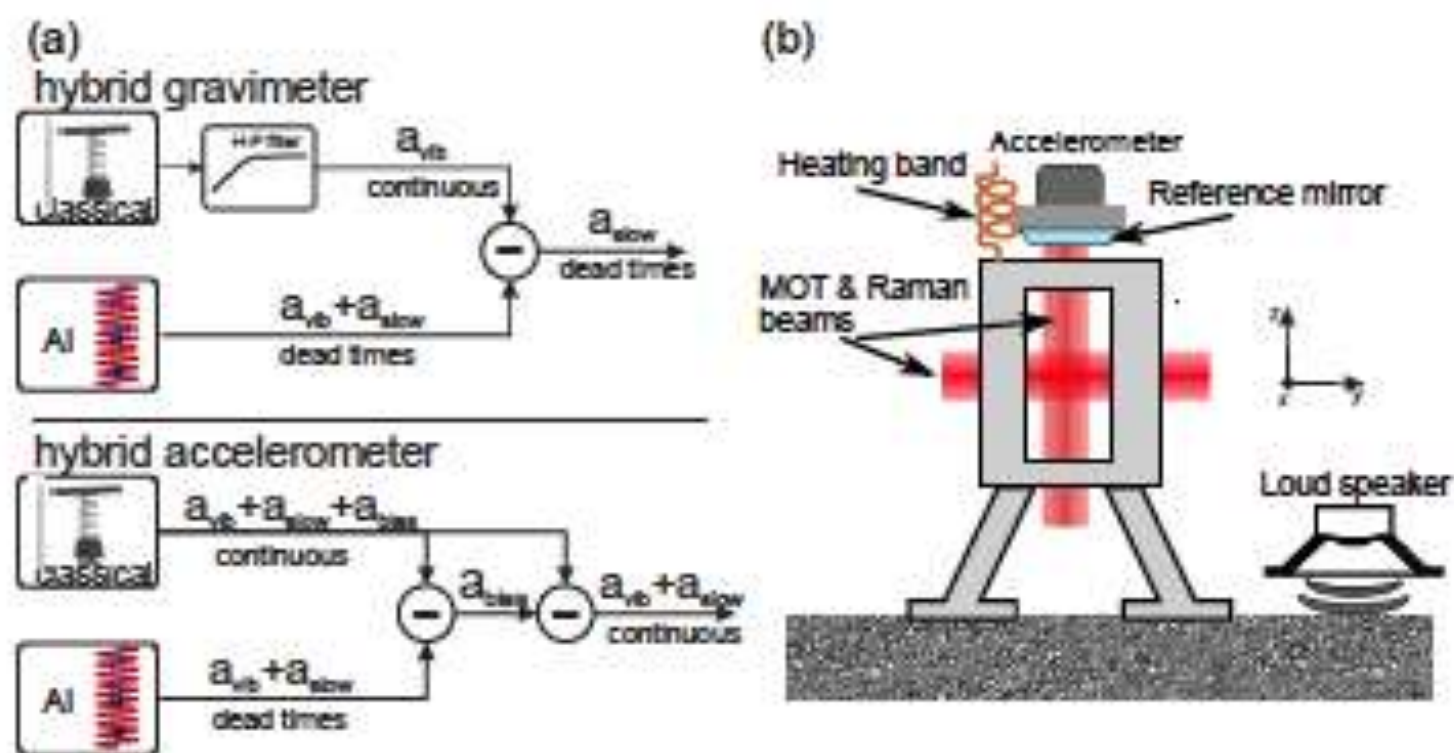


Colibrys
Vibration Sensor

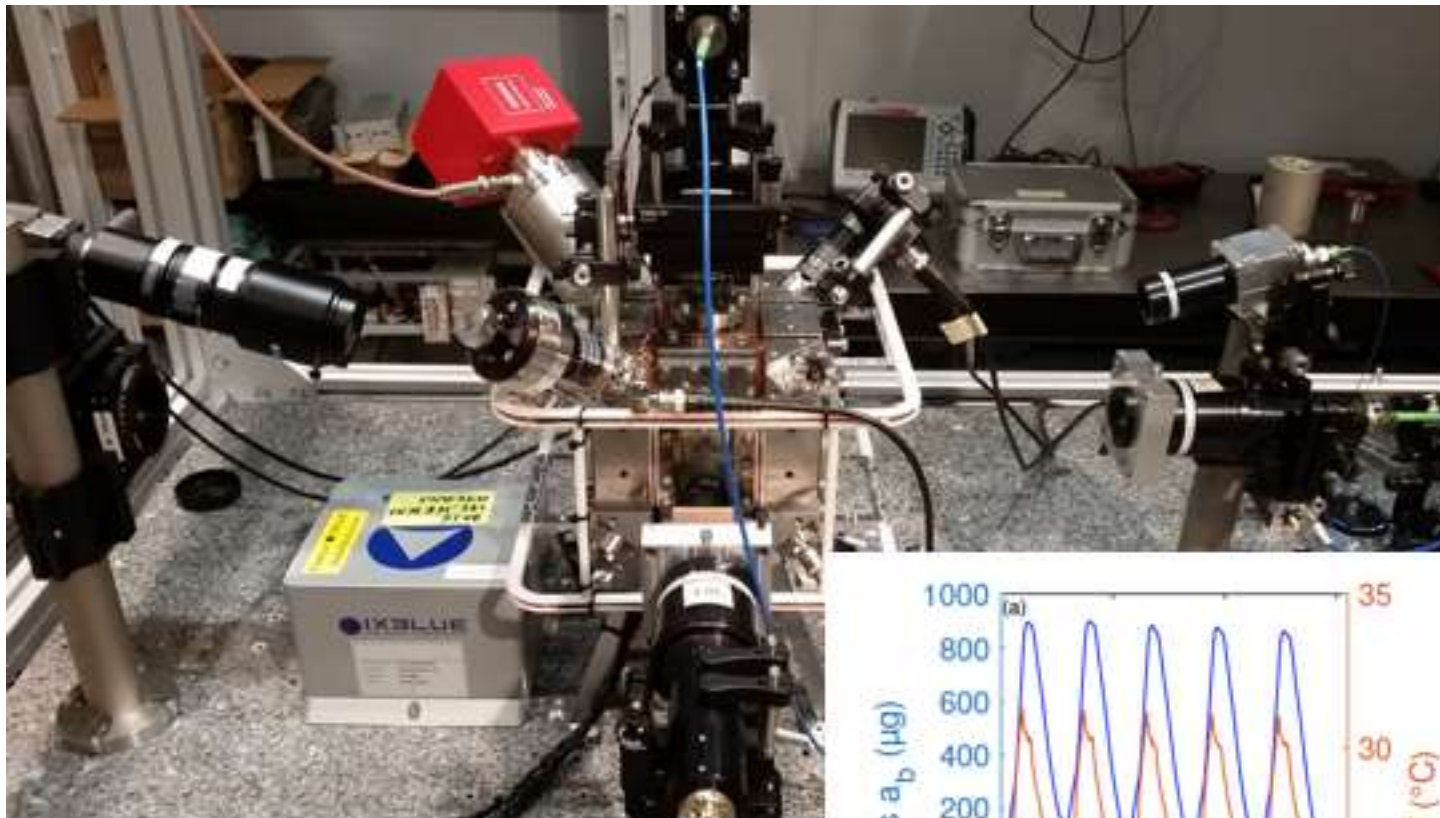




- Test du couplage quantique-hybride dans des conditions “réalistes”

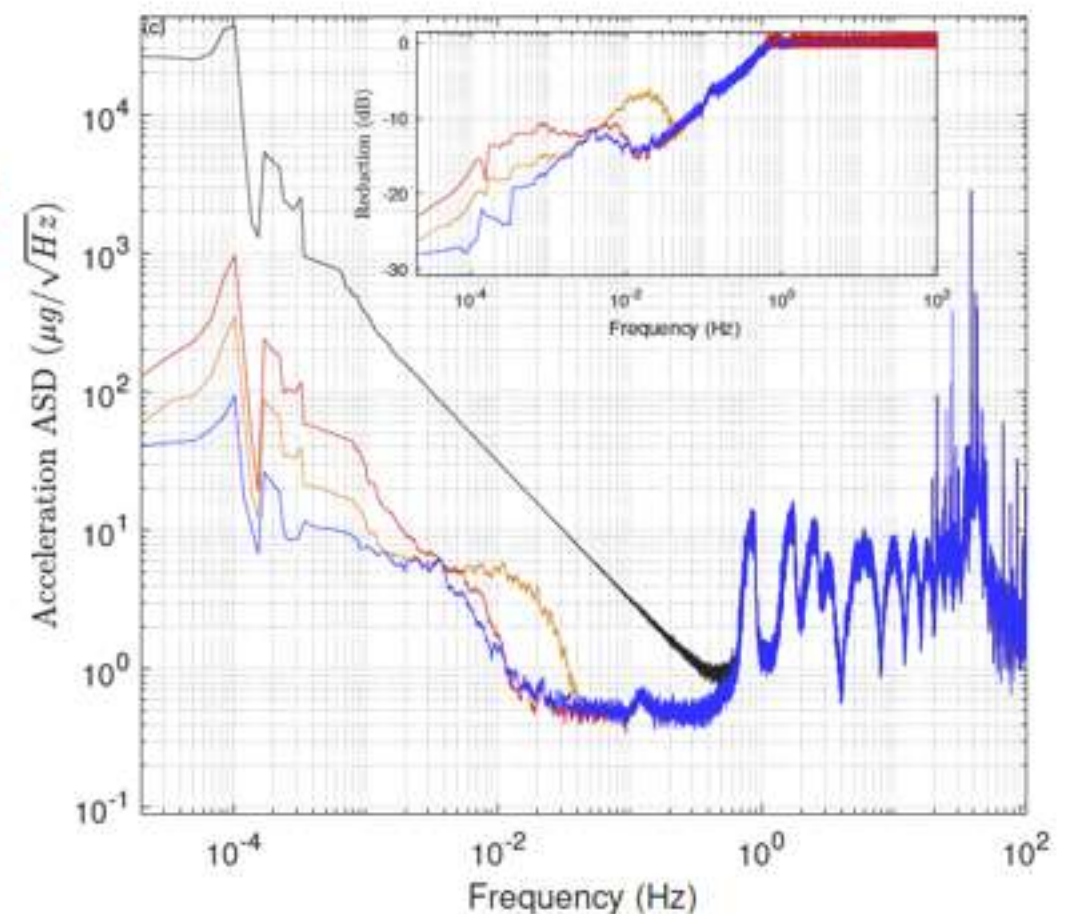
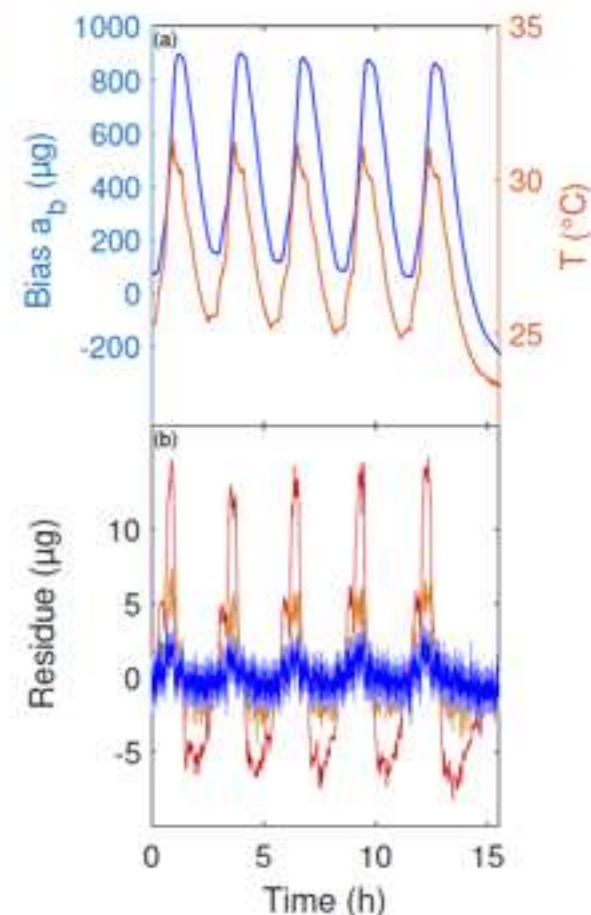


- Détermination et intégration du biais de mesure pour permettre l'intégration des équations du mouvement.

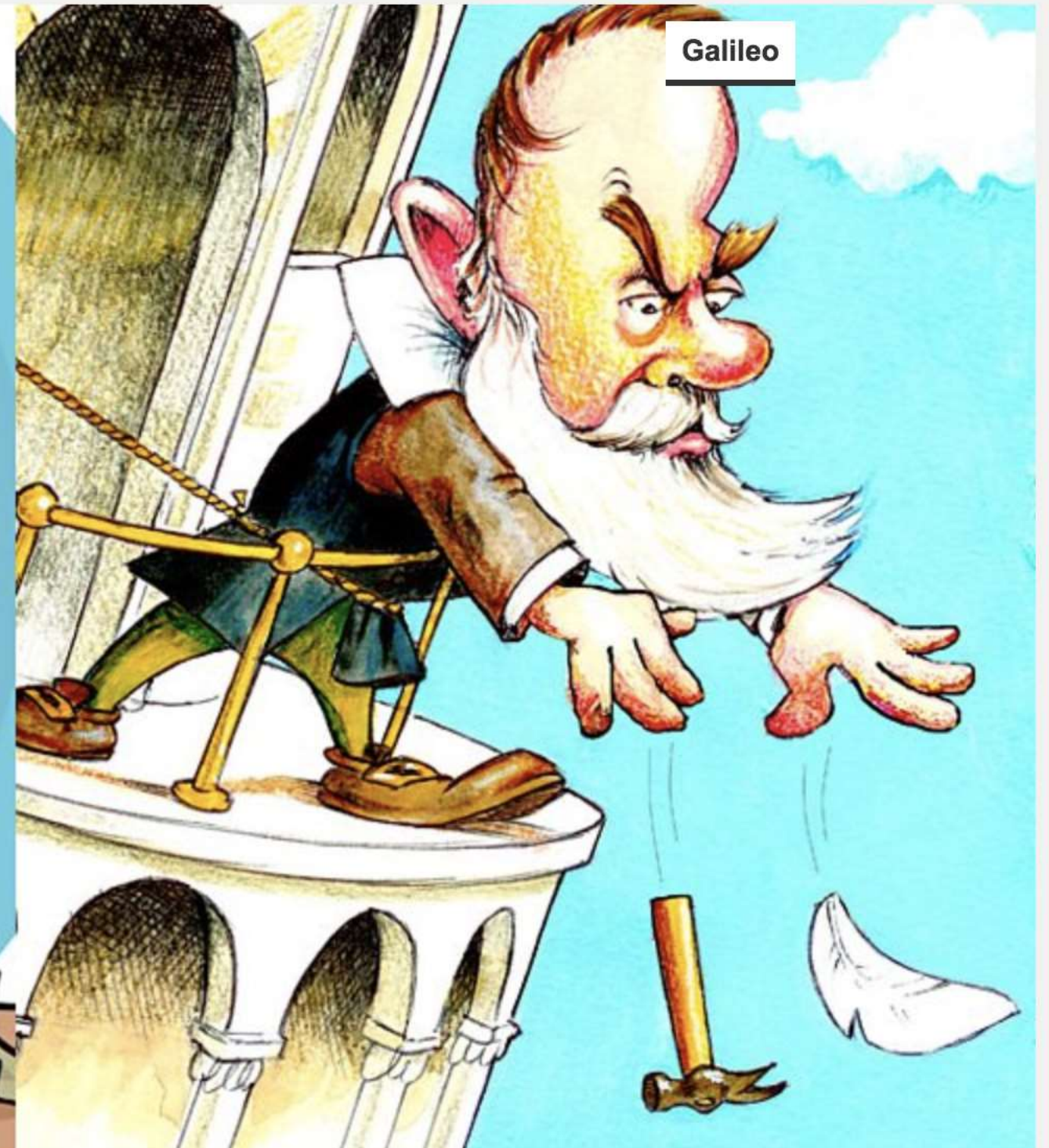


- Compensating the bias of classical accelerometers by coupling with a "perfect" accelerometer

- Use of Kalman filtering techniques for recalibration

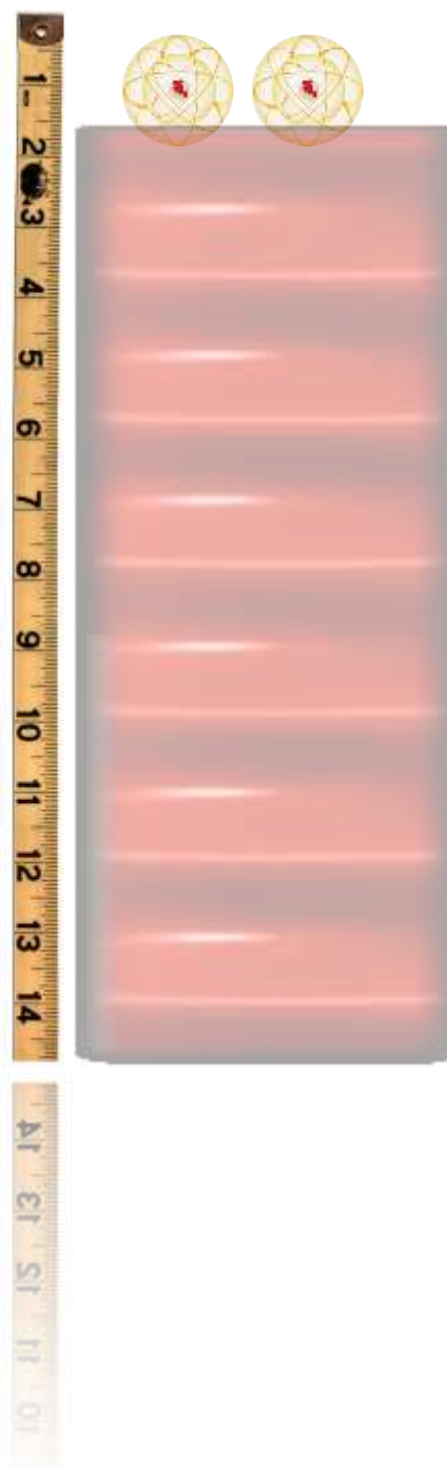


WHAT FALLS FASTER ?

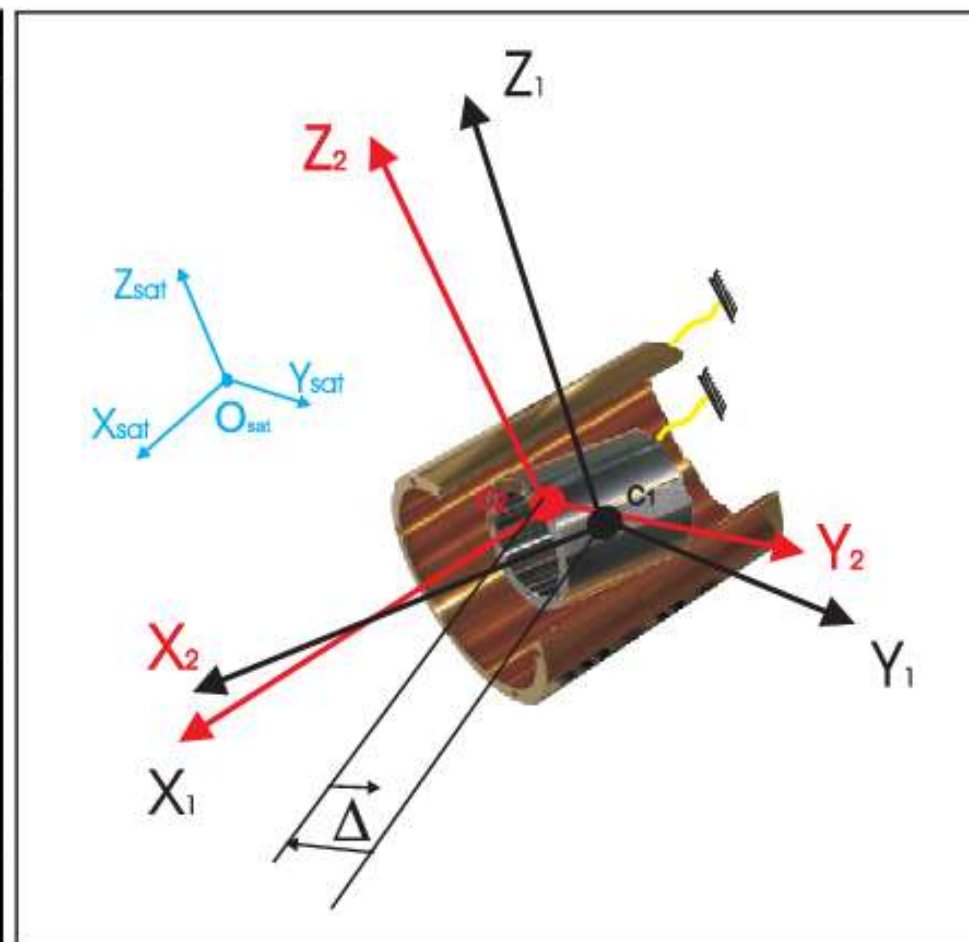
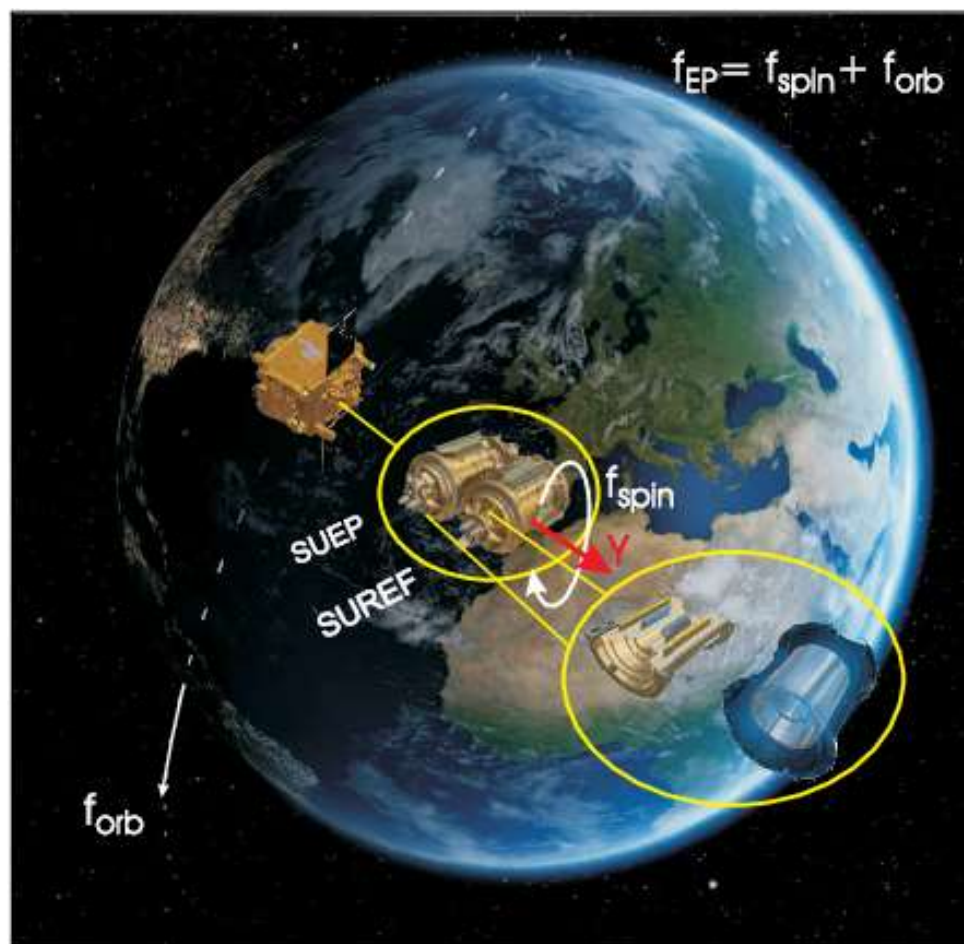


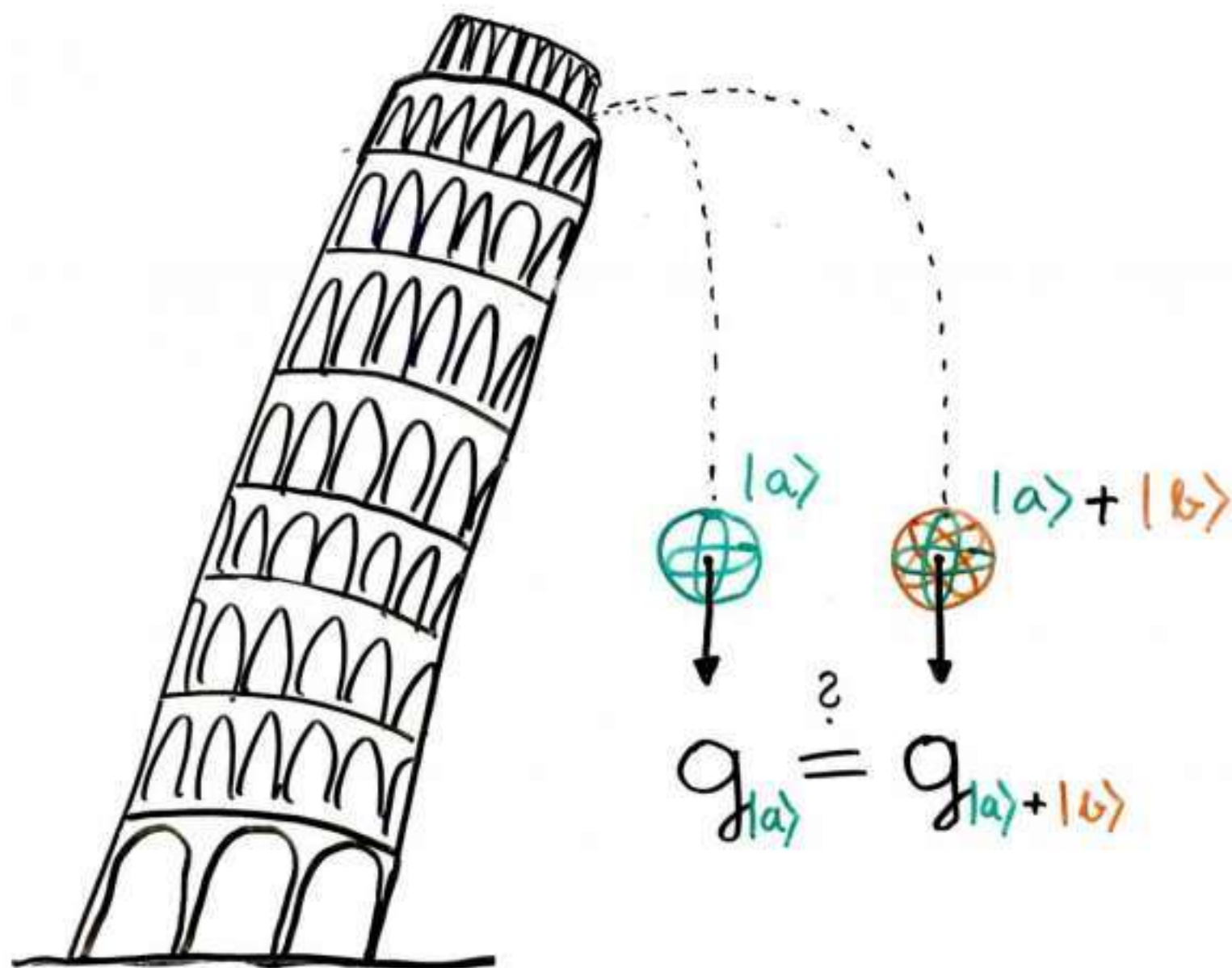
□ How to test and verify general relativity

- ⇒ Is gravity the same for two different atoms (or in other words is inertial mass equal to the gravitational mass) ?
- ⇒ Testing the Einstein Equivalence principle, a cornerstone of Relativity.



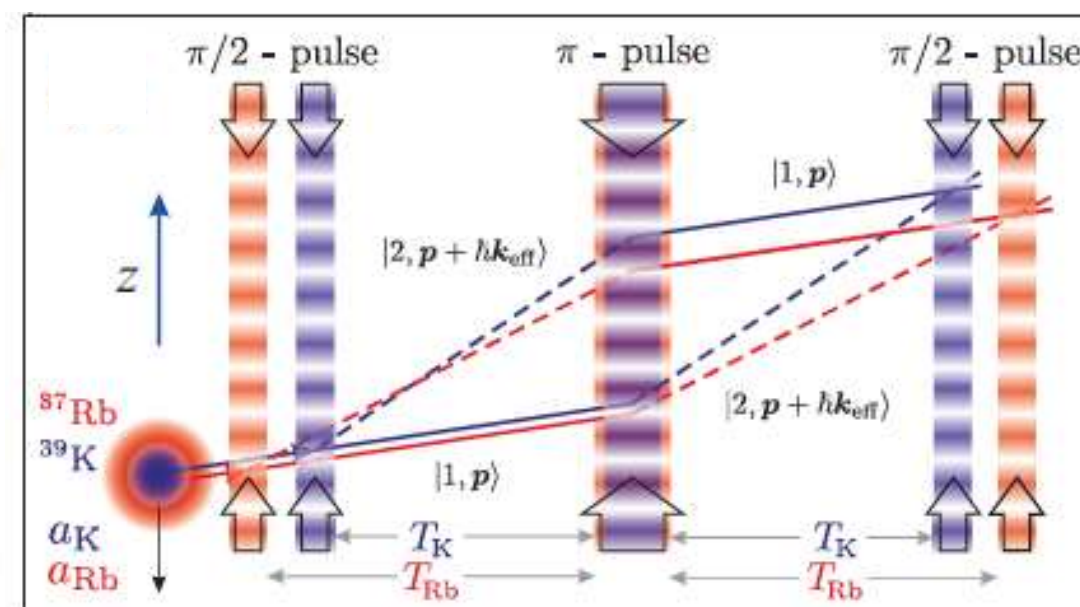
THE WEAK EQUIVALENCE PRINCIPLE





□ Do the same with atoms

- ⇒ Perfect inertial masses
- ⇒ Atoms can follow the exact same geodesic
- ⇒ Change of mass, composition, internal state ...

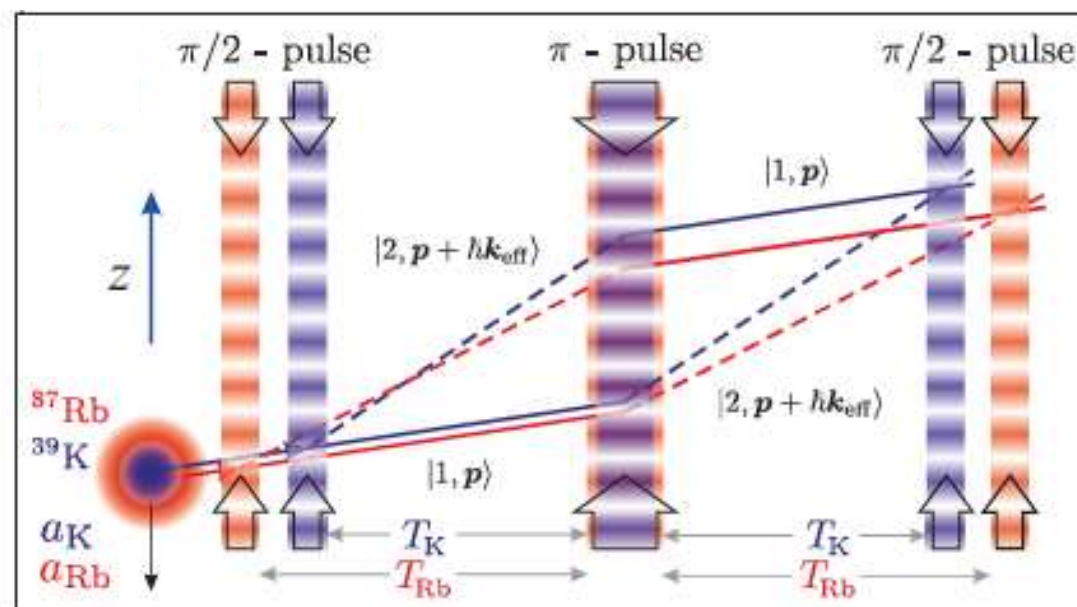
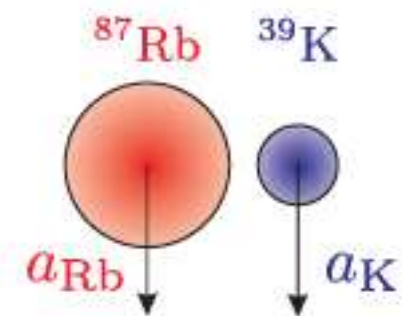


$$\phi_{\text{Rb}} = k_{\text{Rb}} a T^2$$

$$\phi_{\text{K}} = k_{\text{K}} a T^2 + \Delta\phi_{\text{d}}$$

Use of Potassium (767 nm) and Rubidium (780 nm)

- ⇒ High mass and composition difference (39 vs 85 amu, atomic number 19 vs 37)
- ⇒ Similar wavelength allow rejection and simple laser



$$\phi_{\text{Rb}} = k_{\text{Rb}} a T^2$$

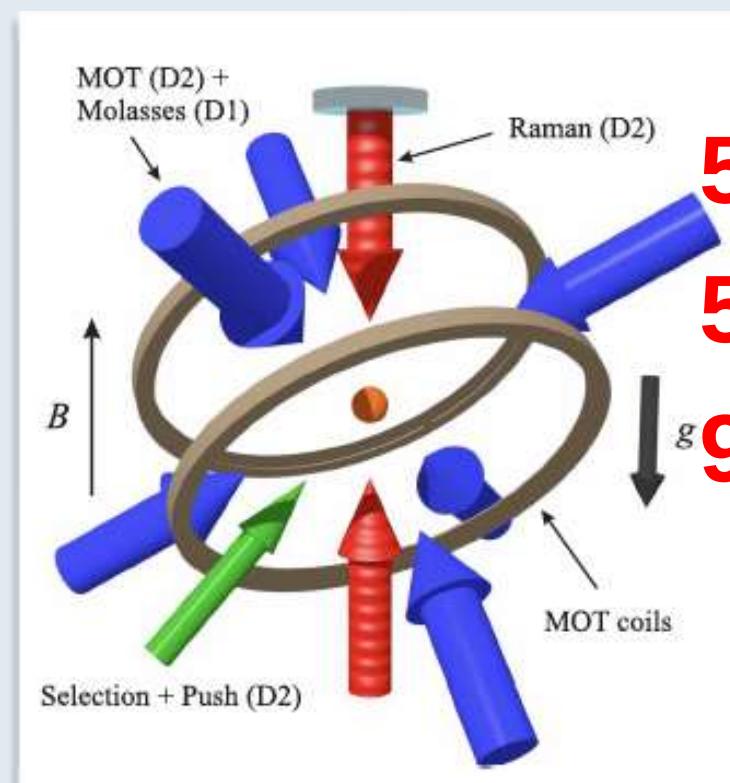
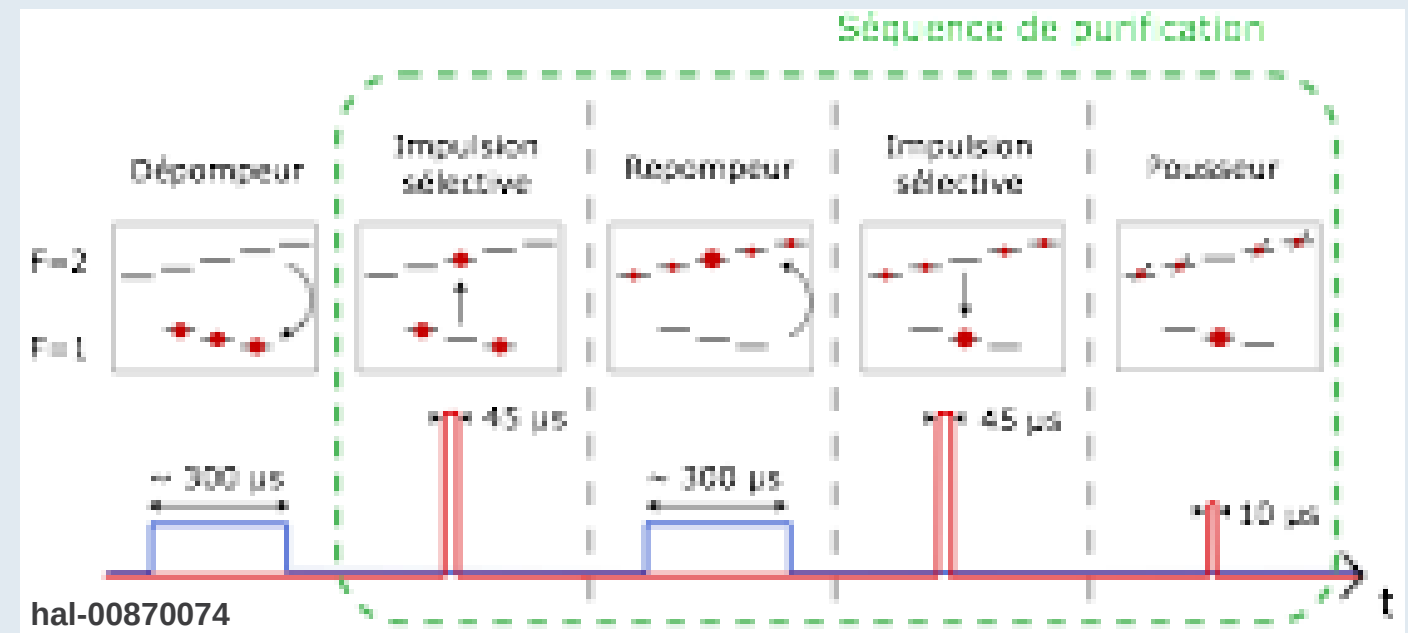
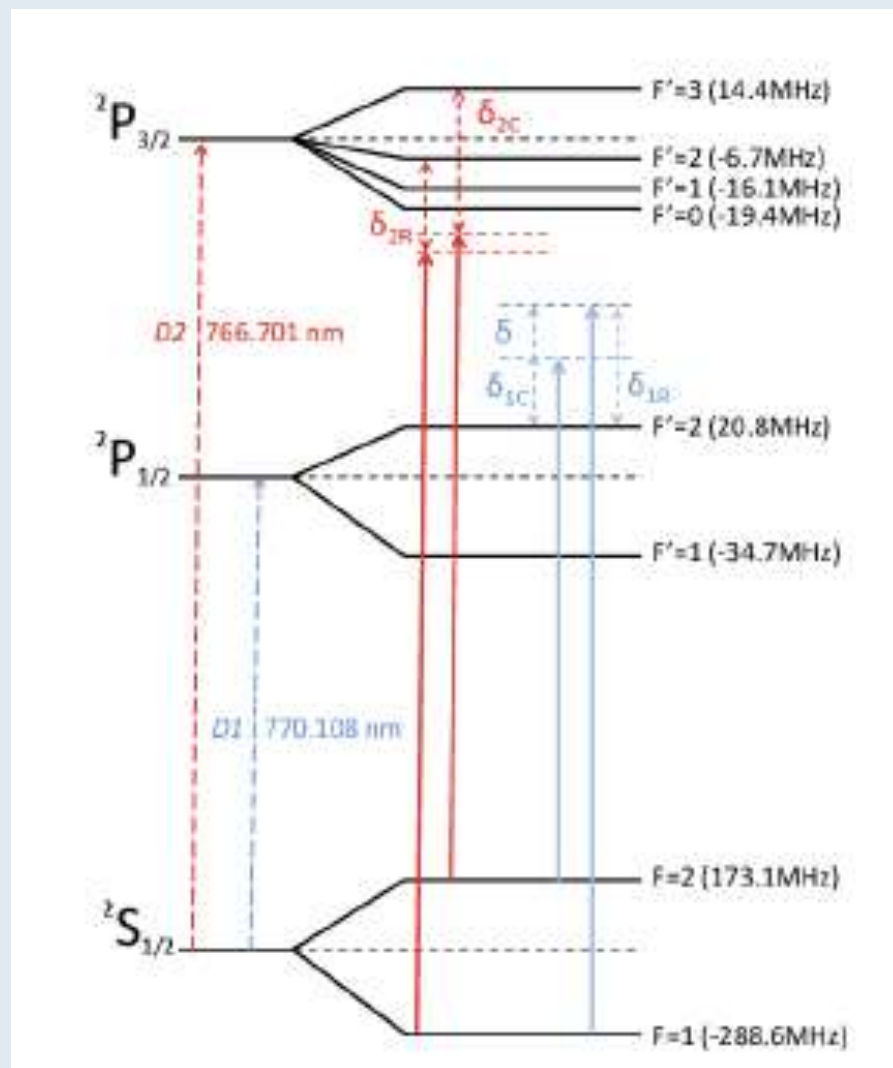
$$\phi_{\text{K}} = k_{\text{K}} a T^2 + \Delta\phi_{\text{d}}$$

$$\eta = \frac{a_{\text{K}} - a_{\text{Rb}}}{g}$$

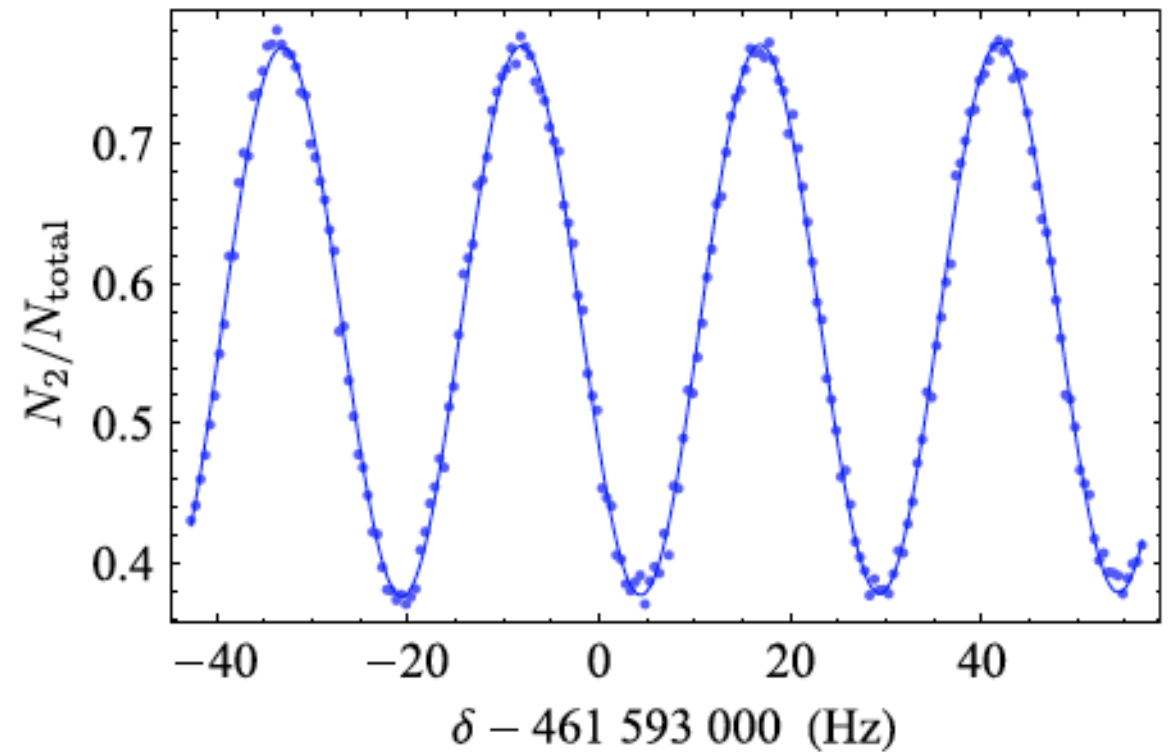
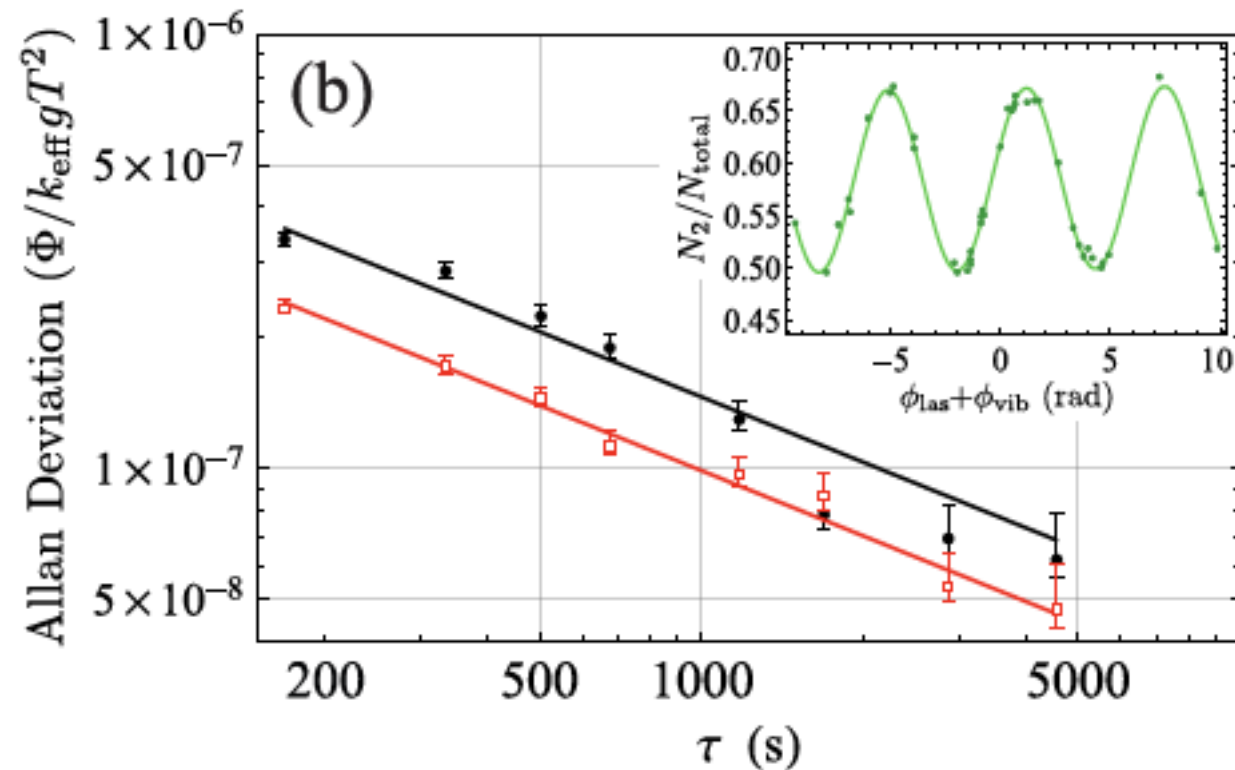
Potassium (^{39}K) is cooled via blue molasses

Salomon et al., EPL 104, 63002 (2014).

^{39}K contrast increased by state purification



5×10^7 atoms
 $5 \mu\text{K}$
95% purity $m_F=0$

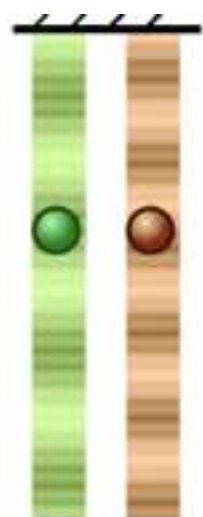


- **Gravity “measurement”**

- Short term 2 mg/ $\sqrt{\text{Hz}}$
- Long term 60 μGal after 1 hr

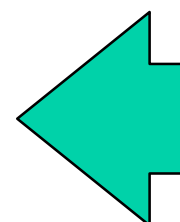
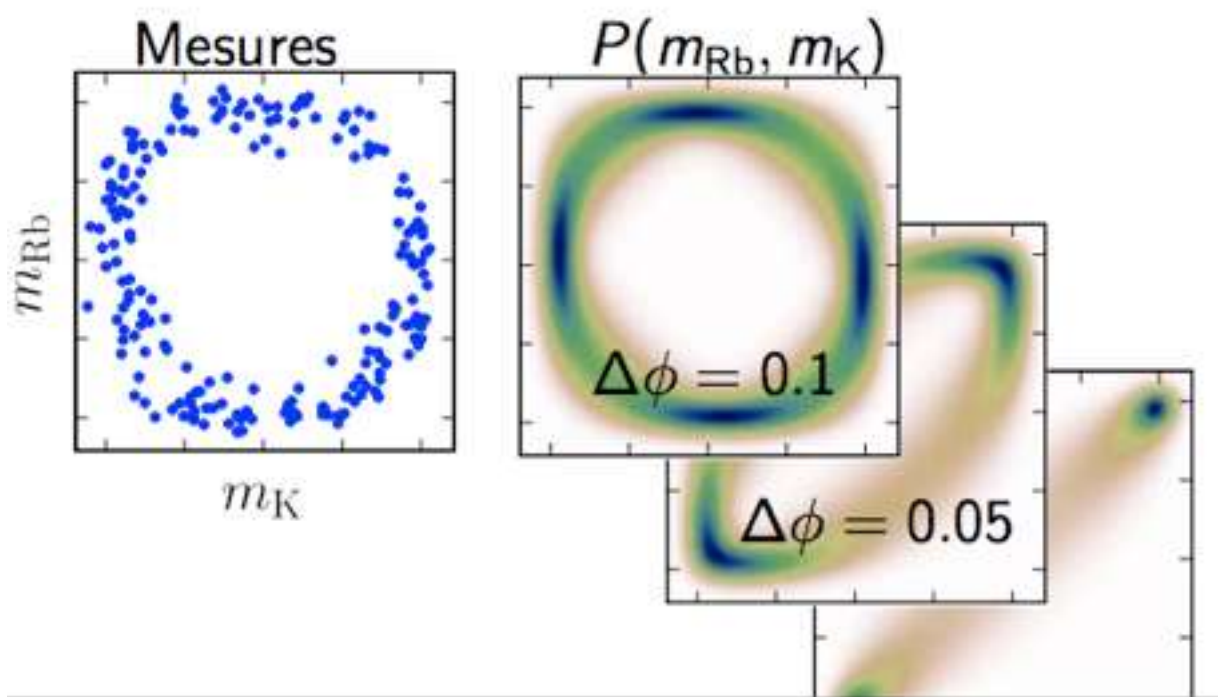
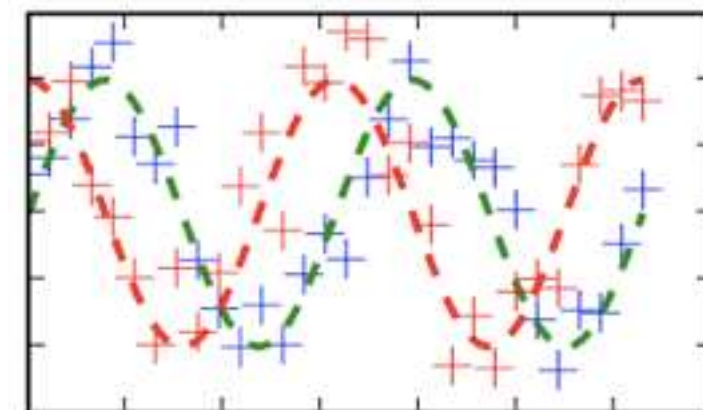
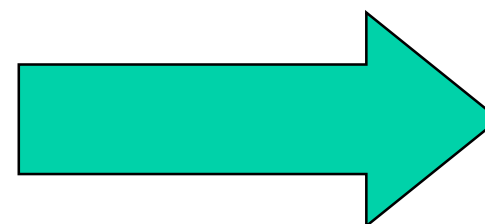
- **Hyperfine spectroscopy**

- Statistical $\Delta f/f$ at 4×10^{-11}
- Current systematic at 10^{-9}



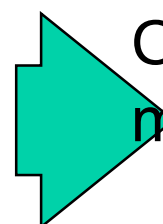
$$\begin{cases} \phi_K &= k_K (a_K T^2 + X_{\text{miroir}}) \\ \phi_{\text{Rb}} &= k_{\text{Rb}} (a_{\text{Rb}} T^2 + X_{\text{miroir}}) \end{cases}$$

$$\phi_{\text{BP}} = k_{\text{BP}} (a_{\text{BP}} T^2 + X_{\text{miroir}})$$

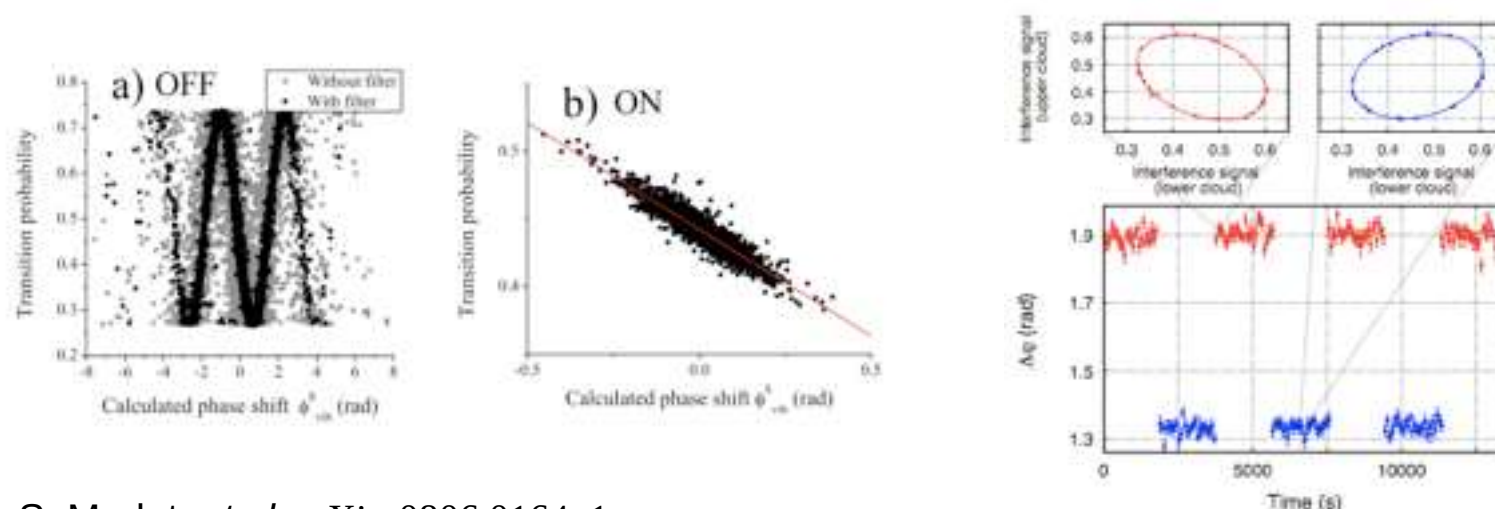


$$\begin{cases} m_K &= A + B \cos(k_K (a_K T^2 + X_{\text{miroir}})) \\ m_{\text{Rb}} &= C + D \cos(k_{\text{Rb}} (a_{\text{Rb}} T^2 + X_{\text{miroir}})) \end{cases}$$

$$\phi_{\text{BP}} = k_{\text{BP}} (a_{\text{BP}} T^2 + X_{\text{miroir}})$$



Comparison of two accelerometer signals to
measure $\Delta\phi \propto a_{\text{Rb}} T^2 \lambda_{\text{Rb}} - a_K T^2 \lambda_K$



S. Merlet, *et al.* arXiv:0806.0164v1

G. Lamporesi, *et al.* Phys. Rev. Lett. 100 050801 (2008)

How to estimate the differential acceleration in a two-species atom interferometer to test the equivalence principle

G Varoquaux^{1,3}, R A Nyman^{1,4}, R Geiger¹, P Cheinet¹, A Landragin² and P Bouyer¹

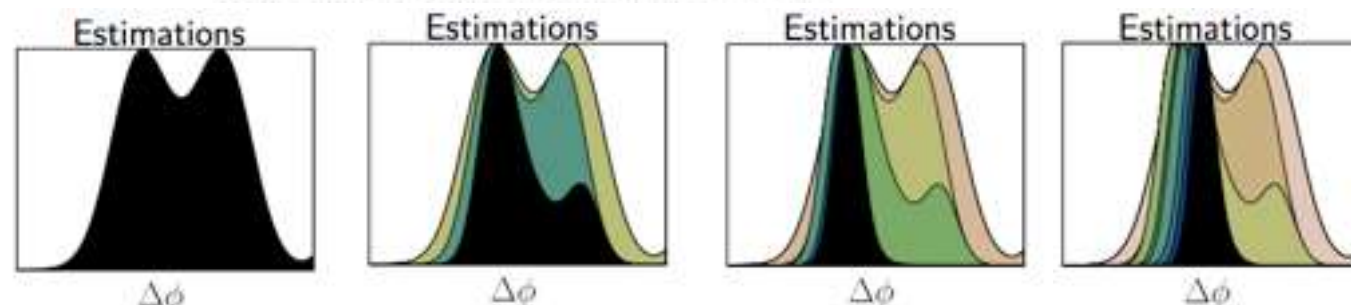
¹ Laboratoire Charles Fabry de l'Institut d'Optique, Campus Polytechnique, RD 128, 91127 Palaiseau, France

² LNE-SYRTE, UMR8630, UPMC, Observatoire de Paris, 61 avenue de l'Observatoire, 75014 Paris, France

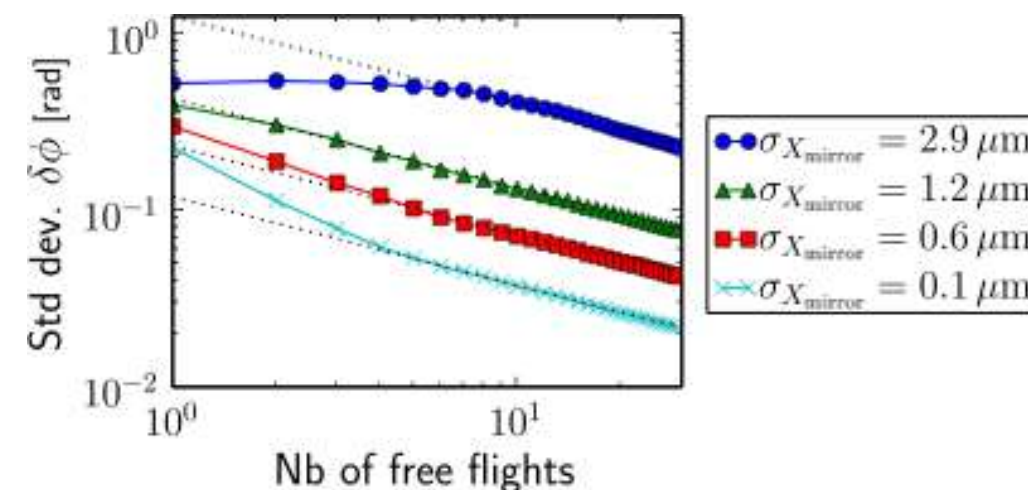
E-mail: philippe.bouyer@institutoptique.fr

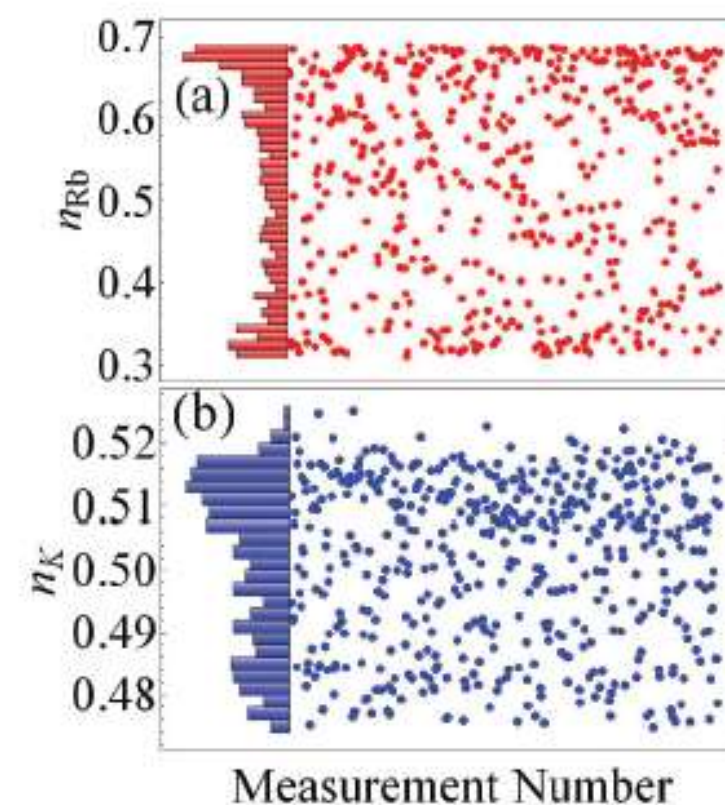
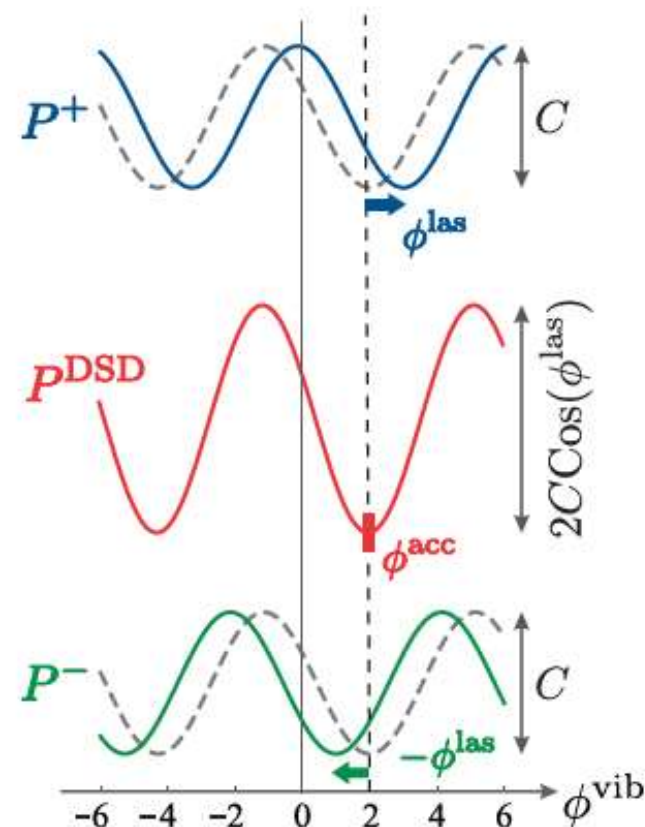
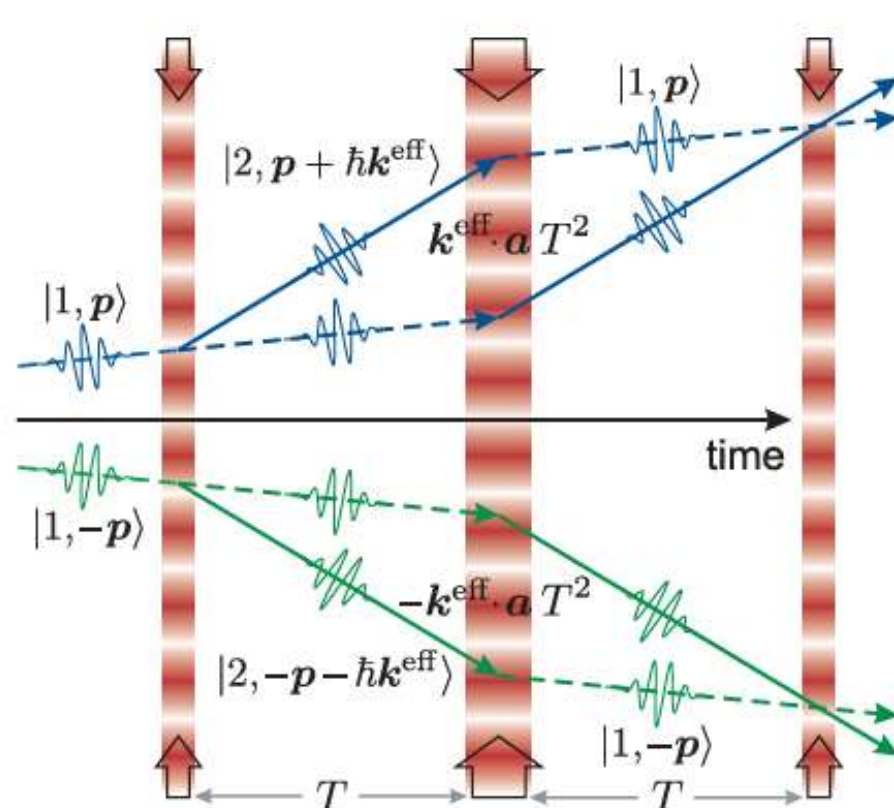
ct

New Journal of Physics 11 (2009) 113010 (14pp)

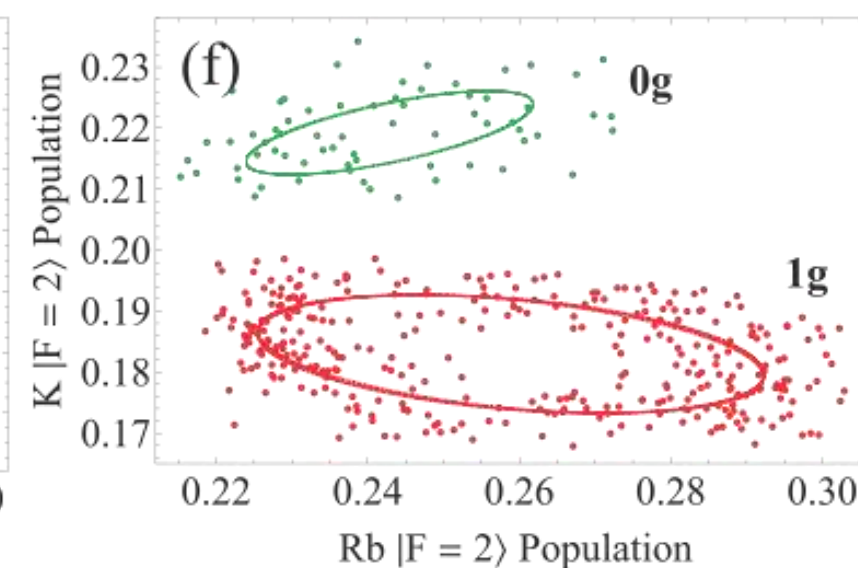
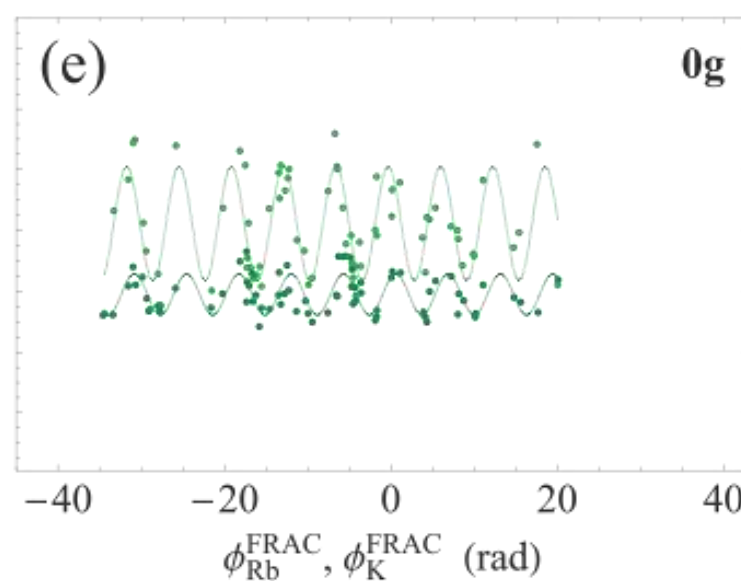
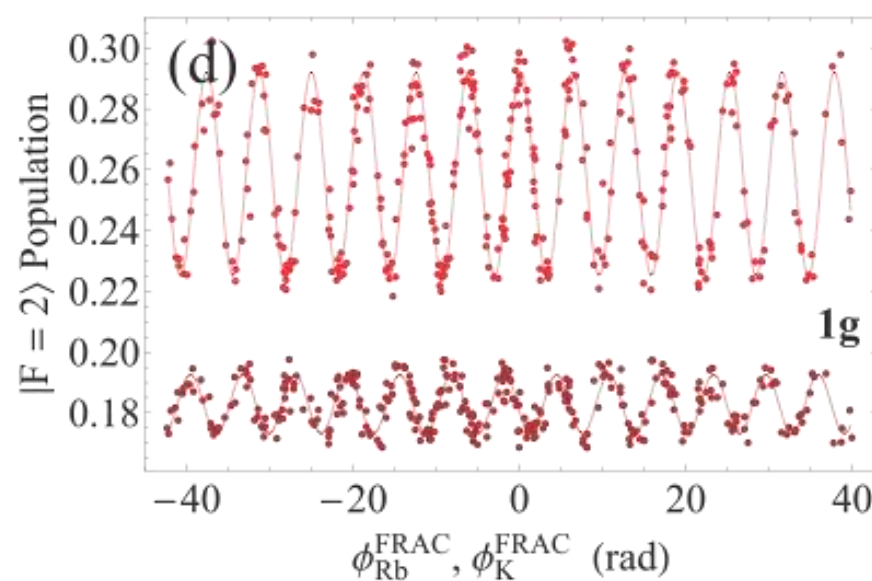


- Differential measurements
- Use of statistical estimators (Stokton *et al.* Phys. Rev. A 76, 033613 (2007))
- Fast convergence (<10 meas.)





Fringe reconstruction by accelerometer correlation



GAIN in 0-g

LOSS in 0-g

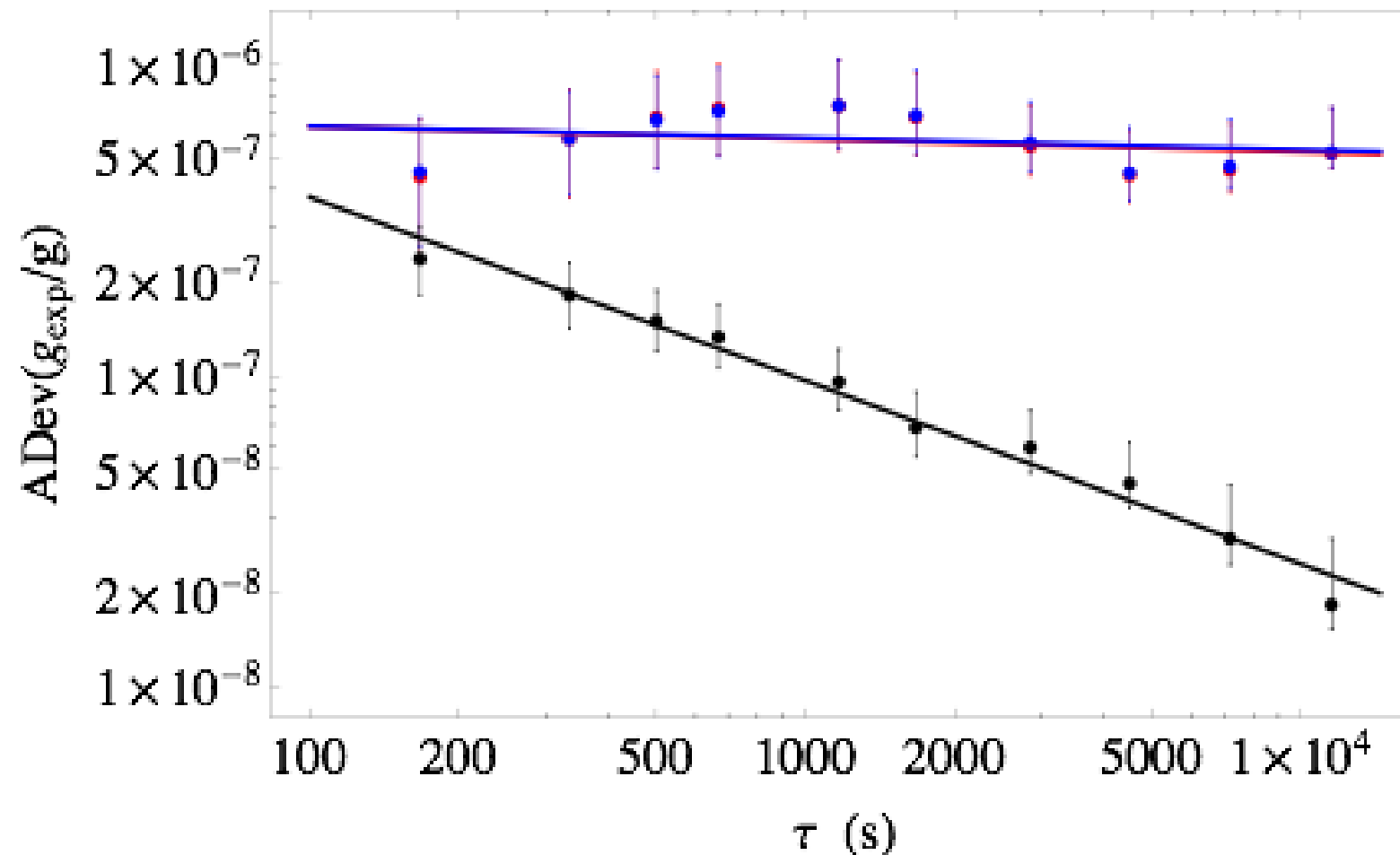
Systematic effect	$+k^{\text{eff}}$ Interferometers in 1g		$\pm k^{\text{eff}}$ Interferometers in 0g				Unit
	$\phi_{\text{Rb}}^{\text{sys}}$	$\phi_{\text{K}}^{\text{sys}}$	$\phi_{\text{Rb}}^{\text{dep}}$	$\phi_{\text{Rb}}^{\text{ind}}$	$\phi_{\text{K}}^{\text{dep}}$	$\phi_{\text{K}}^{\text{ind}}$	
Quadratic Zeeman	2.351(51)	33.78(73)	—	2.355(36)	—	33.83(52)	rad
Magnetic gradient	0.0302(90)	0.91(24)	0.0196(44)	1.90(43)E-5	0.75(0.13)	0.00126(23)	rad
Coriolis effect	-1.06(58)	-1.8(1.0)	13.2(1.5)	-0.600(69)	17.9(2.3)	-1.64(21)	mrاد
One-photon light shift	-2.1(3.6)	-51(81)	—	-2.1(3.6)	—	-51(81)	mrاد
Two-photon light shift	1.3(2.8)	16(68)	1.3(2.0)	—	16(48)	—	—
Extra laser lines	-0.18(10)	—	0.030(26)	0.19(16)	—	—	—
FRAC method	0.0(4.1)	0.0(4.2)	0.0(4.1)	—	0.0(4.2)	—	mrاد
Gravity gradient	0.022(20)	0.061(20)	5.4(5.2)E-6	0.015(14)	1.60(59)E-5	0.039(14)	μrad
Total	2.379(52)	34.65(78)	0.0341(65)	2.352(36)	0.78(14)	33.78(52)	rad

Atoms stay still

Large rotation

EP sensitivity : 10^{-5} , EP accuracy 10^{-4}

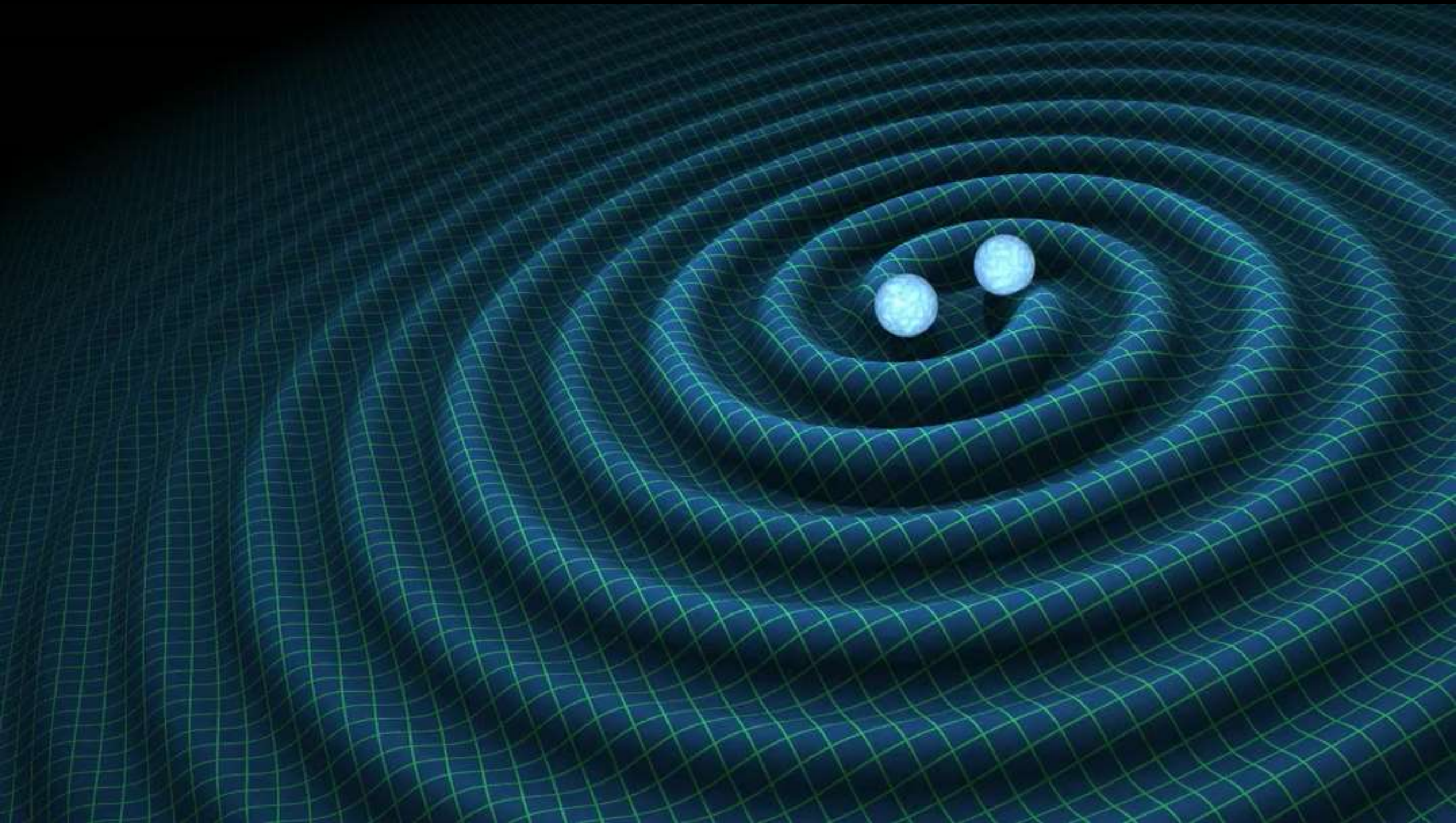
	$+k^{\text{eff}}$ Interferometers in 1g			$\pm k^{\text{eff}}$ Interferometers in 0g			Unit
	Rb	K	Difference	Rb	K	Difference	
ϕ_j^{FRAC}	3.291(18)	1.345(32)	1.192(37)	2.854(75)	3.71(10)	0.85(13)	rad
$\phi_j^{\text{acc}} + \phi_j^{\text{las}}$	0.778	1.255	0.476	0.4667	0.4672	3.33E-5	rad
ϕ_j^{sys}	2.379(52)	34.65(78)	0.85(78)	0.0341(65)	0.78(14)	0.74(14)	rad
ϕ_j^{DSD}	—	—	—	-0.0480(34)	-0.0469(49)	1.15(59)E-3	rad
Φ_j	0.134(55)	-34.56(78)	-0.13(78)	2.401(76)	2.50(18)	0.10(20)	rad
η	—	—	-0.2(1.2)E-3	—	—	1.5(3.1)E-4	



UFF test on ground (20 ms)
: statistical error at 1.5×10^{-8}

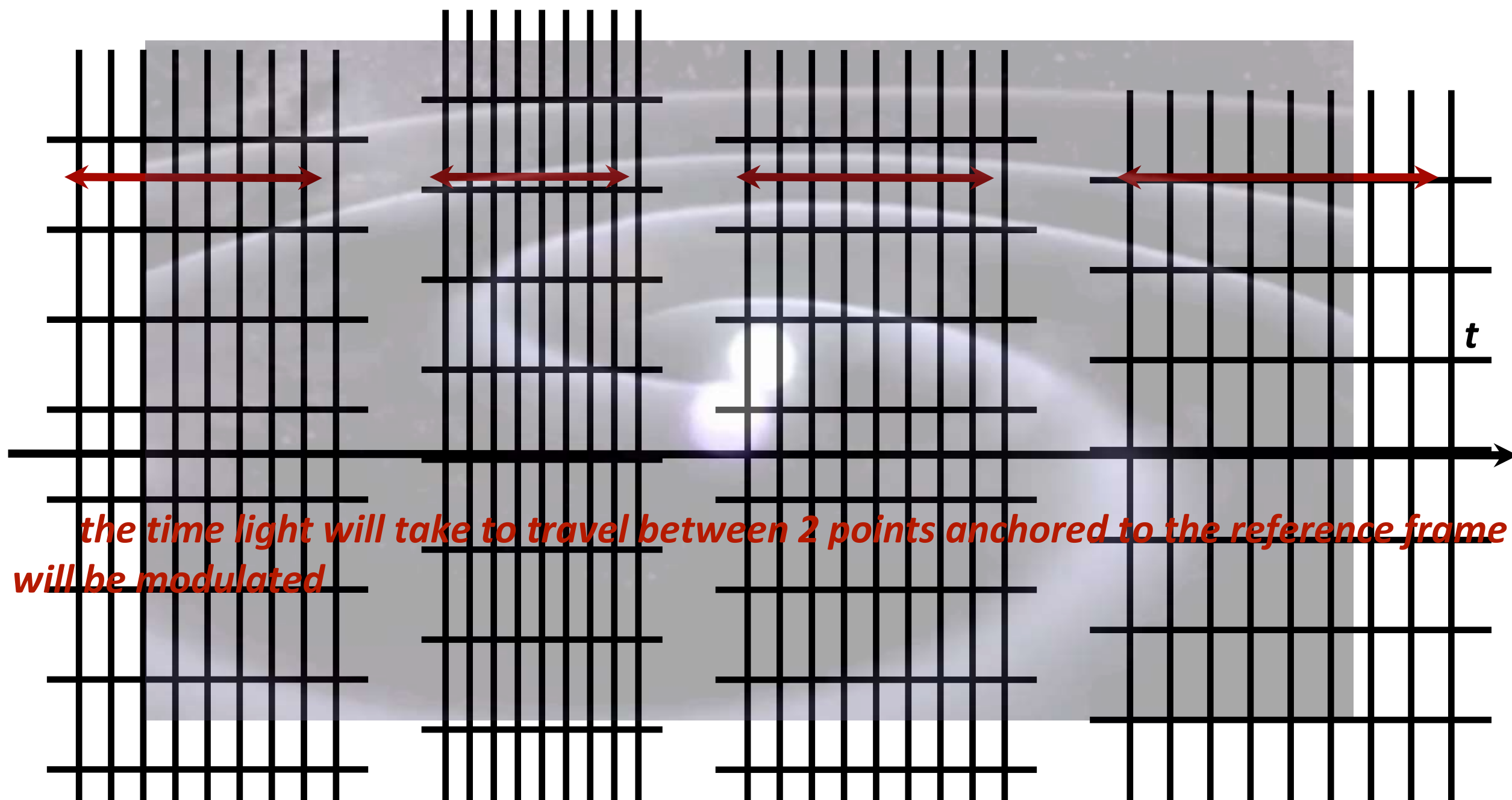
Still some systematic effects to track

Systematic effect of $\Delta\eta=10^{-6}$
Calibrated at 10% for
Quadratic Zeeman effect
and 2-photon light shift



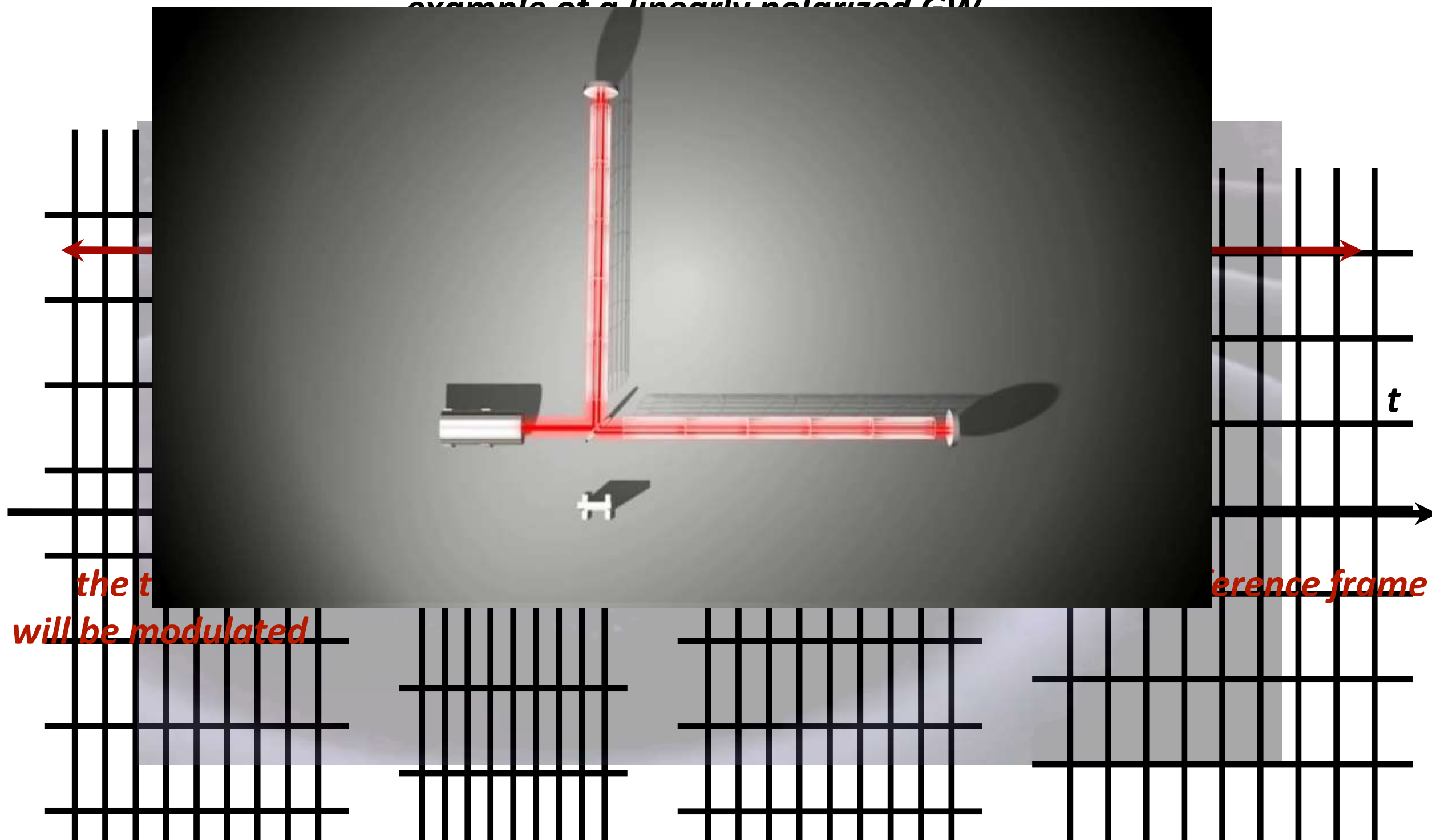
Gravitational waves

Gravity waves will distort space time *example of a linearly polarized GW*



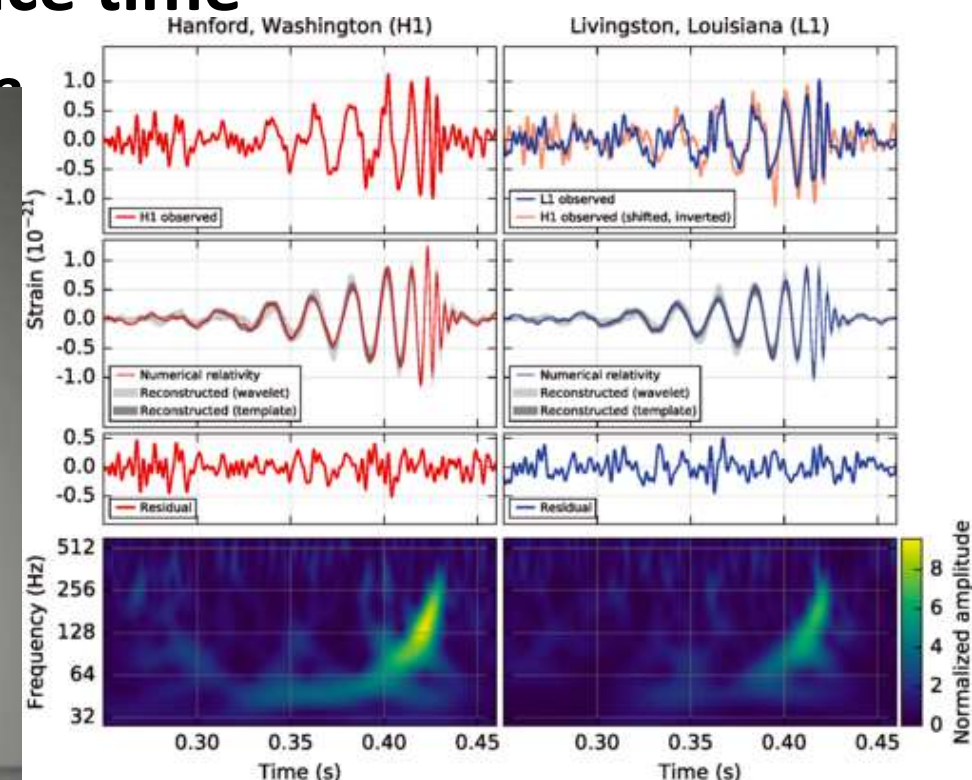
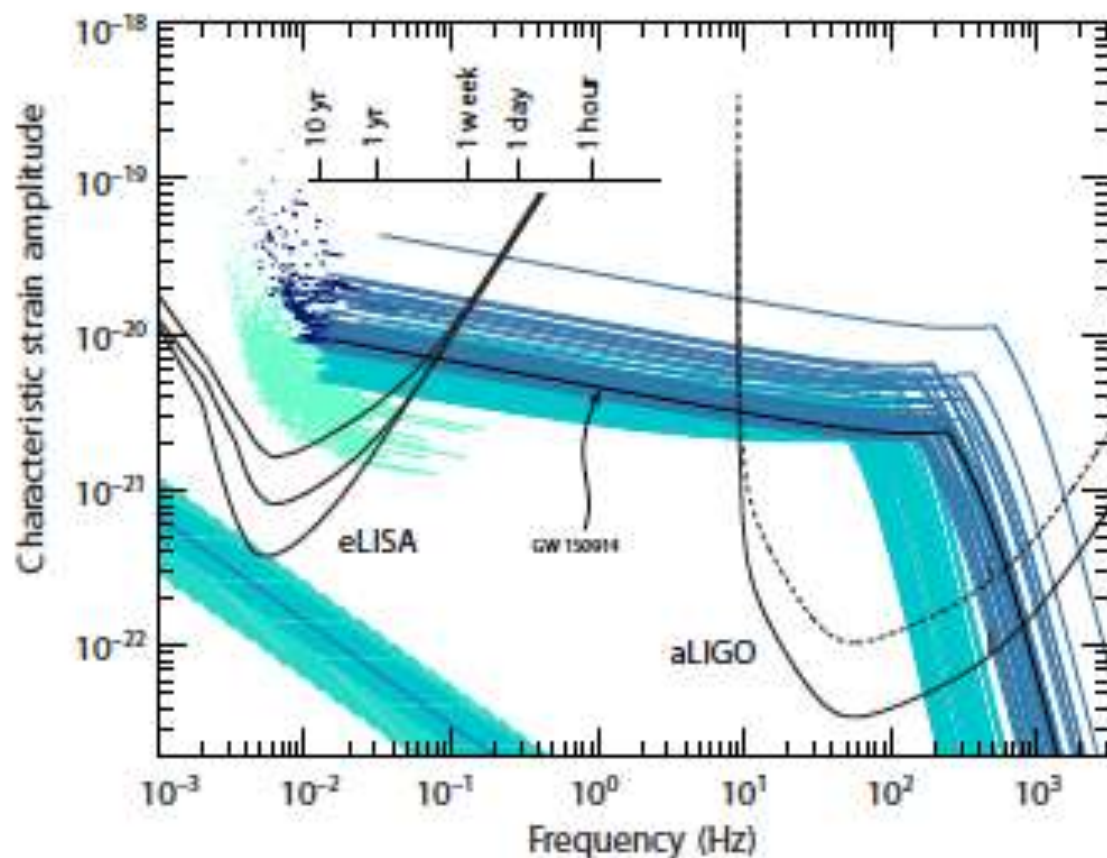
Gravity waves will distort space time

example of a linearly polarized GW



Gravity waves will distort space time

example of a linearly polarized



reference frame

EQUIPEX 2011 : a new initiative to prepare the next generation of gravitation antenna

Underground infrastructure
for geophysics and
gravitational wave detection

15 institutes - 3 compagnies

Geoscience

Gravity sensitivity of 10^{-10} g/Sqrt(Hz) @ 2Hz

Gradient sensitivity of 10^{-13} s²/Sqrt(Hz) @ 2Hz

(resolves motion of 1m³ of water at of 100m). Applications in geology, hydrogeology...

Gravitational wave physics

Different limitations with respect to optical detectors.

Interesting astrophysics sources in the subHz region.

www.matterwave-antenna.org

Laboratoire(s)/ Laboratory
Laboratoire Photonique, Numérique et Nanosciences – LP2N
Laboratoire Souterrain Bas Bruit – LSBB
Systèmes de Référence Temps – Espace - SYRTE
Astrophysique Relativiste Théories Expériences Métrologie Instrumentation Signaux - ARTEMIS
Centre Lasers Intenses et Applications - CELIA
Laboratoire Kastler-Brossel – LKB
Astroparticule et Cosmologie – APC
GEOAZUR
Géologie des Systèmes et des Réservoirs Carbonatés - GSRC Environnement Méditerranéen et Modélisation des Agro- Hydrosystèmes - EMMAH Institut Pluridisciplinaire de Recherche Appliquée dans le domaine du génie pétrolier - IPRA IDES
Laboratoire d'Electronique Antennes et Télécommunication - LEAT Geosciences Montpellier
Institut de Physique du Glode de Strasbourg - IPGS
Entreprise(s) / company
ALPHANOV
MUQUANS
SOLETANCHE BACHY TUNNELS

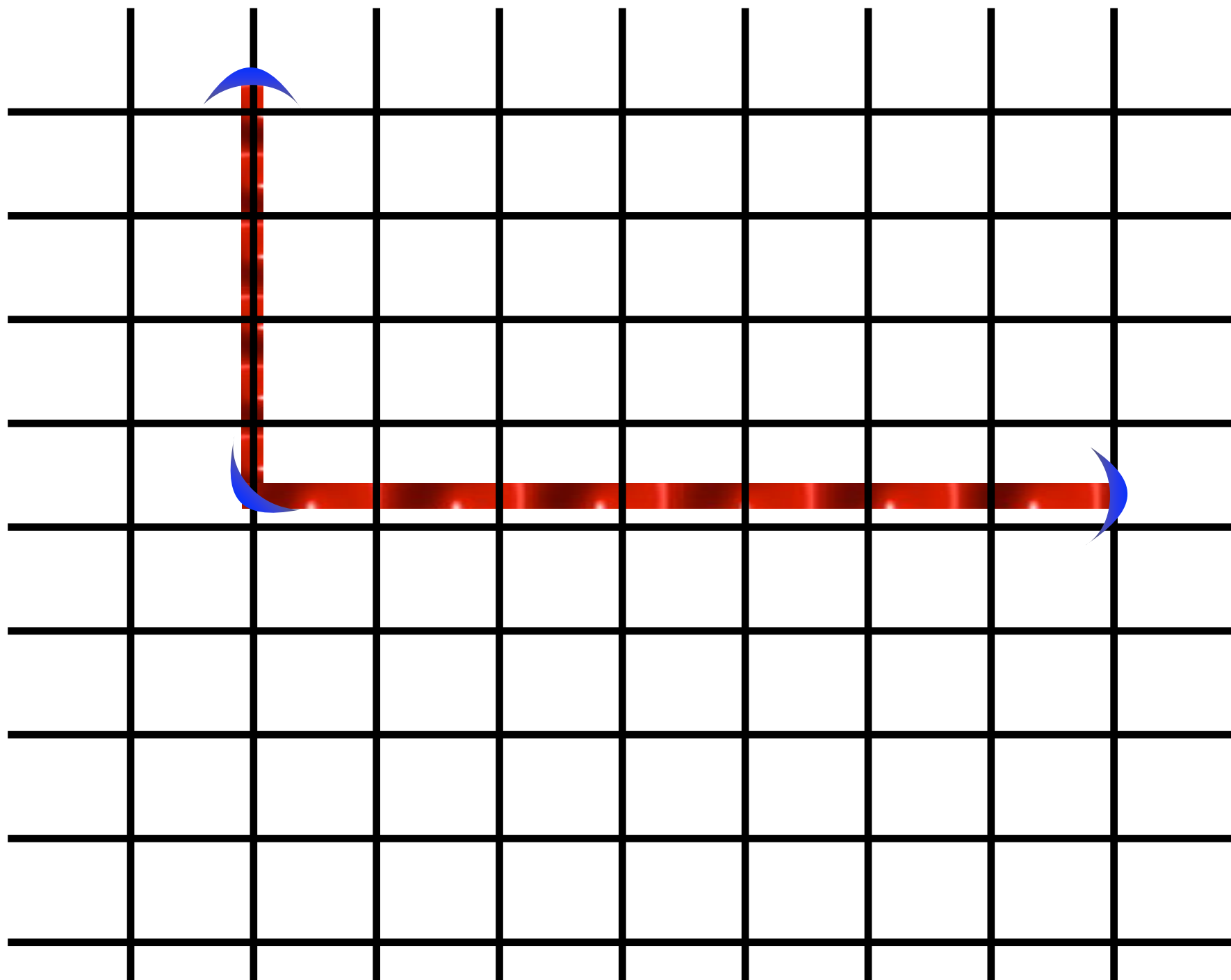


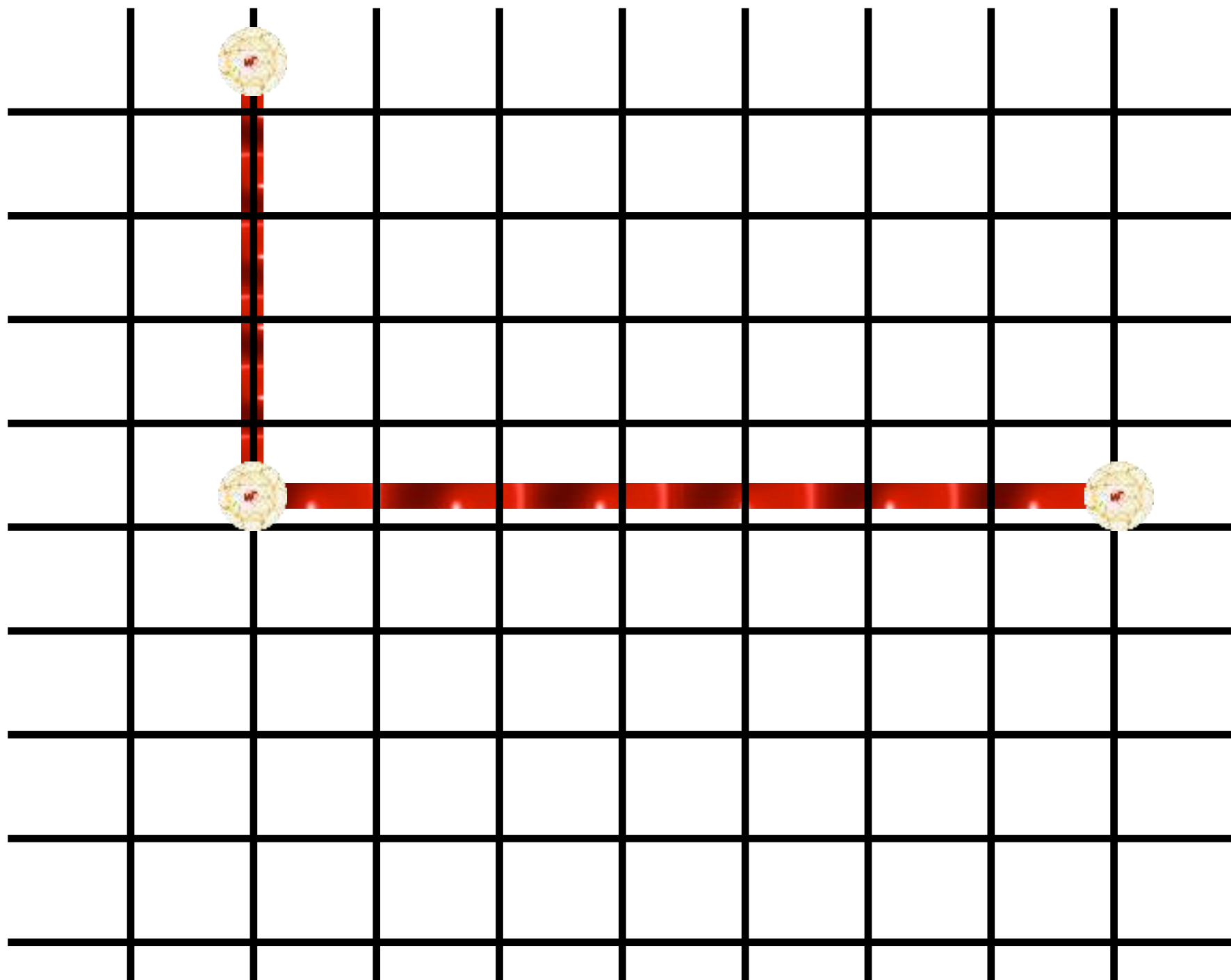
Observatoire
de la COTE d'AZUR

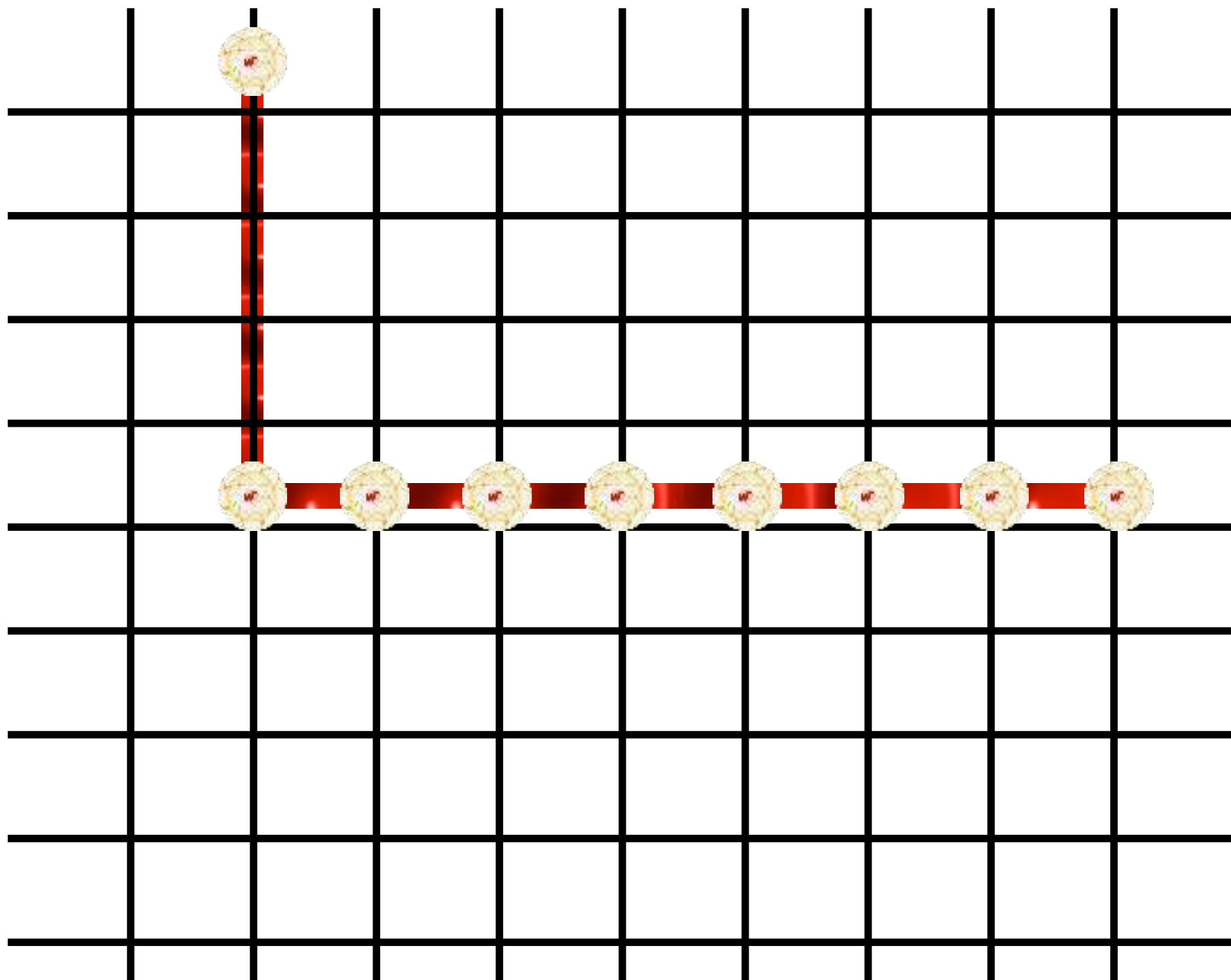


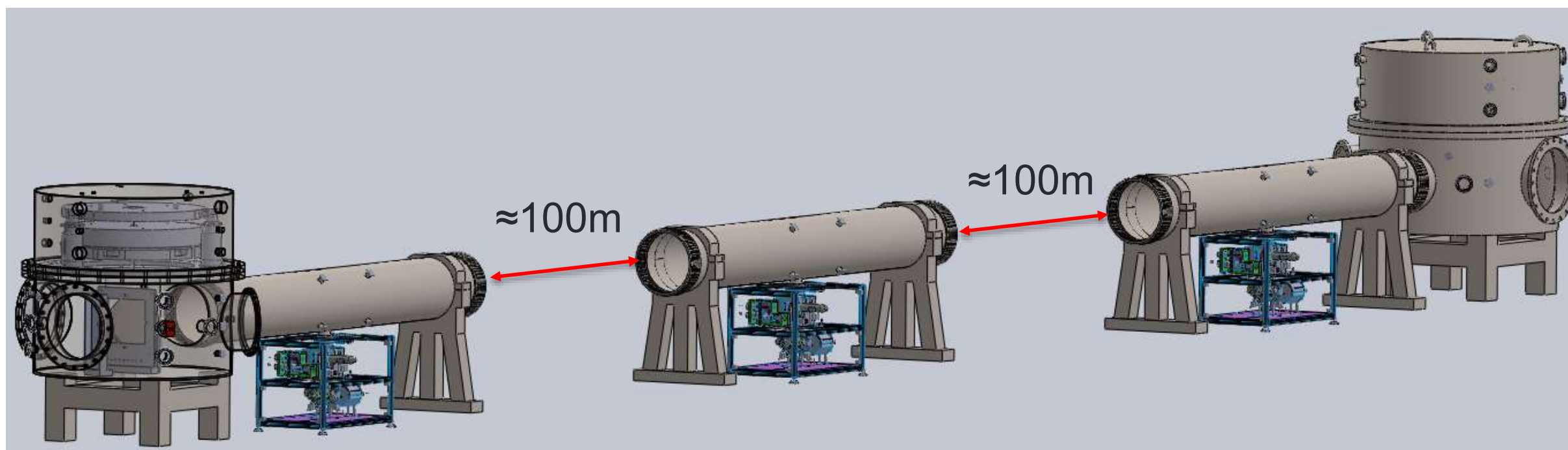
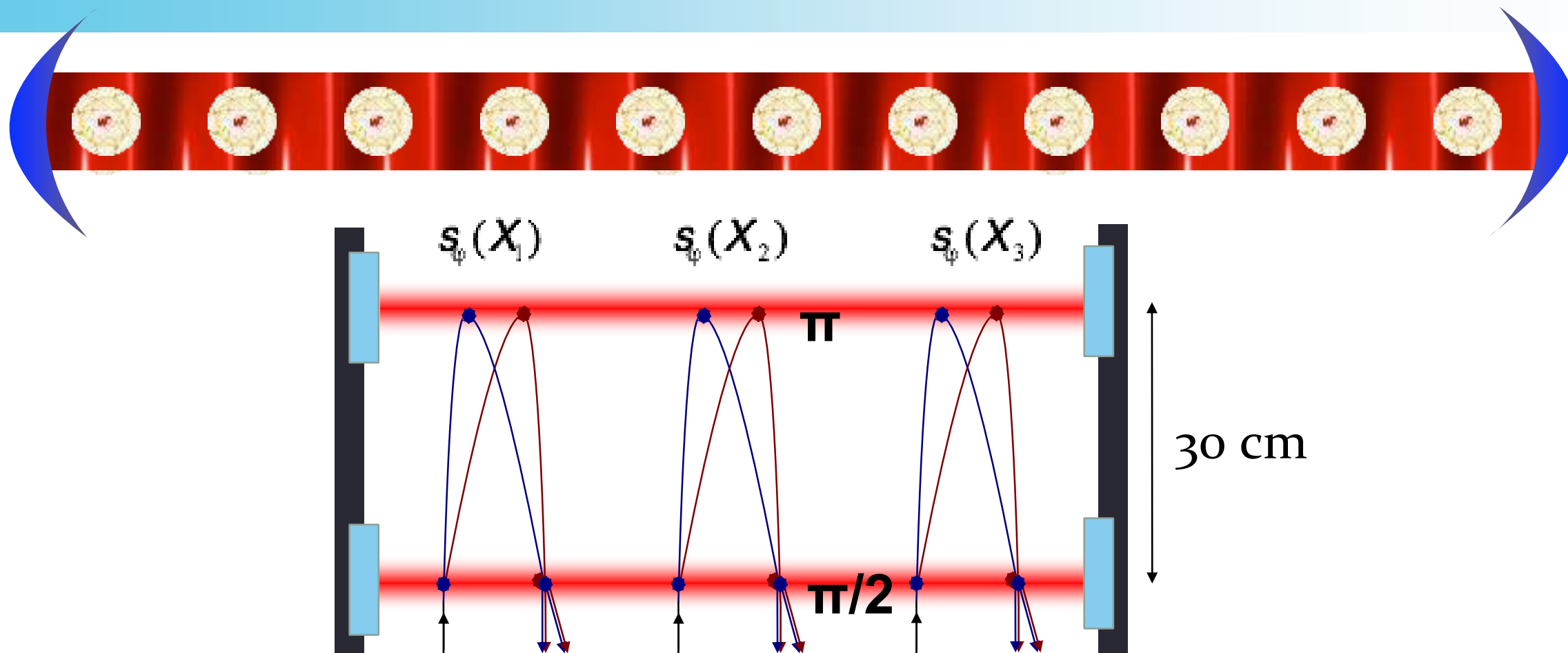
Systèmes de Référence Temps-Espace

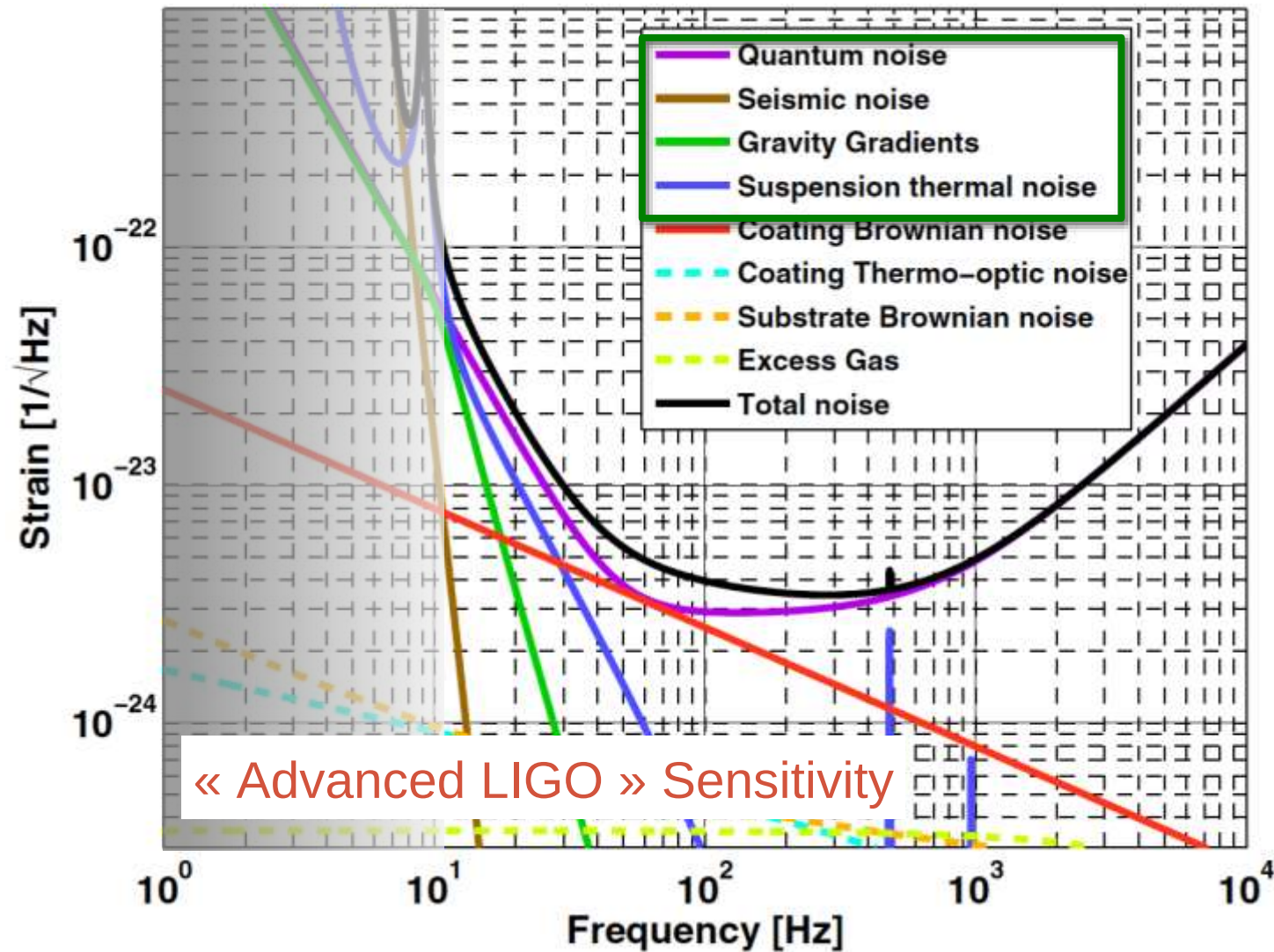






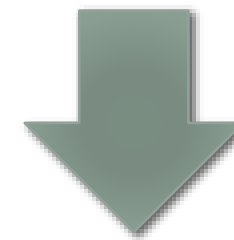






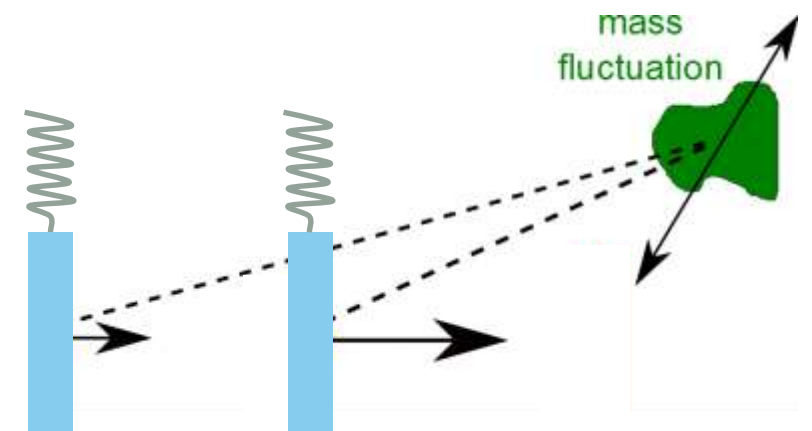
Limitations for $f < 10$ Hz:

- Radiation pressure noise
- Imperfections of Mirror suspensions
- « Gravity gradient » noise

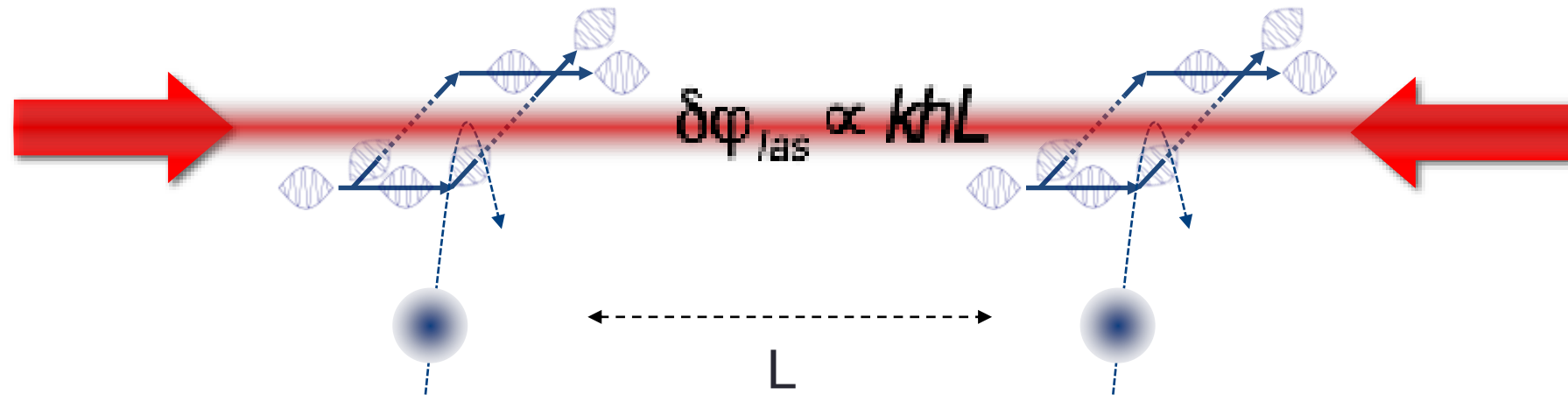


« Gravity Gradient » noise

Fluctuations of the Earth gravity field



Let's use free falling atoms as “test masses” instead of mirrors



PHYSICAL REVIEW D **78**, 122002 (2008)

Atomic gravitational wave interferometric sensor

Savas Dimopoulos,^{1,*} Peter W. Graham,^{2,†} Jason M. Hogan,^{1,‡} Mark A. Kasevich,^{1,§} and Surjeet Rajendran^{1,2,||}

¹Department of Physics, Stanford University, Stanford, California 94305, USA

²SLAC, Stanford University, Menlo Park, California 94025, USA

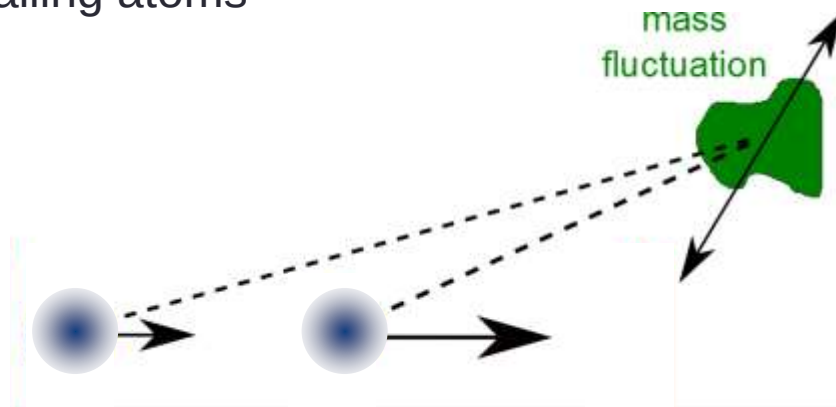
(Received 28 August 2008; published 19 December 2008)

Enable to overcome:

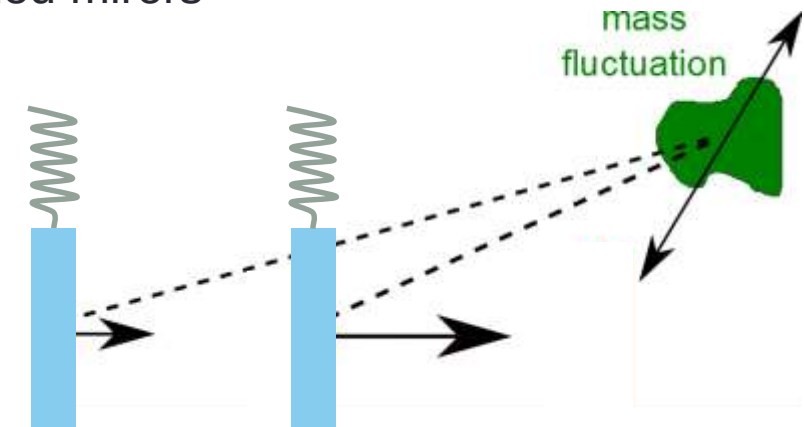
- Limitations related to suspension systems.
- Radiation pressure noise.

Sensitivity to Gravity Gradient Noise is the same !

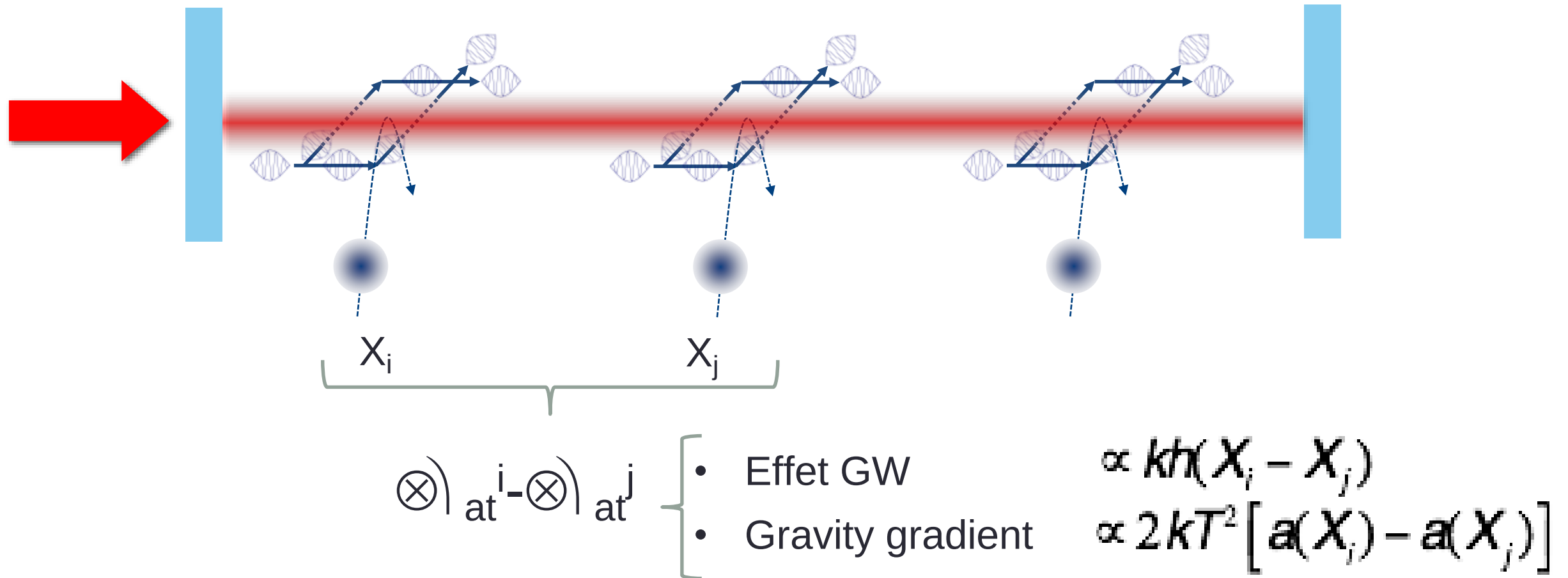
Free falling atoms



Suspended mirrors



Example of the MIGA Geometry



Discrimination between GW effects and gravity gradients using the spatial resolution of the antenna

PHYSICAL REVIEW D 93, 021101(R) (2016)

Low frequency gravitational wave detection with ground-based atom interferometer arrays

W. Chaibi,^{1,*} R. Geiger,^{2,†} B. Canuel,³ A. Bertoldi,³ A. Landragin,² and P. Bouyer³

¹ARTEMIS, Université Côte d'Azur, CNRS and Observatoire de la Côte d'Azur, F-06304 Nice, France

²LNE-SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ. Paris 06, 61 avenue de l'Observatoire, 75014 Paris, France

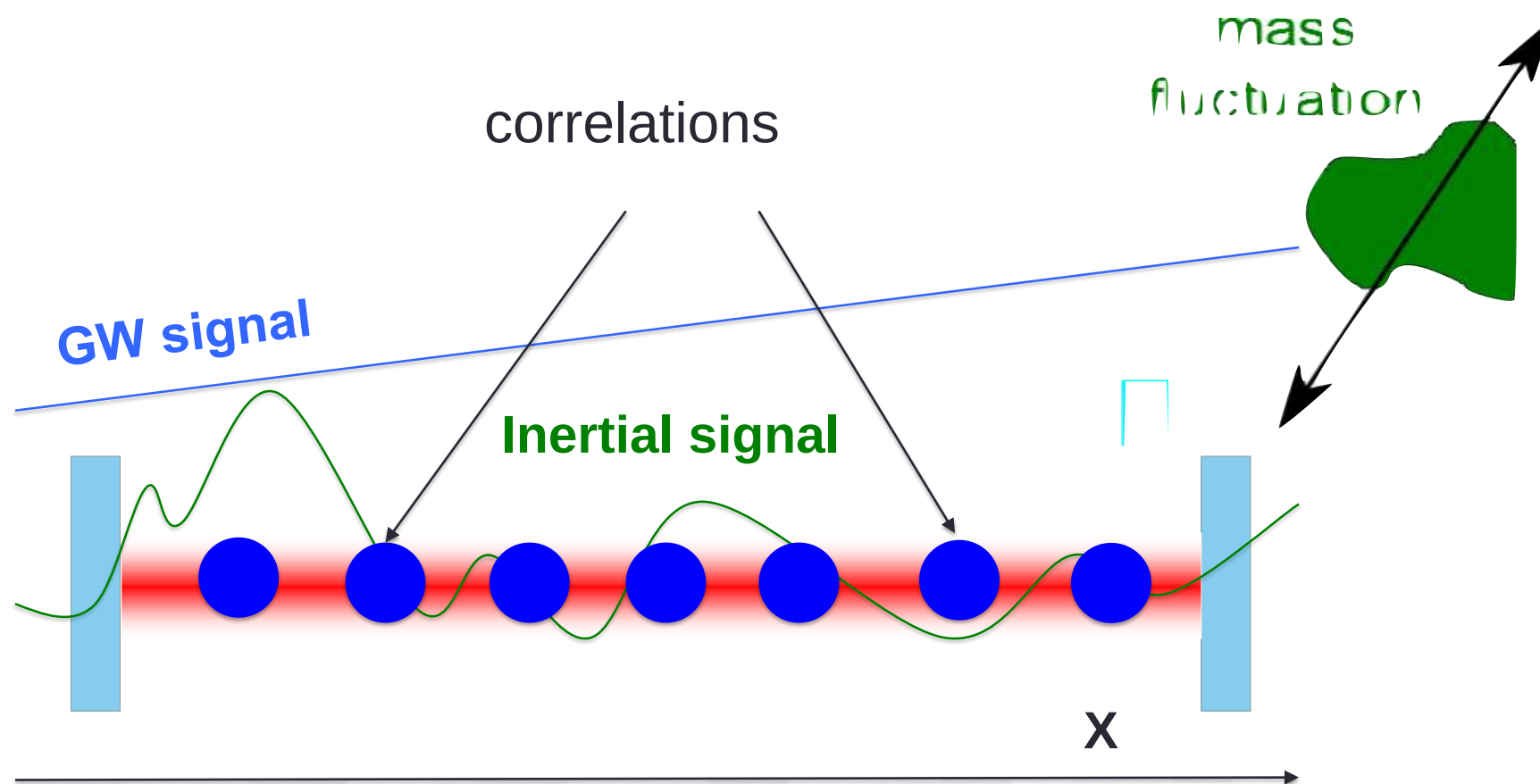
³LP2N, Laboratoire Photonique, Numérique et Nanosciences Université Bordeaux-IOGS-CNRS:UMR 5298, rue Mitterrand, F-33400 Talence, France

(Received 23 June 2015; published 15 January 2016)

- Low frequency (10^{-2} -10 Hz) GW detection limited by detection noise
- Measures of the local gravity field = Geoscience

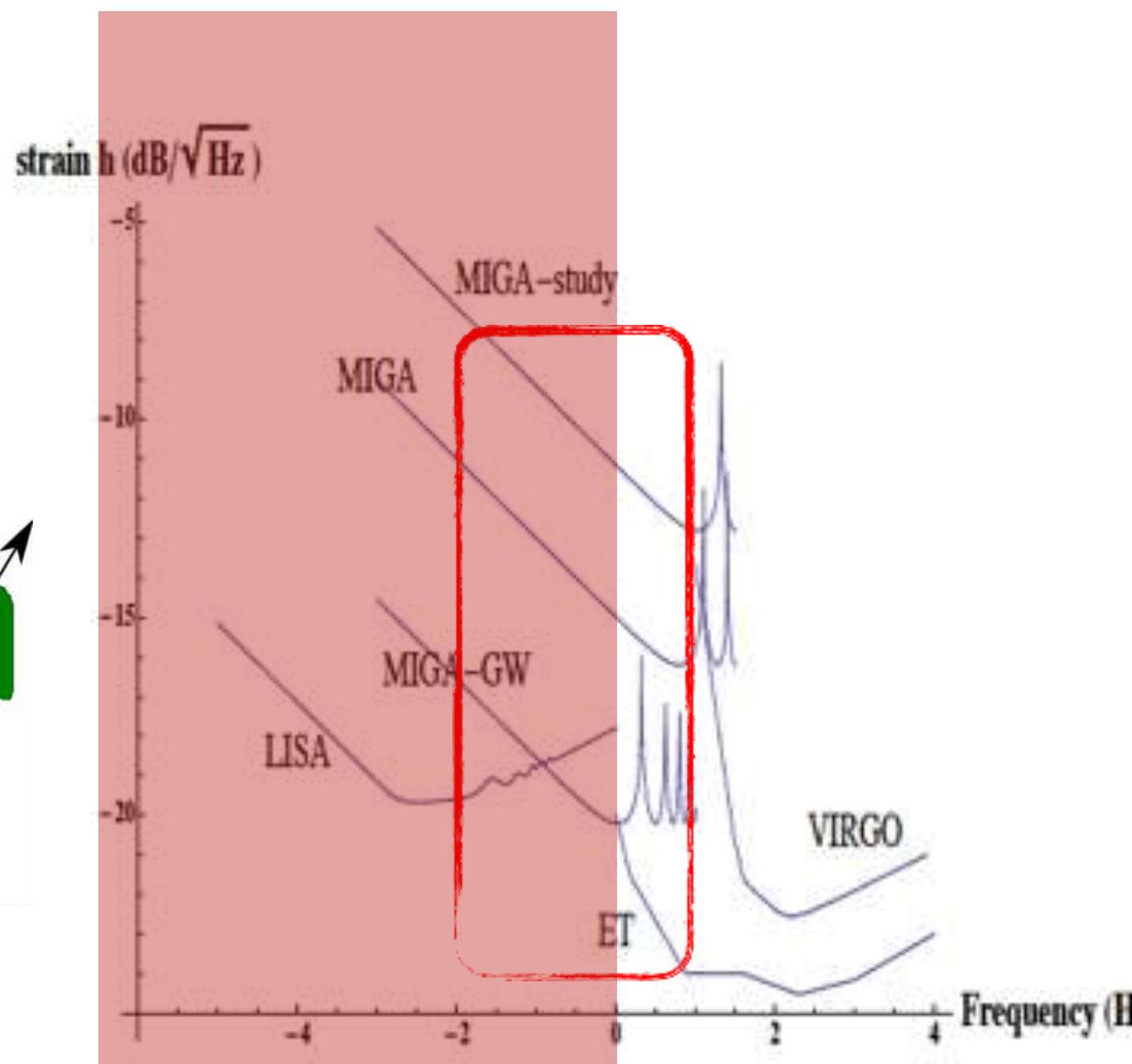
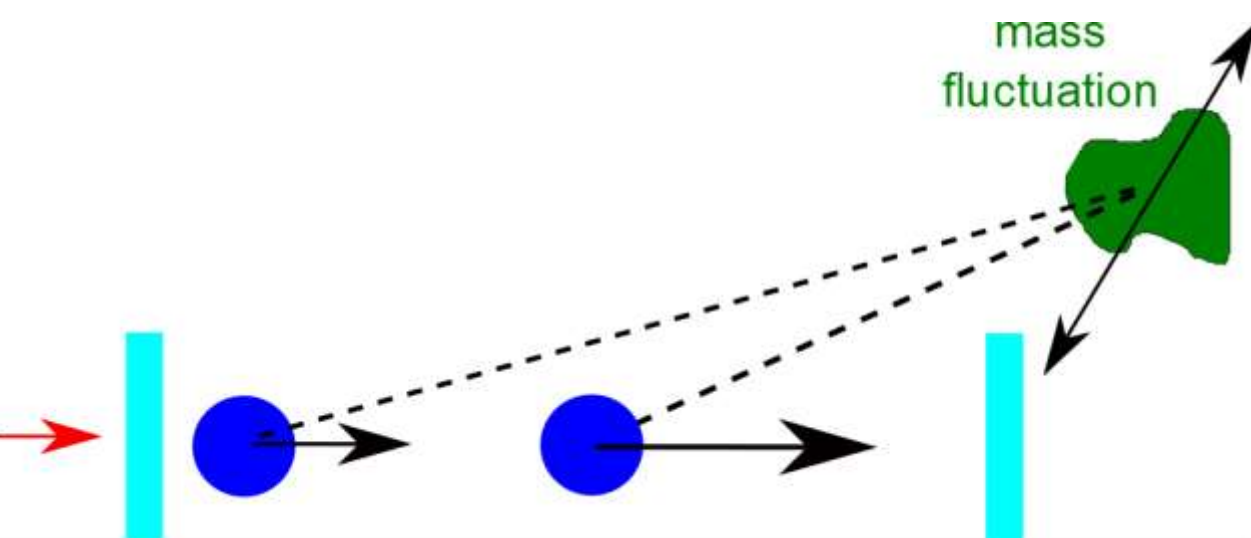
Use of AI offers possibility to spatially resolve gravity

- GW have long wavelength while GG have short characteristic length of variation (1 m – few km)
- **Correlations between distant sensors provide information on the GG noise and allows to discriminate it from the GW signal**



GGN is a fundamental limit for ground based GW detectors

- Noise due to nearby mass fluctuation
- Limiting factor under 1-10Hz

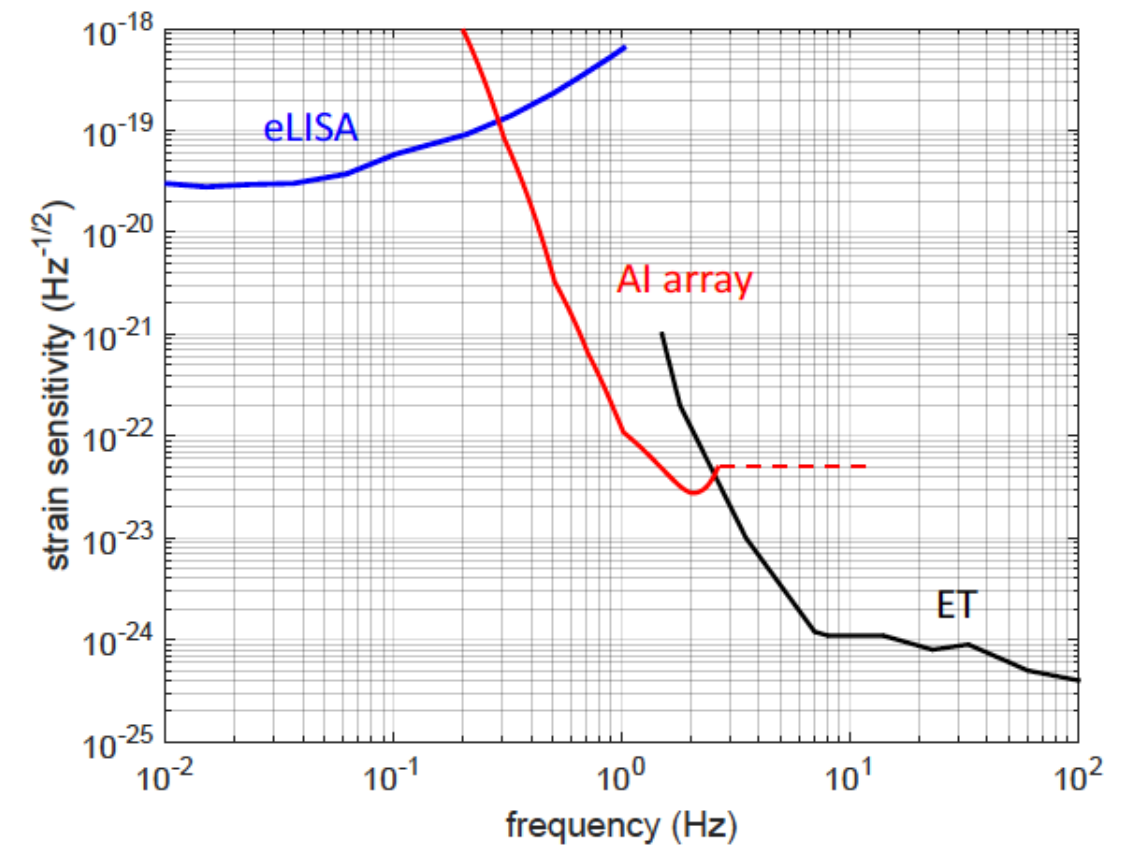
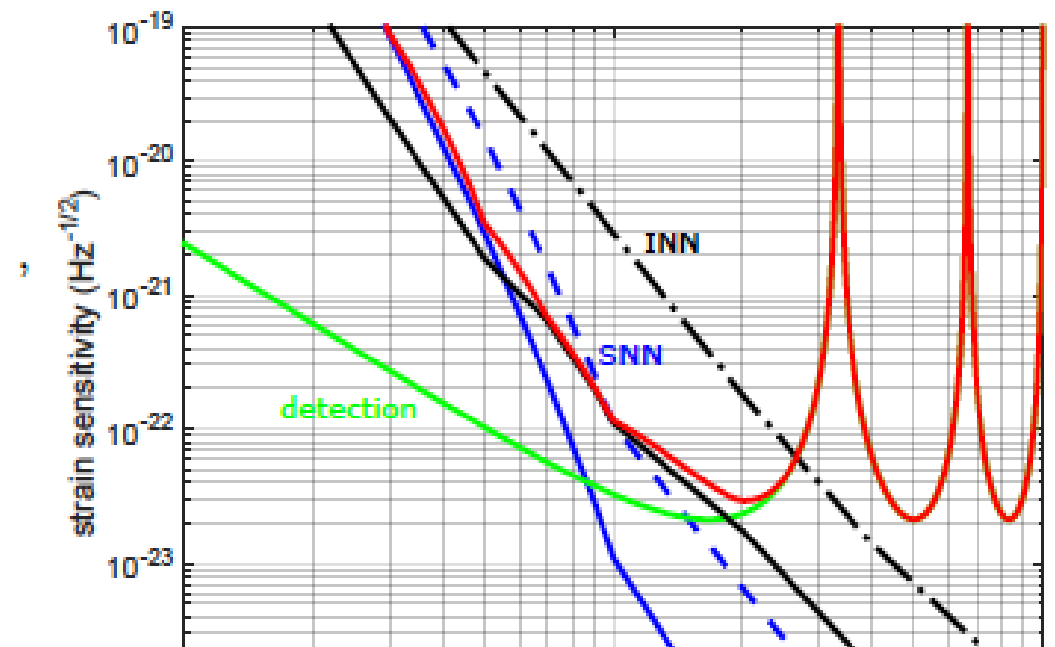
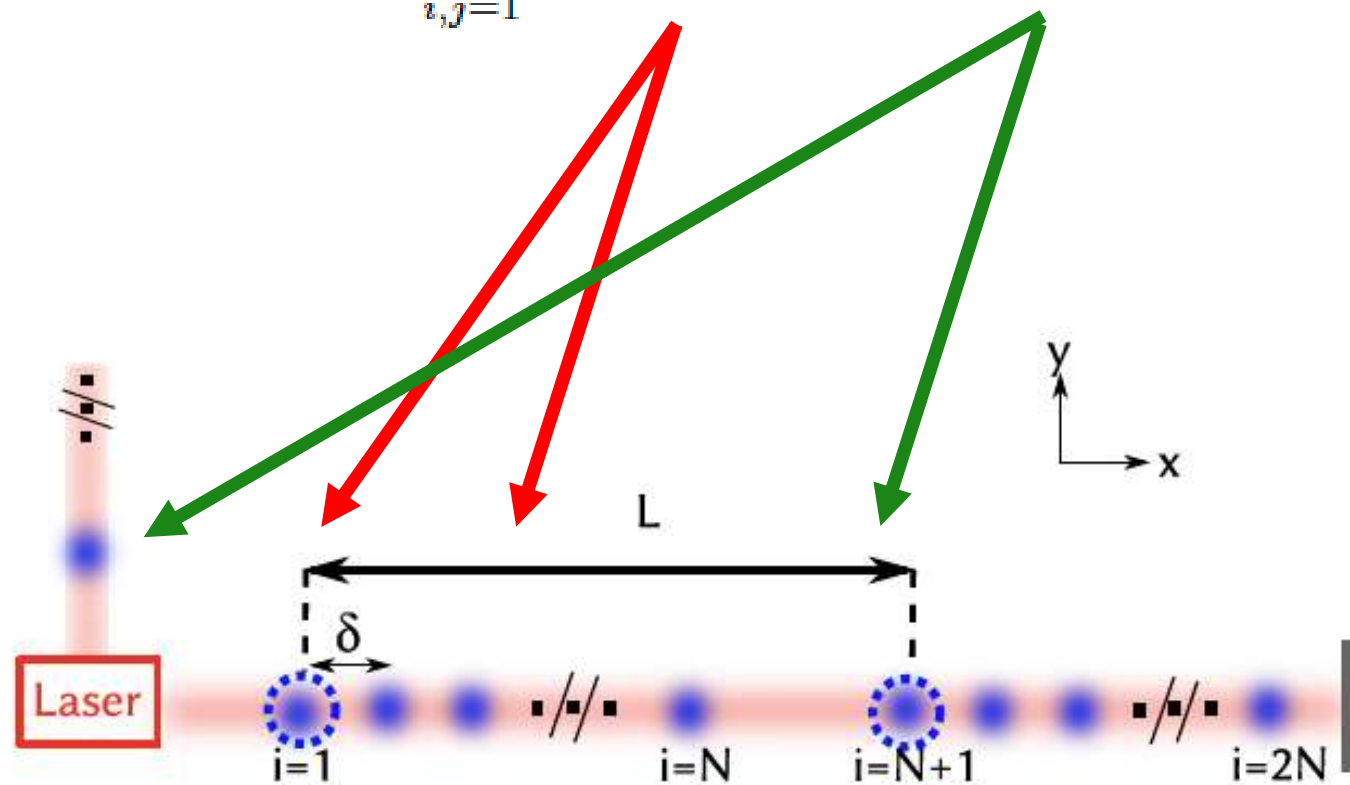


→ Strain sensitivity

$$S_h(\omega) = \frac{S_a(\omega)}{\omega^4 L^2} + \frac{4S_\epsilon(\omega) - \text{Shot noise}}{16NL^2(2nk)^2 \sin^4(\omega T/2)}$$

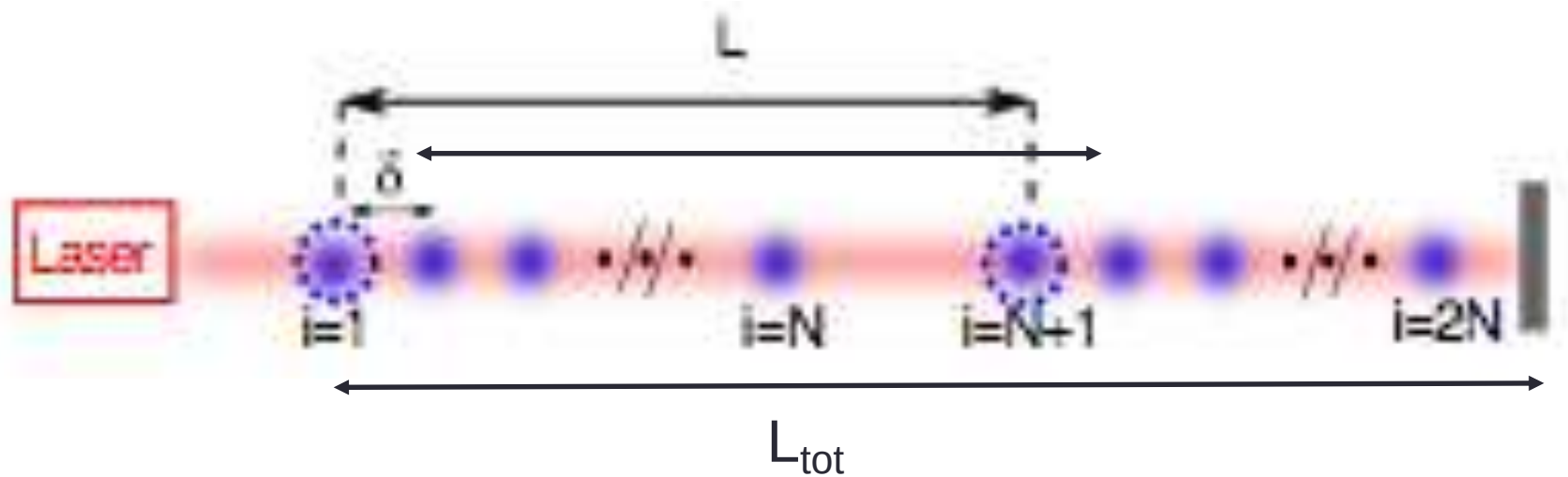
→ Seismic noise

$$S_a(\omega) = \frac{1}{N^2} \sum_{i,j=1}^{2N} (\mathcal{C}_{\parallel}(X_i, X_j, \omega) + \mathcal{C}_{\perp}(X_i, Y_j, \omega))$$



W. Chaibi, et al. Phys. Rev. D 93, 021101(R), 2016

Dense arrays of Atom Interferometers could be used as future GW detectors

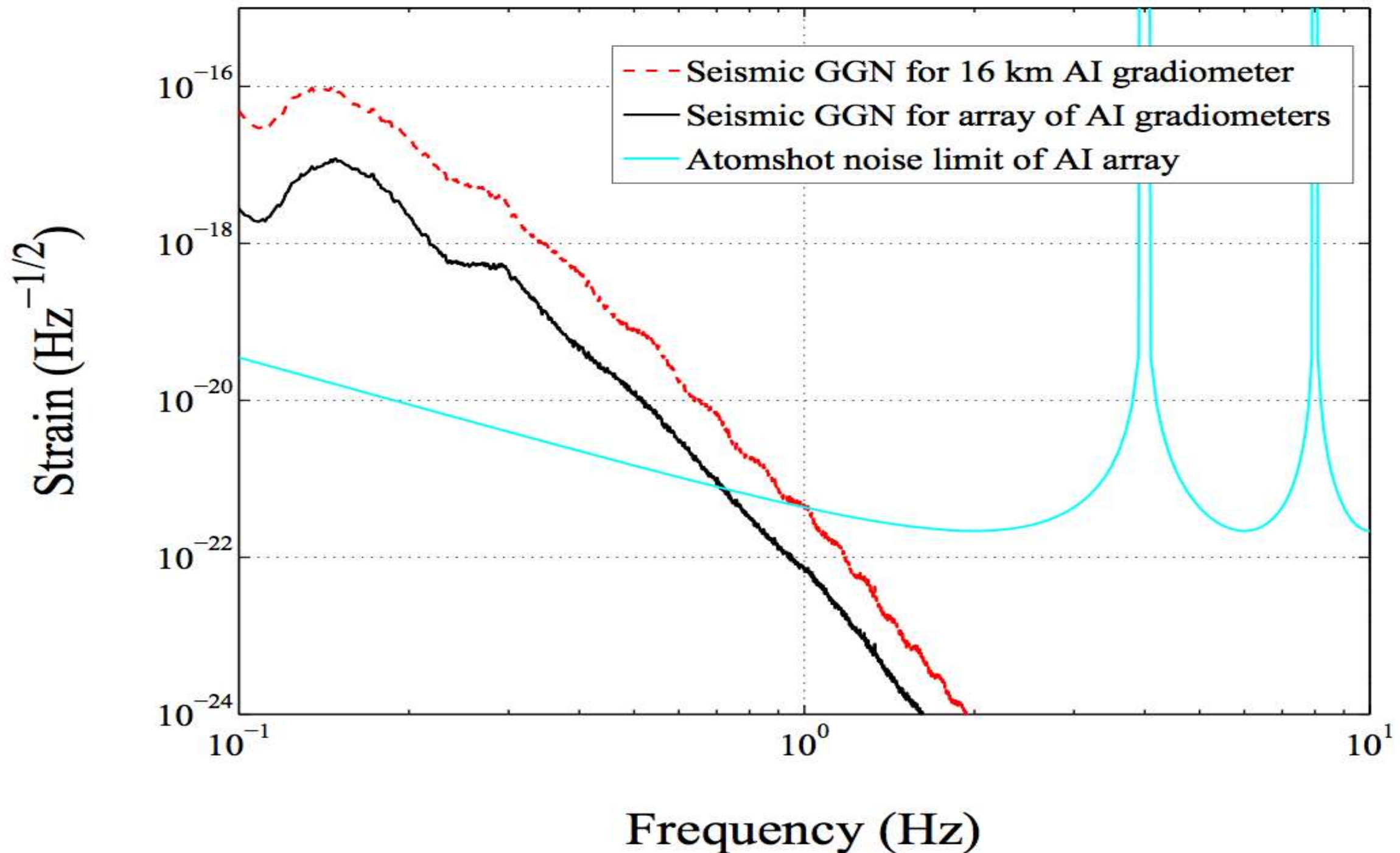


- $L_{\text{tot}} = 32$ km
- $N = 80$ gradiometers
- baseline $L = 16$ km

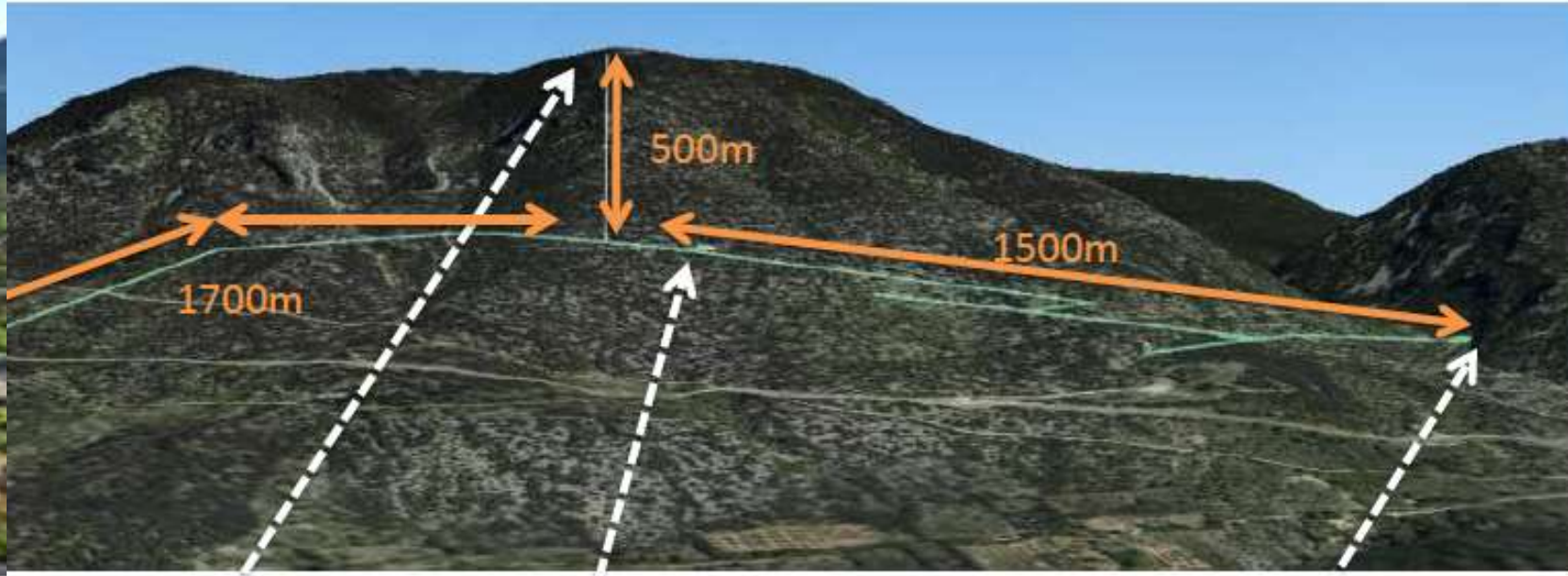
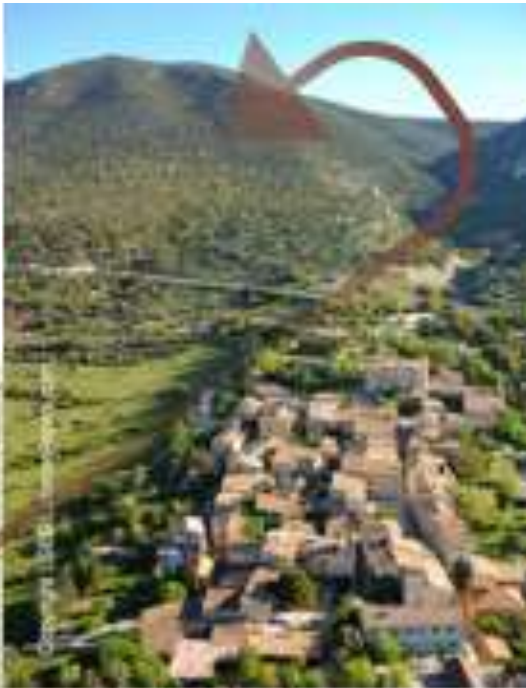
- Gravitational Wave signal can be extracted using a spatial averaging method
- N Correlated gradiometers enable to average the GGN over several realizations

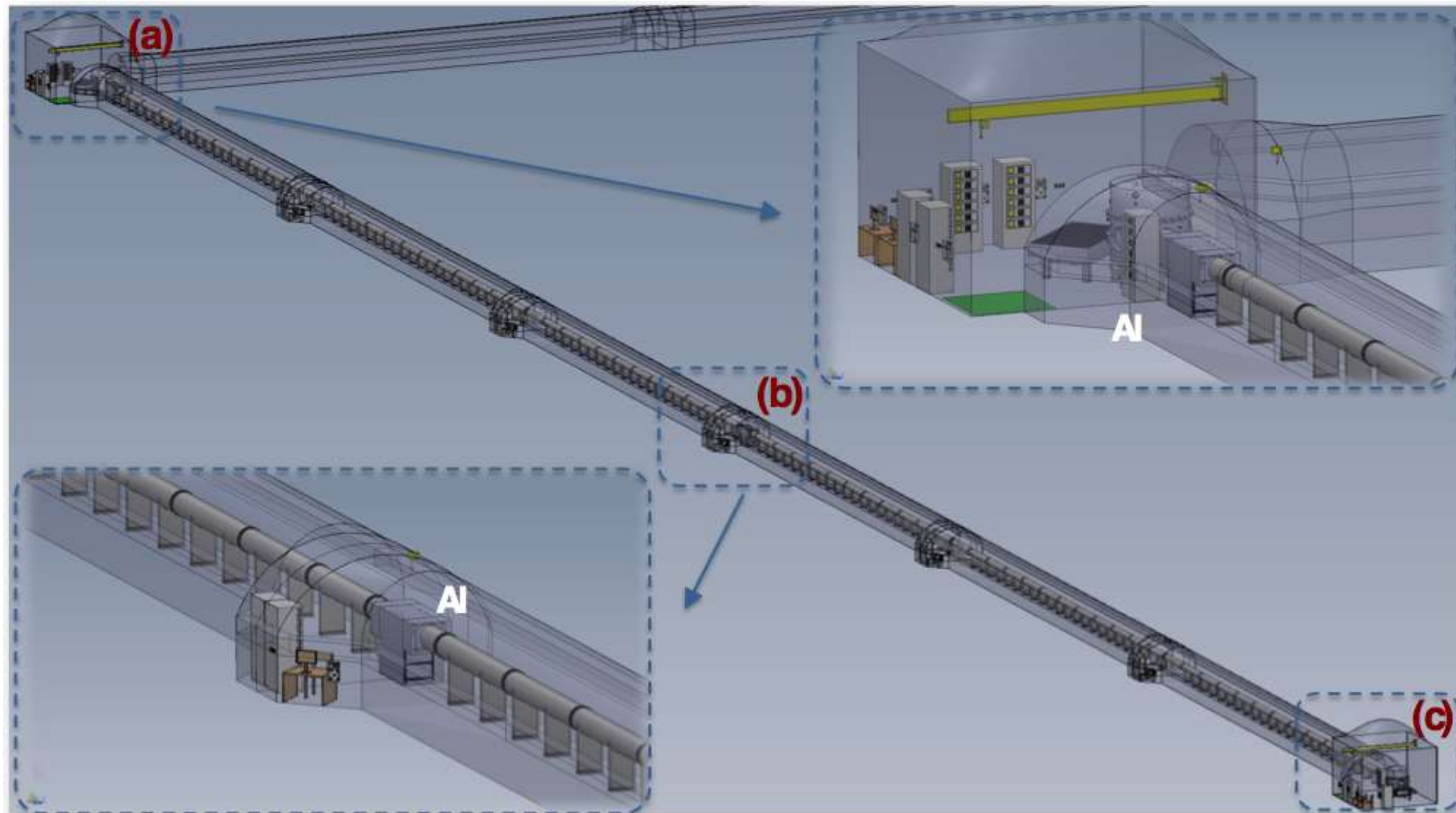
$$H_N(t) = \frac{1}{N} \sum_{i=1}^N \psi_i(t)$$

- The geometry of the detector ($^{\text{TM}}, L$) is chosen with respect to the spatial correlation properties of the GGN.

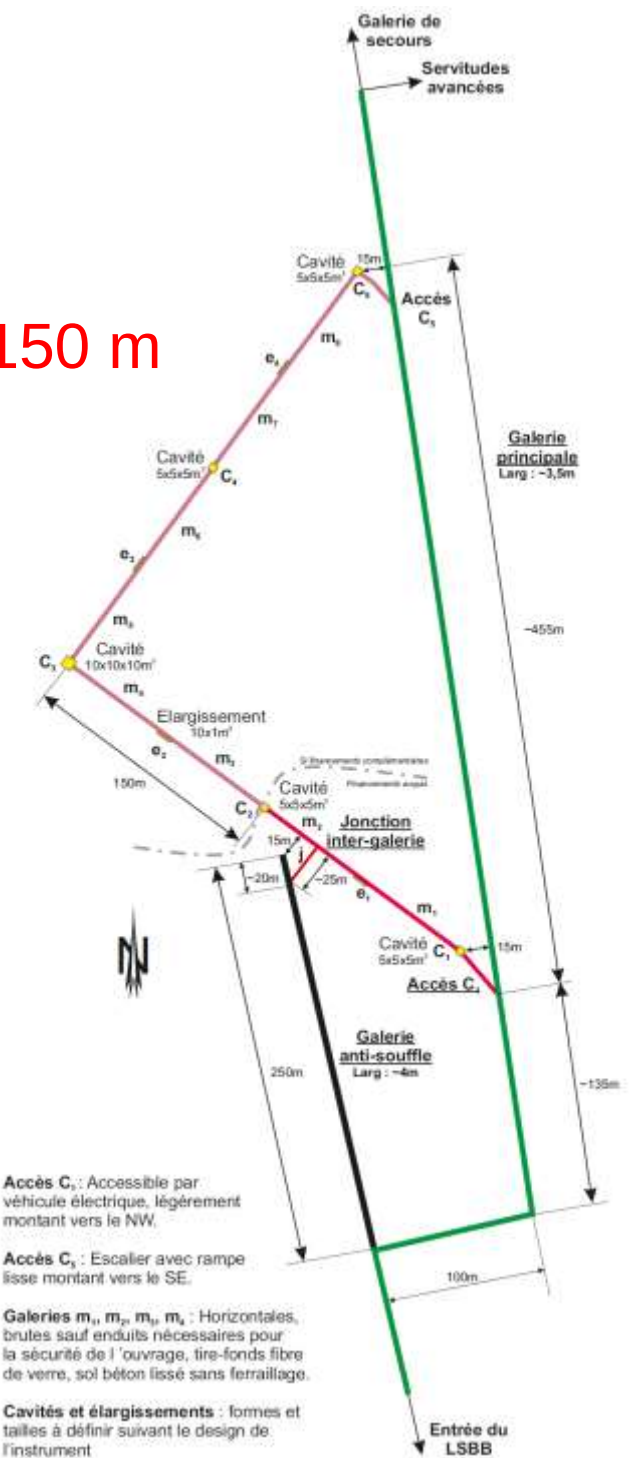


- Gain of about factor 10 in the 100 mHz 1 Hz band
- Space for improvement using all spatial information of the network (use different baseline L in the numerical treatment)





150 m



Construction of different subsystems in on the way

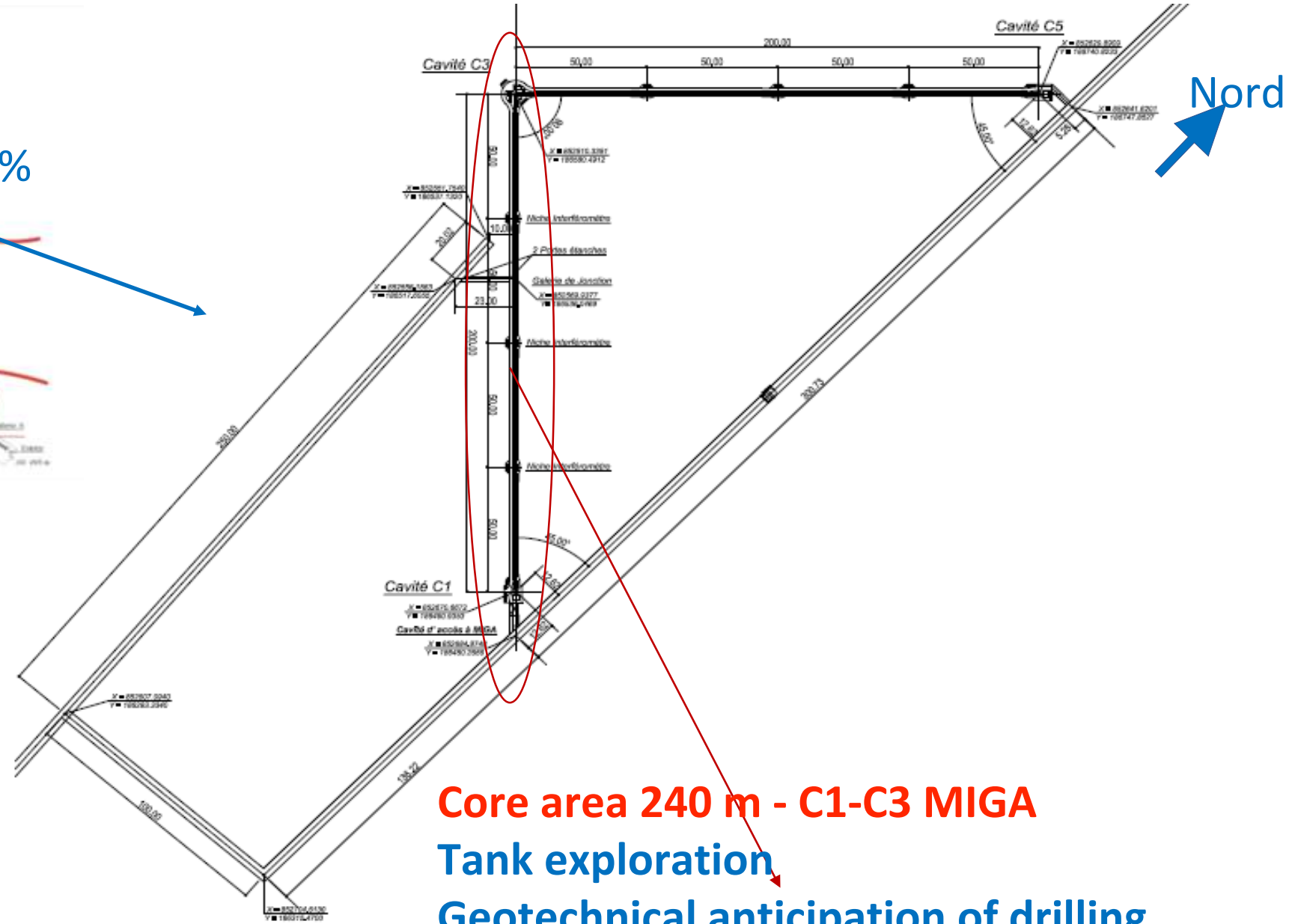
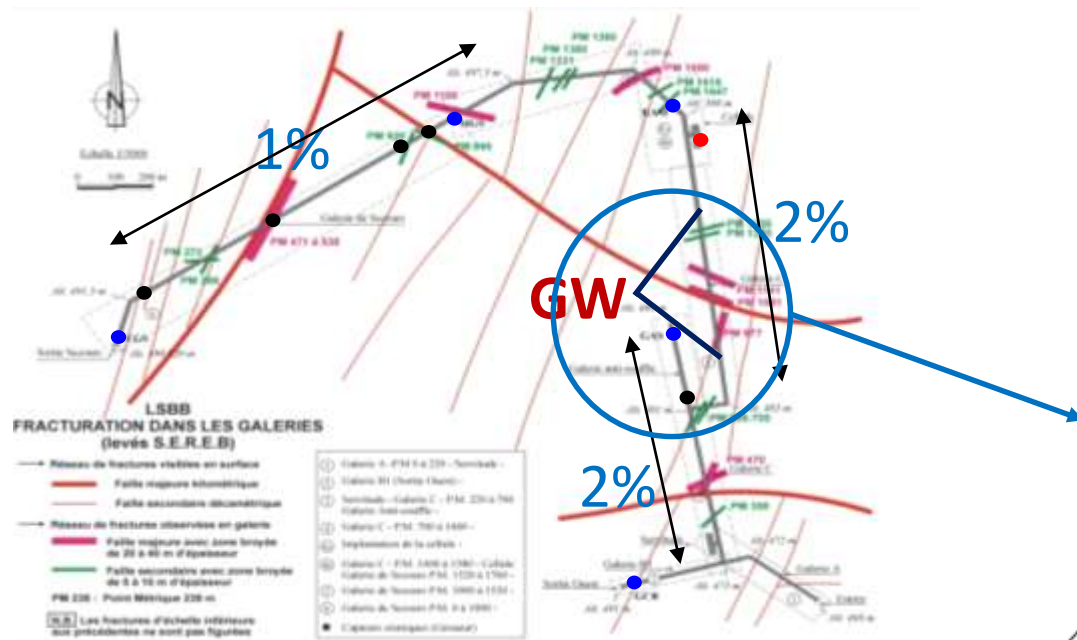
A prototype AI and a full suspension is available at LP2N

Start commissioning of the 10 m prototype in 2018

Start Gallery preparation mid 2018

MIGA installation mid 2019

First operation 2020



Core analysis and wall imaging

Geological modeling

Carbonated reservoir prediction

Environment

hydro-geological analysis around

Core area 240 m - C1-C3 MIGA

Tank exploration

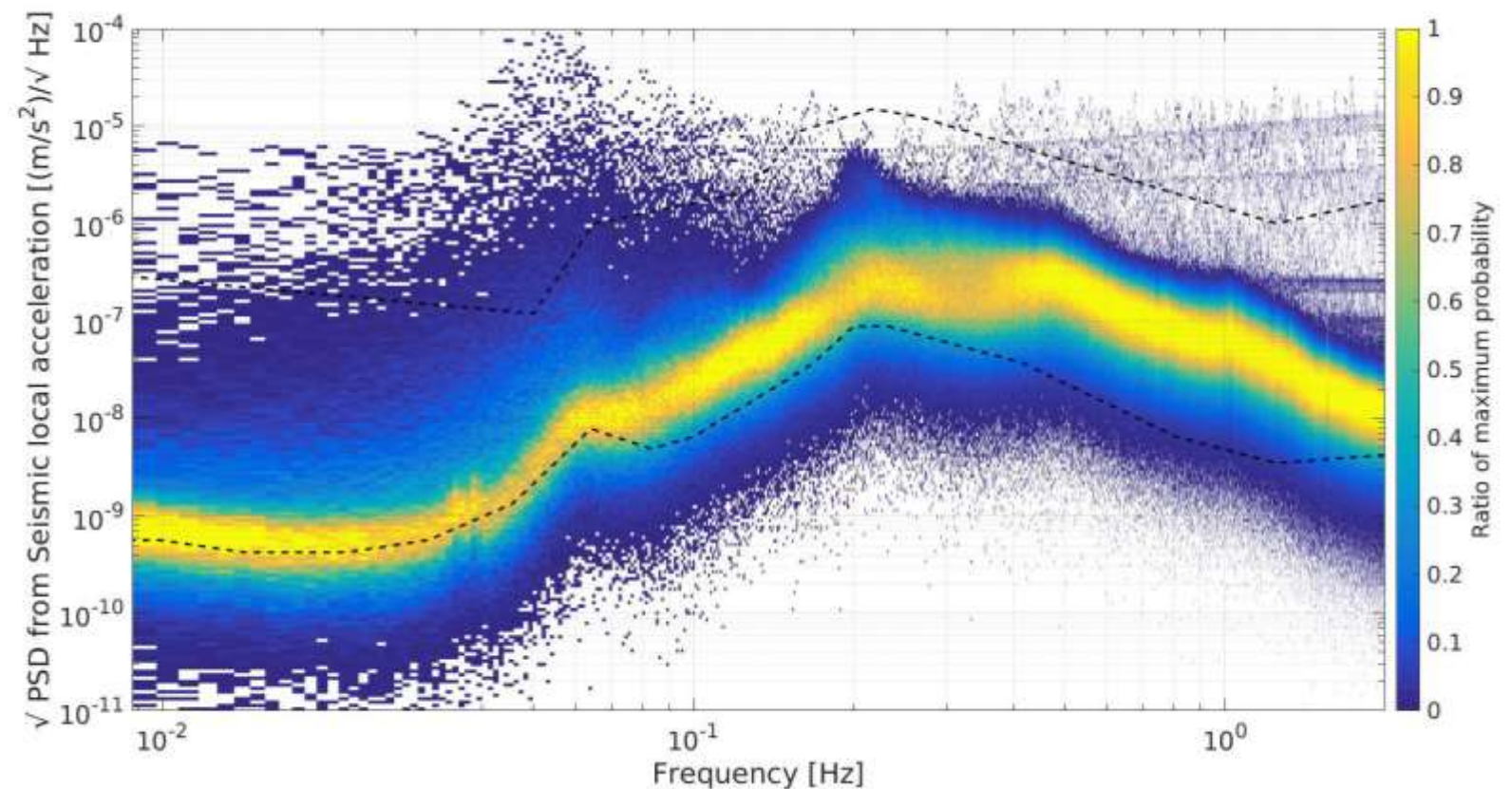
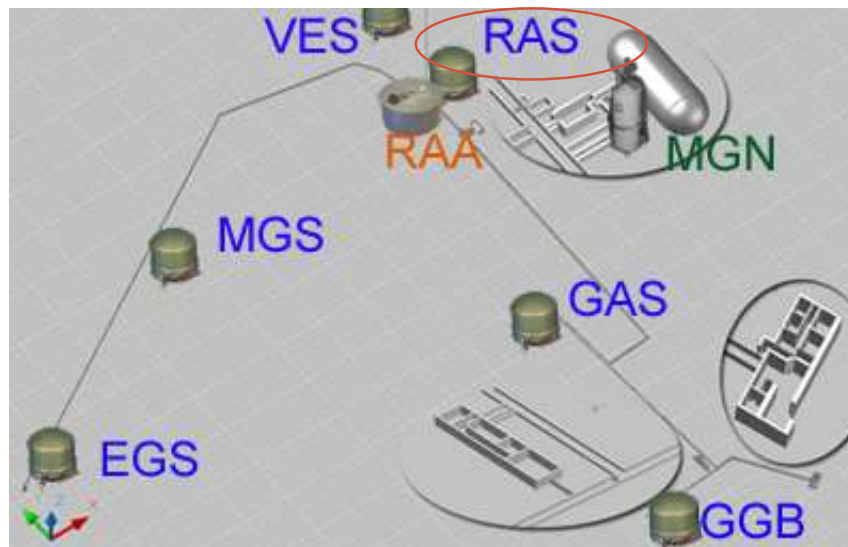
Geotechnical anticipation of drilling

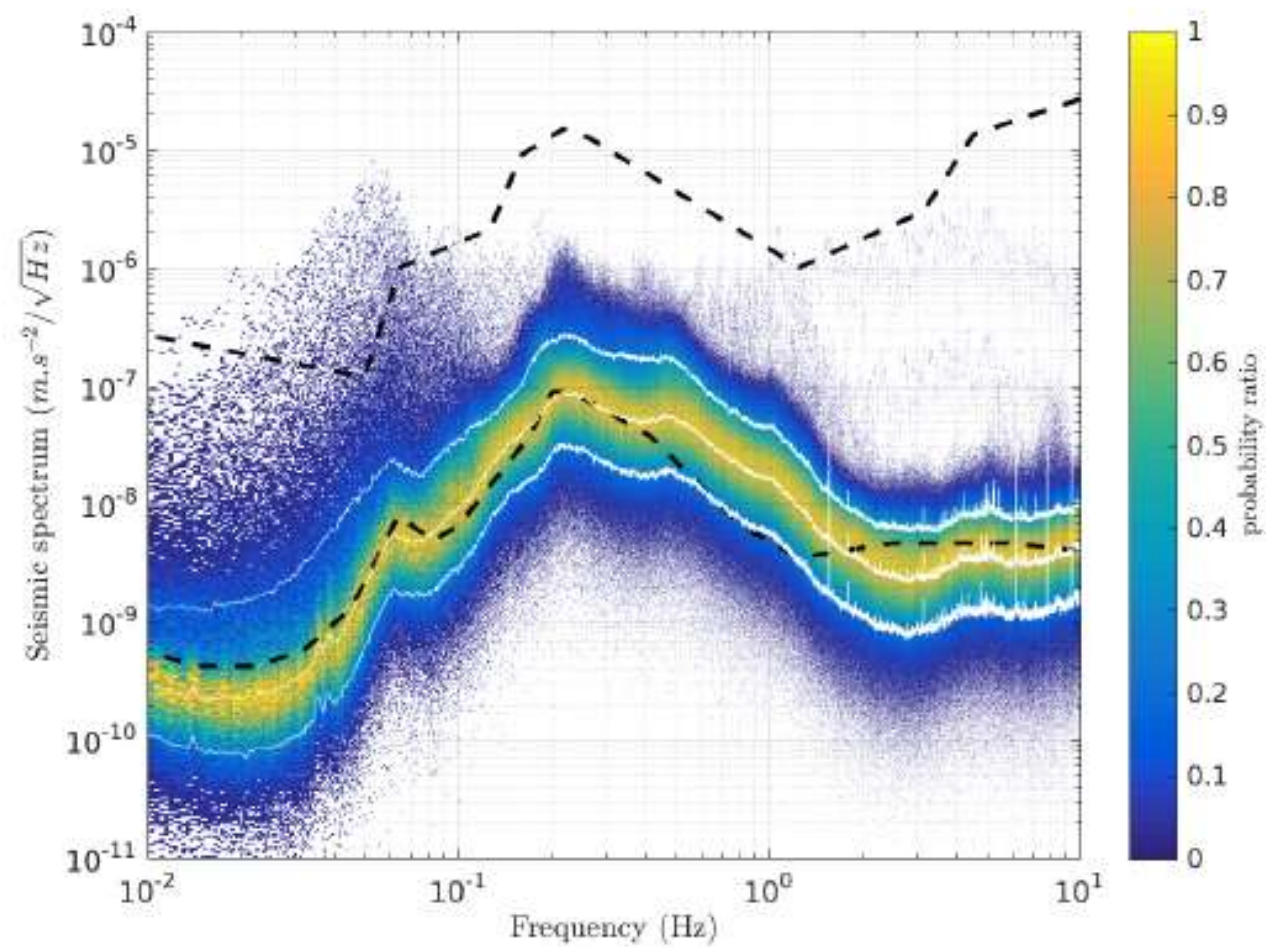
Sources Gravity Gradient noise on detector site (10^{-2} -10 Hz)

- Seismic GGN
- Atmospheric GGN
- Other : geophysical properties (hydrology), linked to human activity

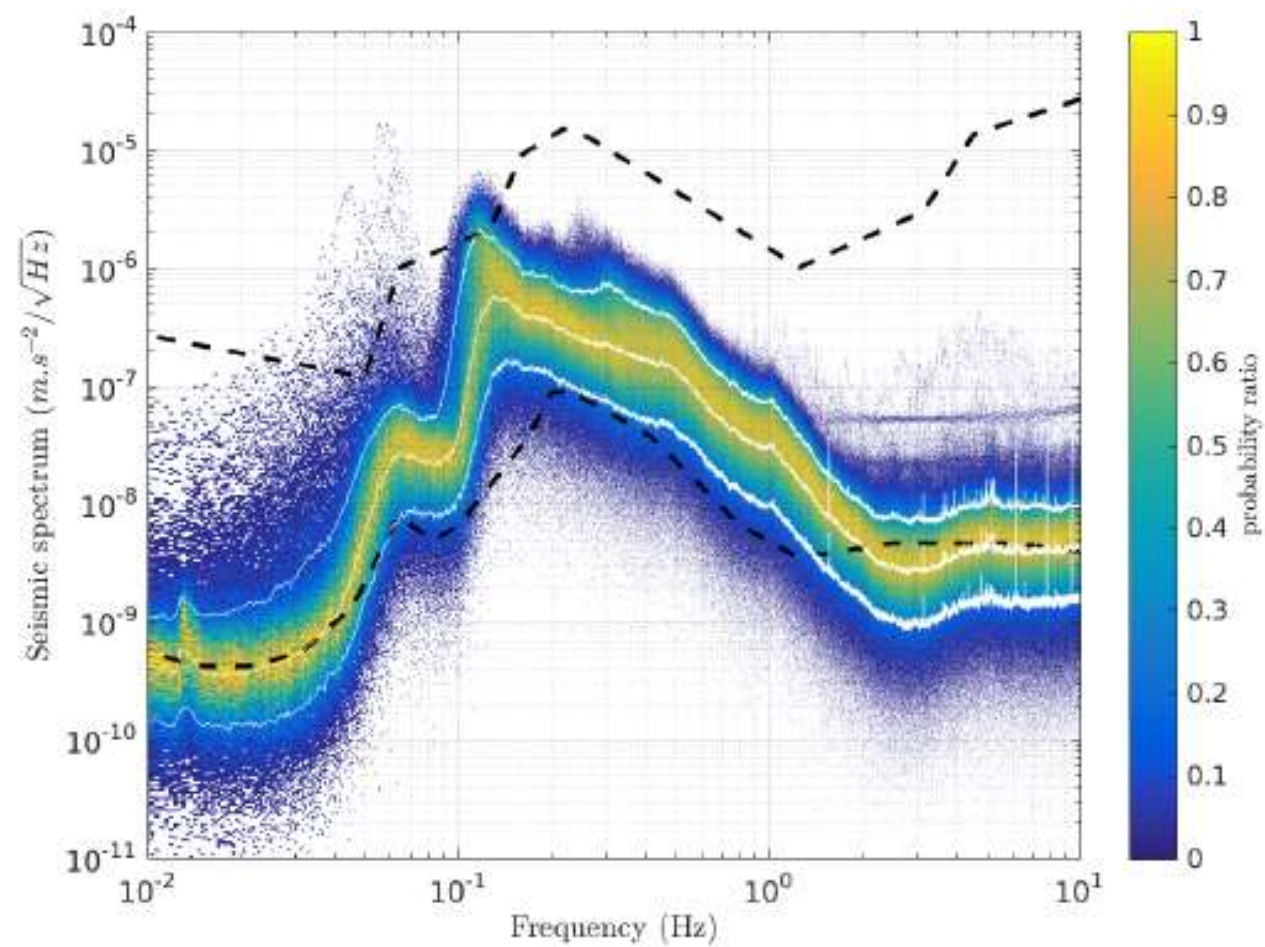
Seismic GGN for MIGA at LSBB

- STS-2 sensor

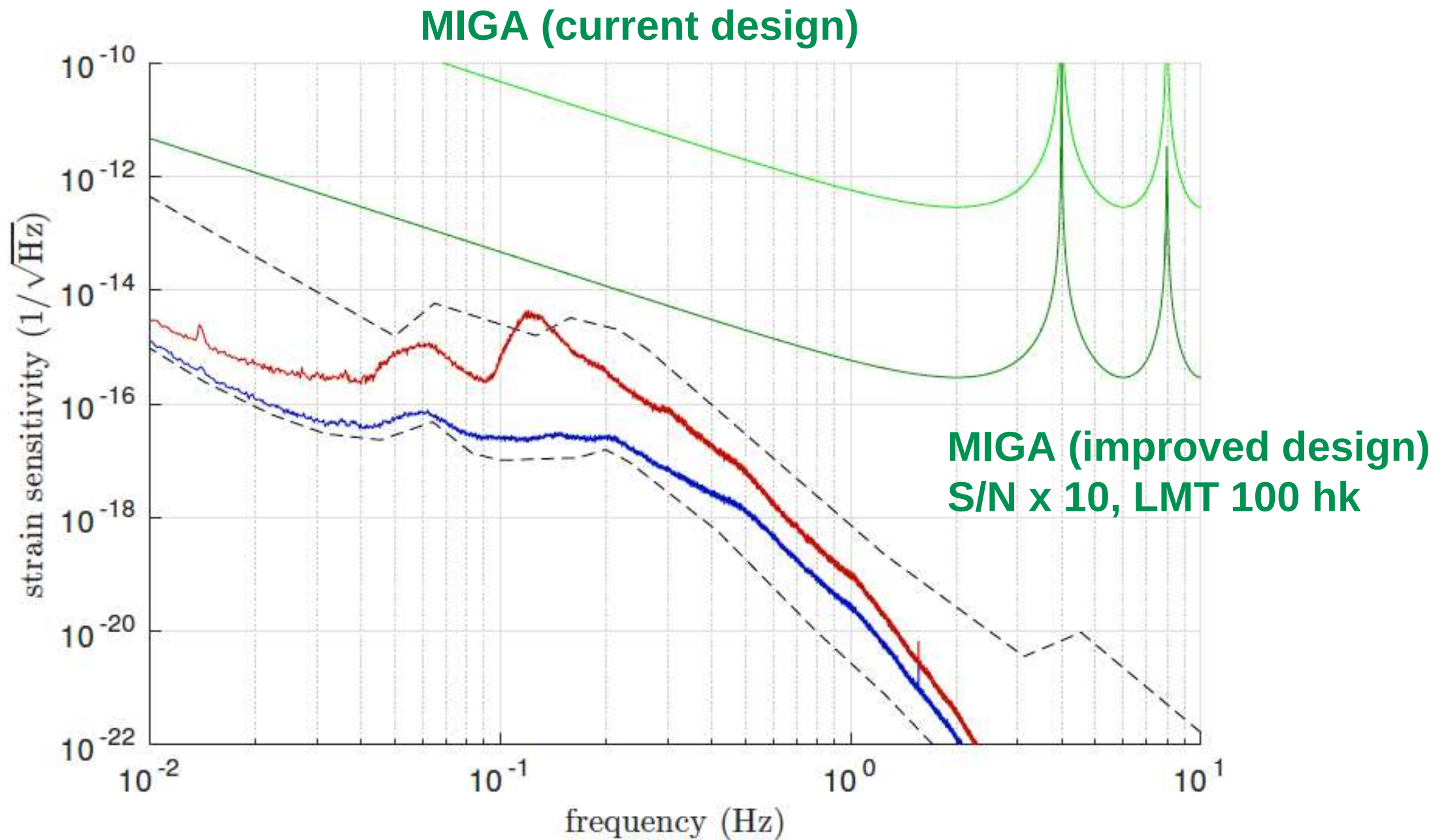




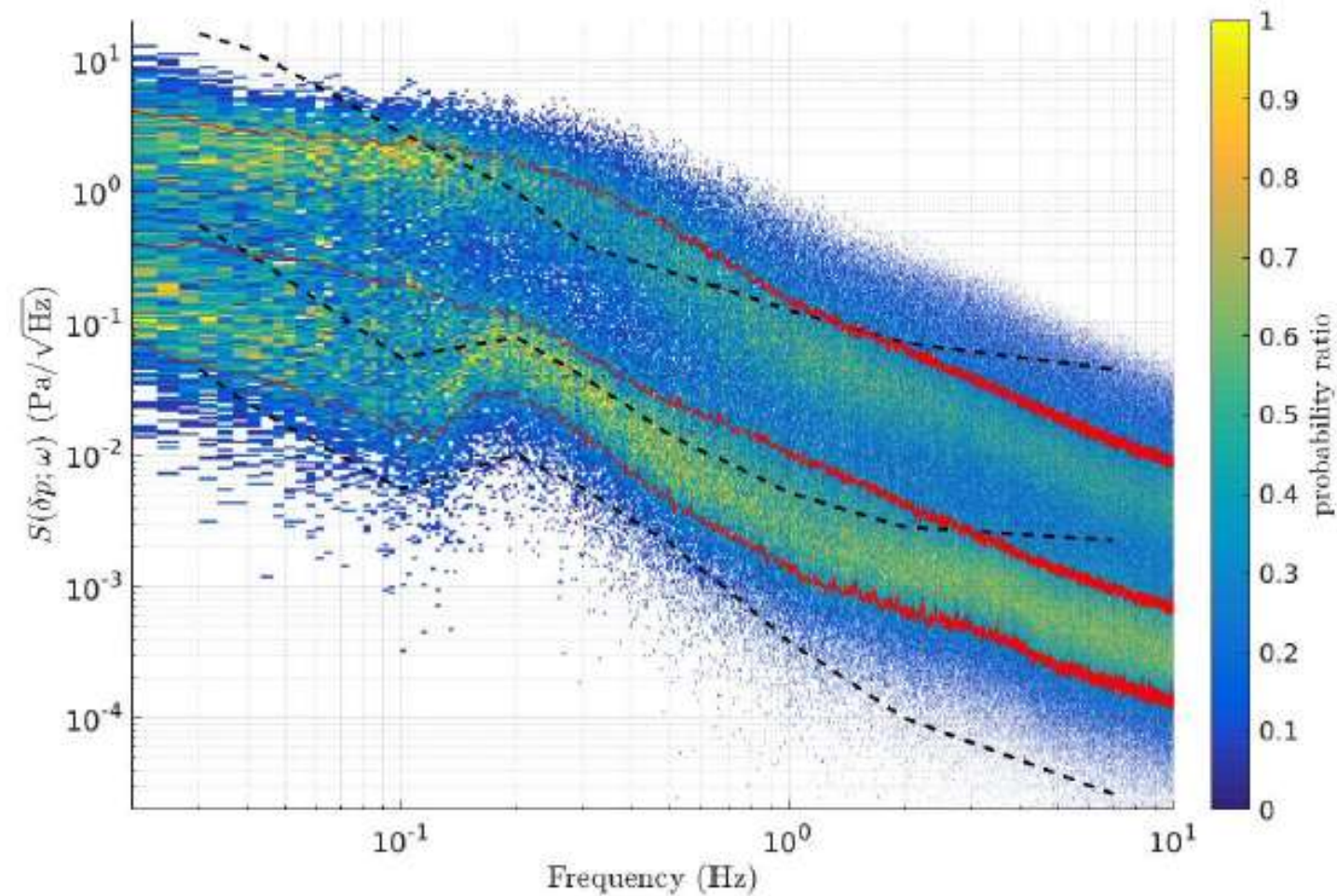
(a) on a "quiet" month (august)



(b) on a "noisy" month (February)

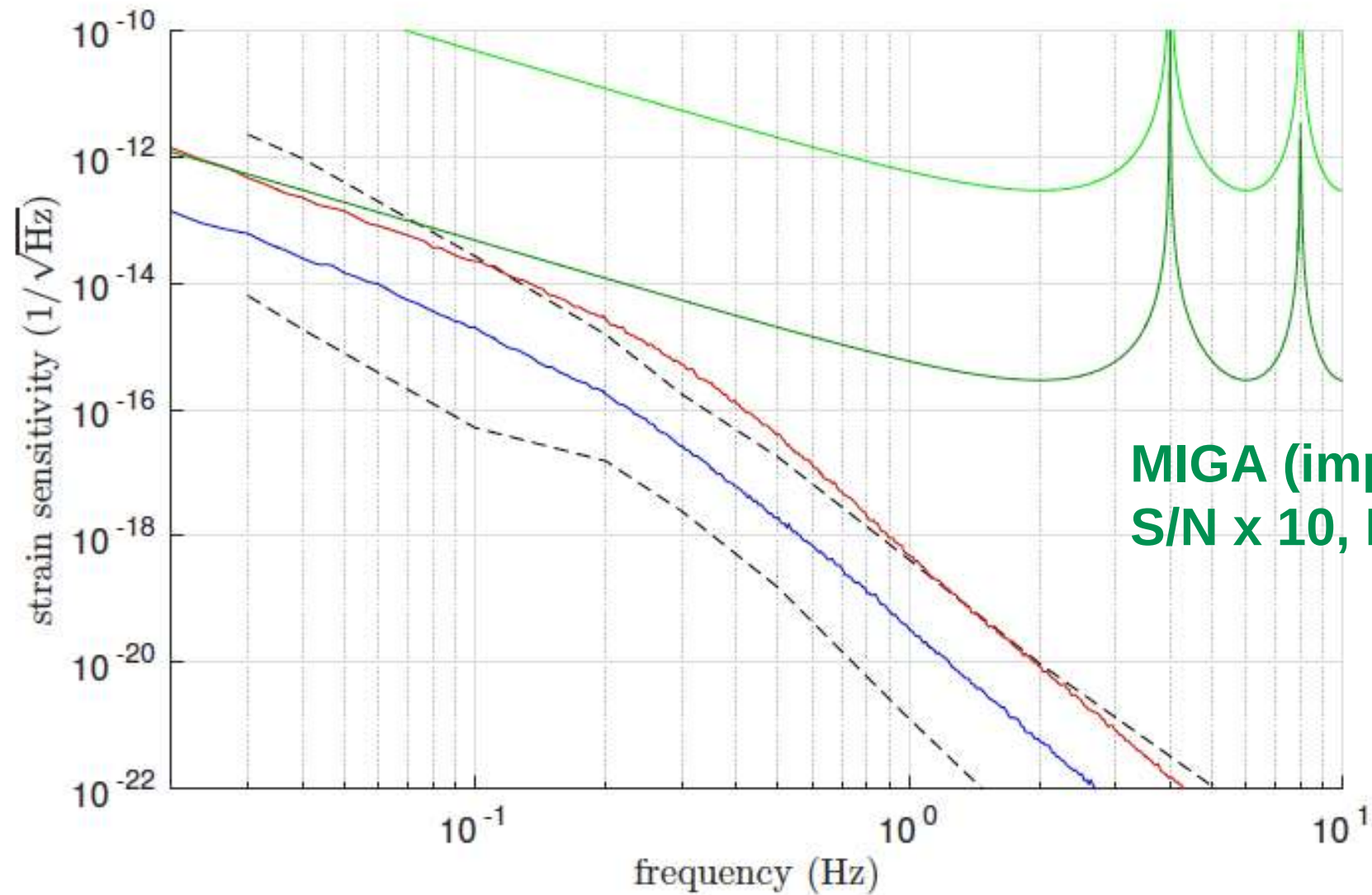


MIGA strain sensitivity. In blue (resp. red) seismic GGN projection from Lsbb 50th percentile of a quiet week (resp. 90th percentile of a noisy week), in dashed black from Peterson model data, in green detection noise for MIGA in its initial configuration (light green) and for an improved version (dark green).



Histogram of the outside pressure variations 500 m above the future MIGA gallery with 10th, 50th and 90th percentile in red and Bowman low, mid and high model in dashed black.

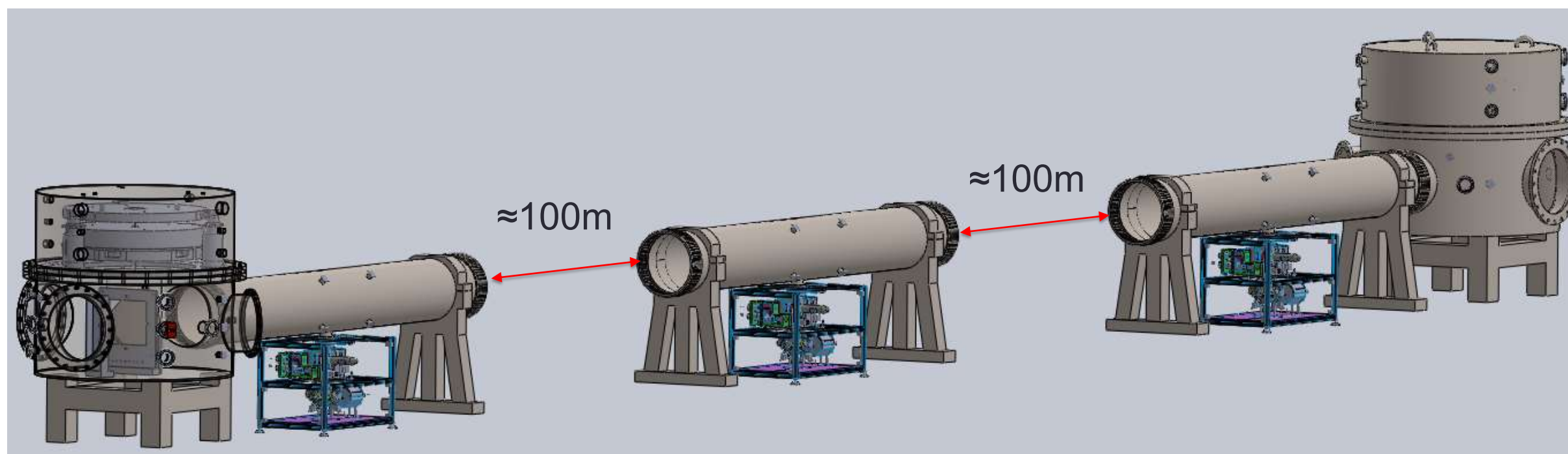
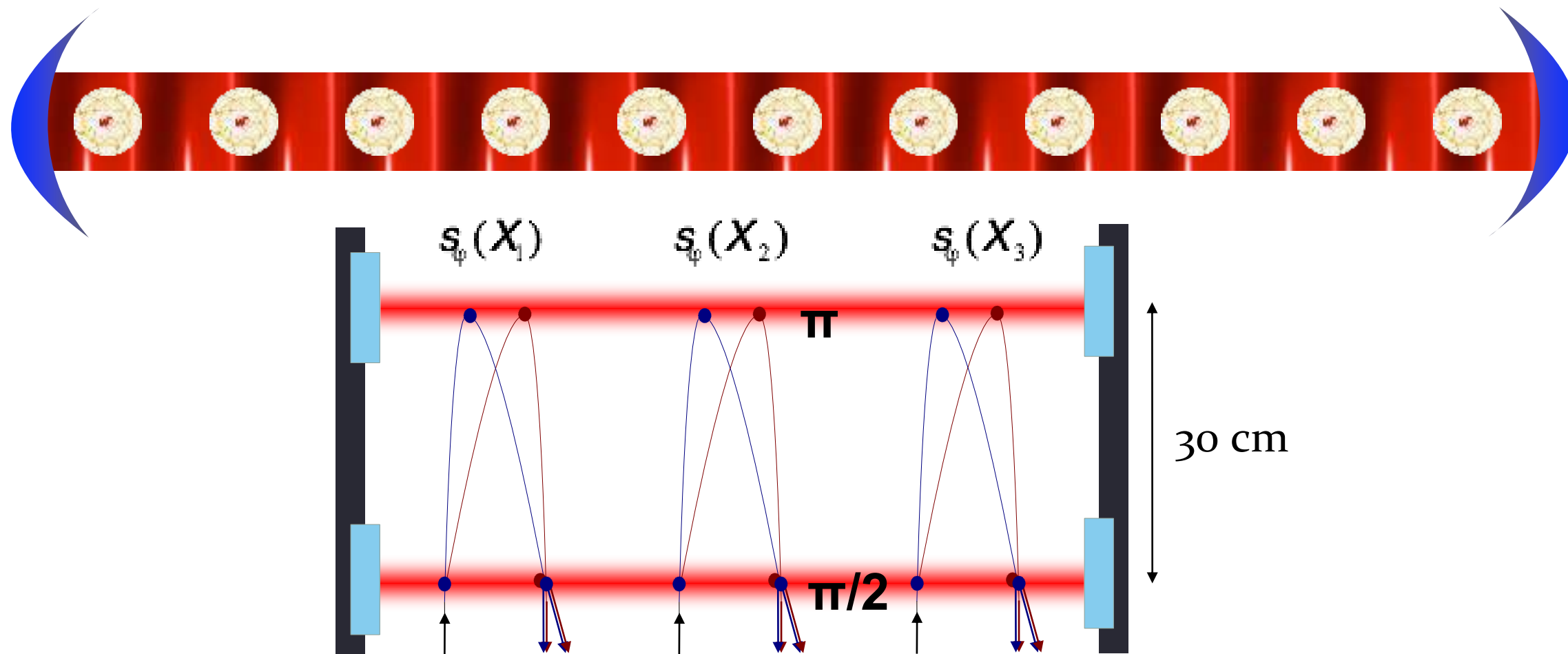
MIGA (current design)



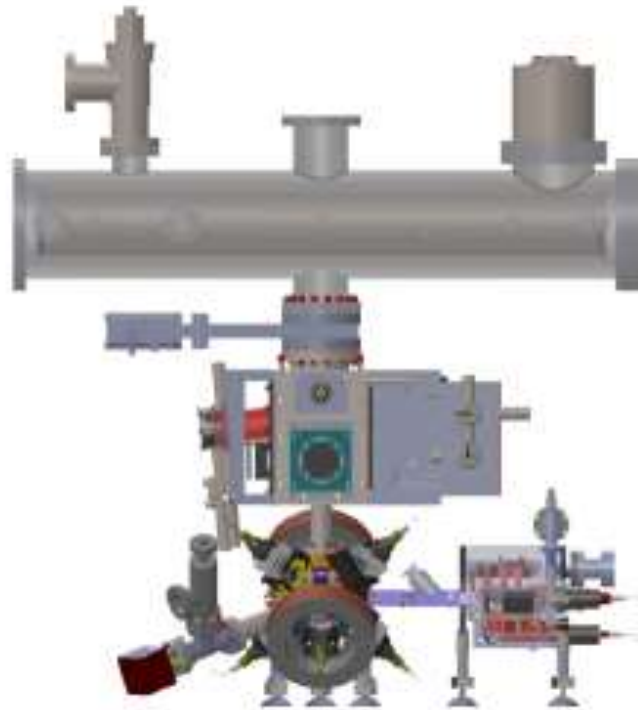
MIGA (improved design)
S/N x 10, LMT 100 hk

MIGA strain sensitivity for the infrasound GGN, with data from Bowman model (dashed black) and from data gathered on site at the Lsbb (50th percentile in blue and 90th percentile in red).





Accelerometer

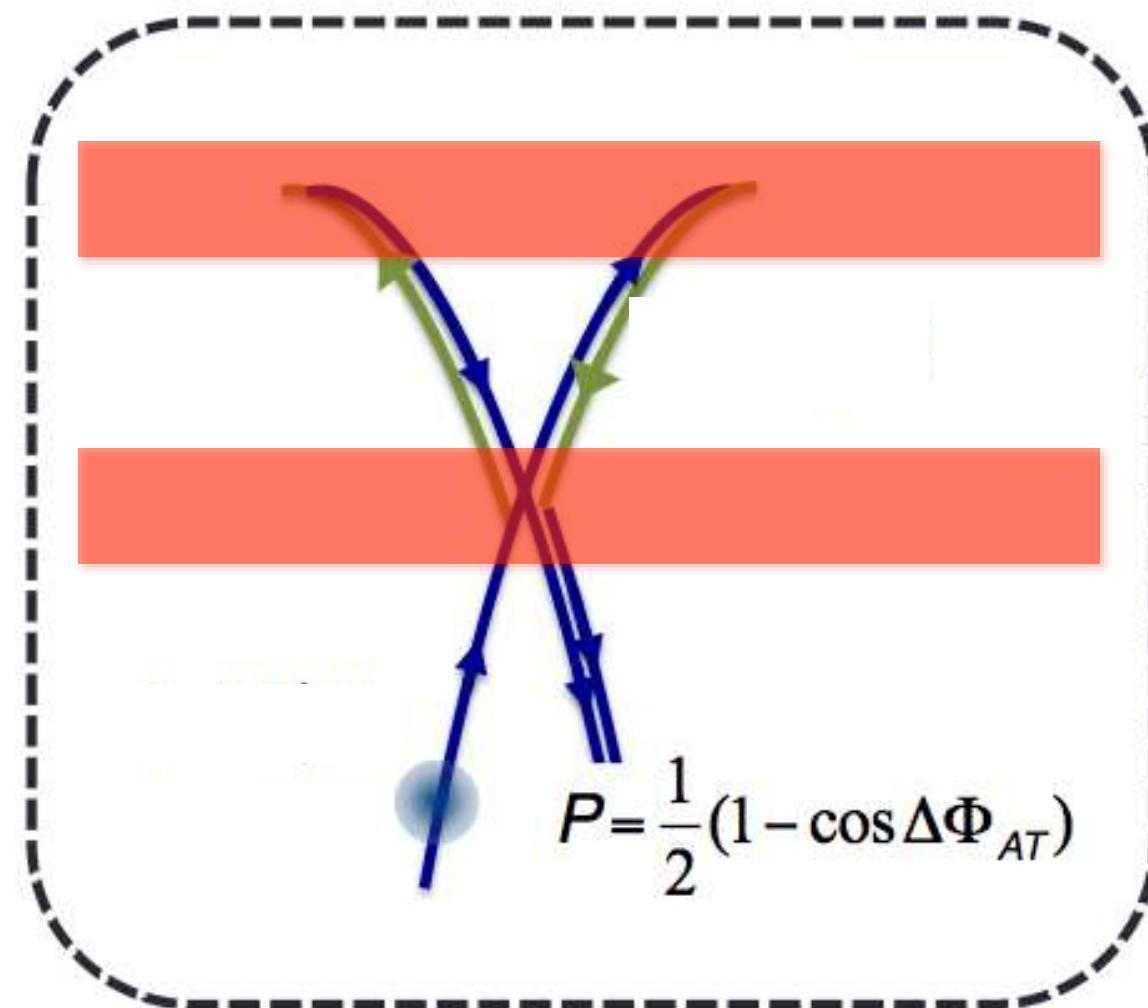
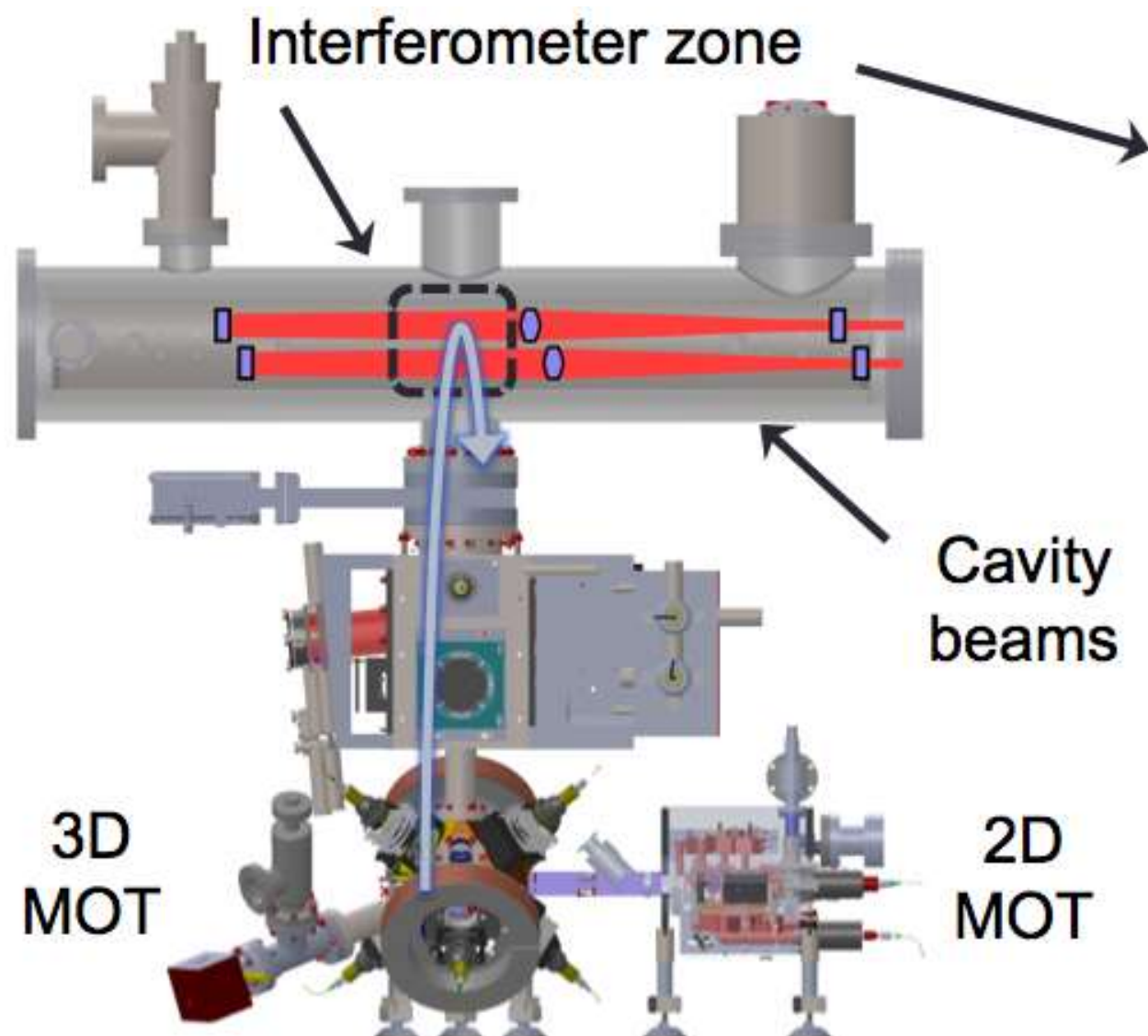


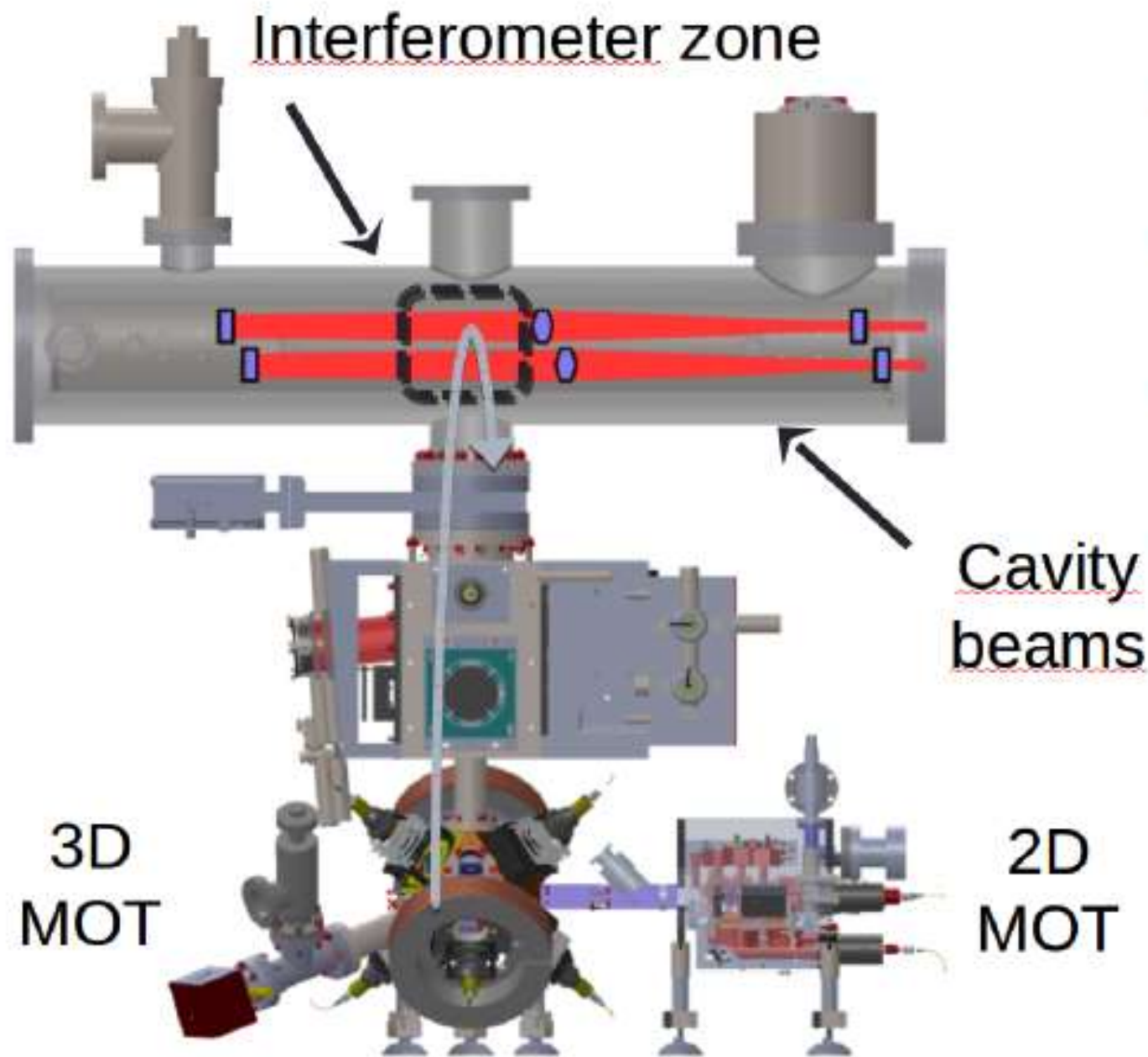
- One cold atom source
- two parallel 80 cm long cavities
- Study cavity enhanced Bragg pulses

Gradiometer

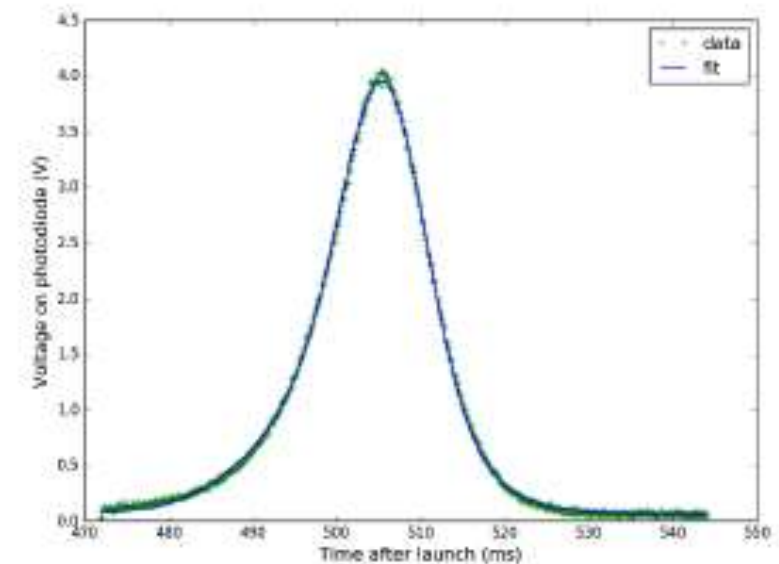


- Two cold atoms sources
- Two parallel 5.7 m long cavities
- Study of differential measurements (gradiometer)
- Testing the equipment

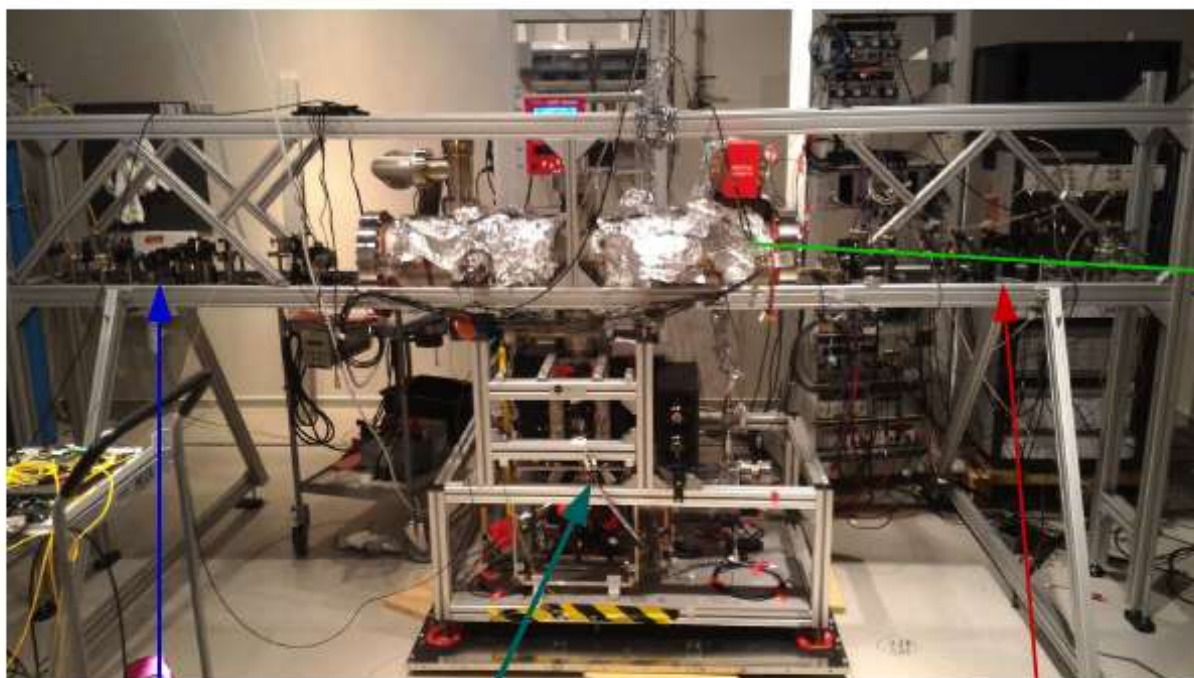




- Loading a 3D Magneto-optical trap (MOT) with a 2D MOT
- Preparation : velocity an magnetic selection with Raman beams
- Interferometry with intracavity Bragg pulses
- Detection by fluorescence with sheet of lights

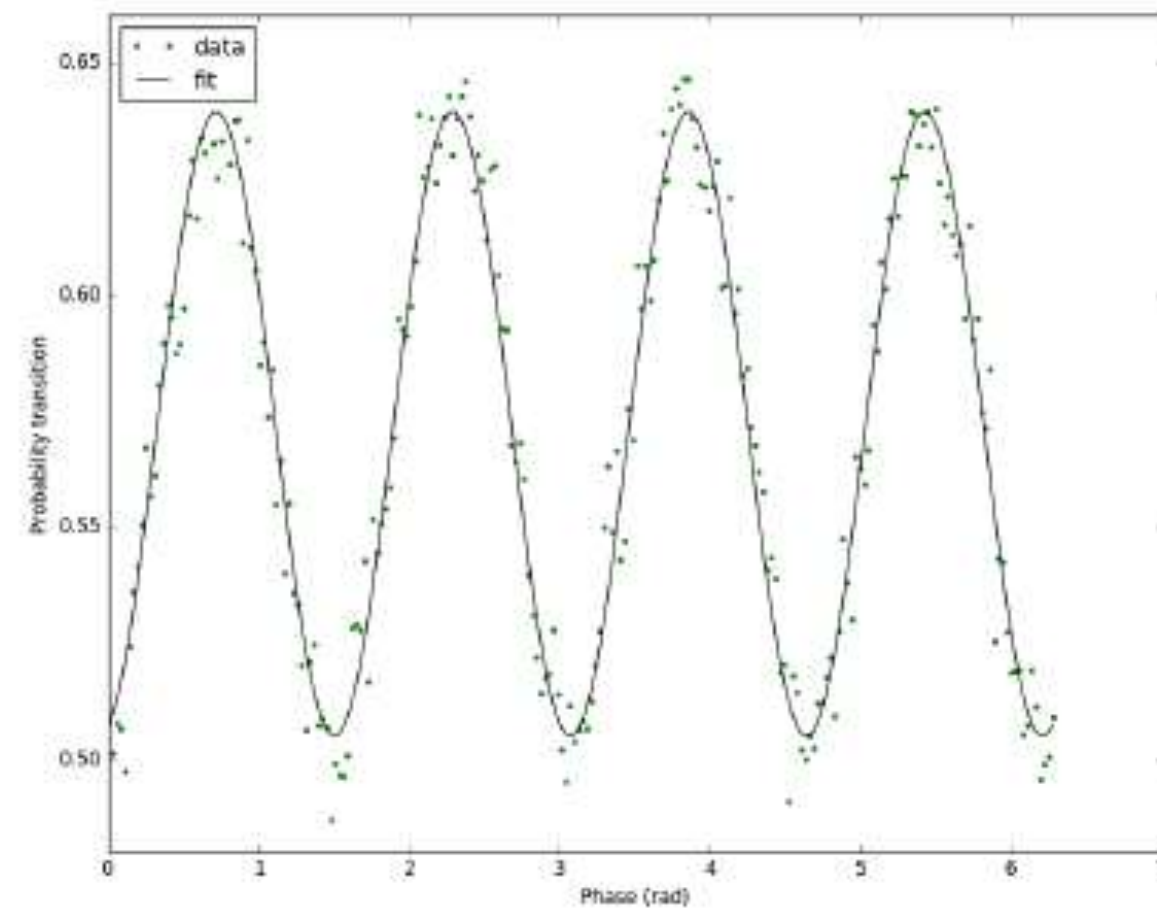
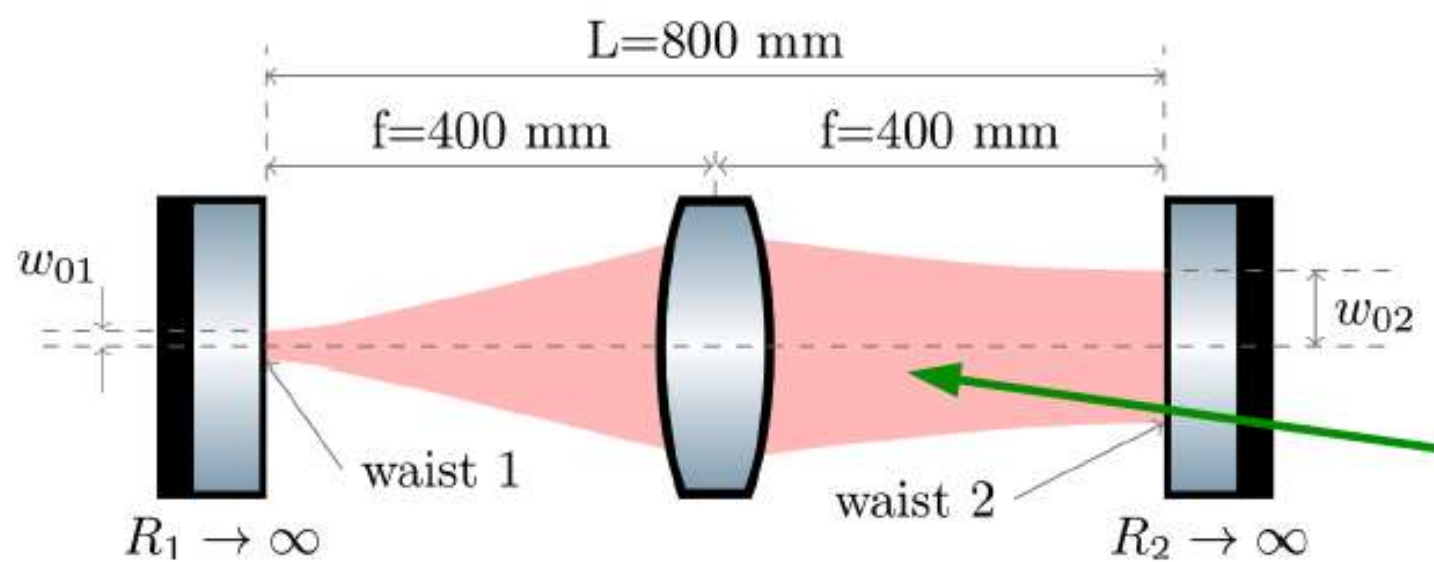
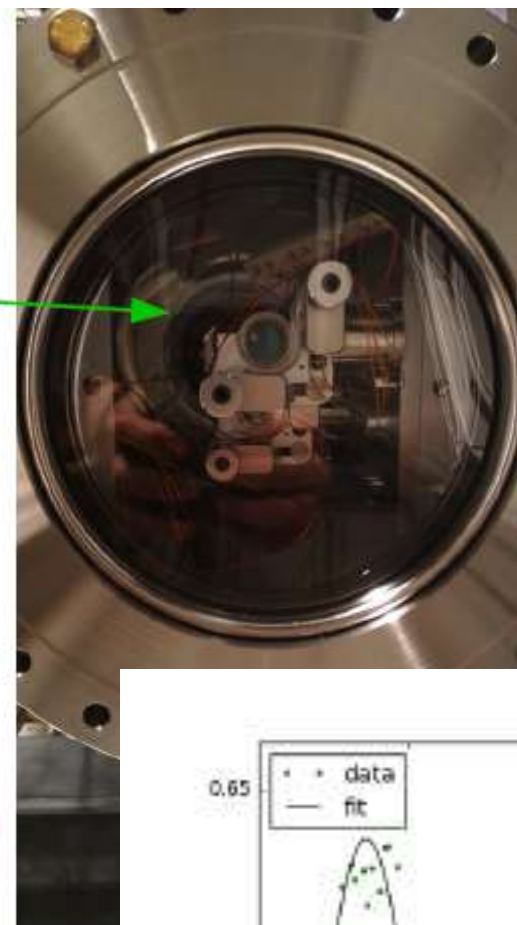


$T \sim 4 \mu\text{K}$



1560 nm injection bench

780 nm injection bench



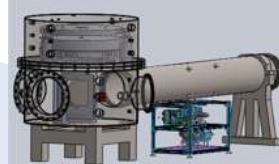
2012

2013

**2013 - Project manager
hired from VIRGO**

**2015 – First
suspension and sensc.
prototype**

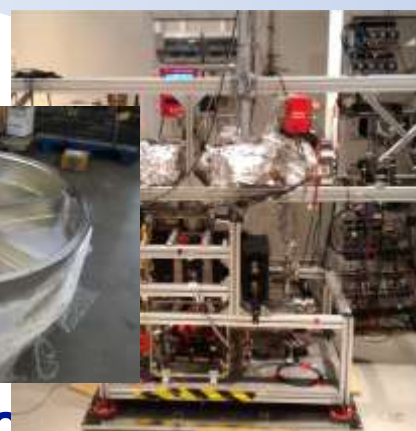
2014 – First design of the instrument



2014

2015 – Gravimeter

2015

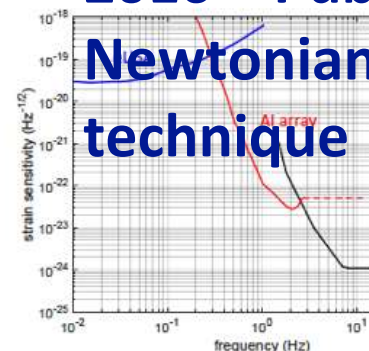


2016

2016 – GW discovery



**2016 – Publication (PRD) of the
Newtonian Noise suppression
technique**



2017

2017 – 3 sensors ready

2018

2018 - prototype

**2019
Instrument online**



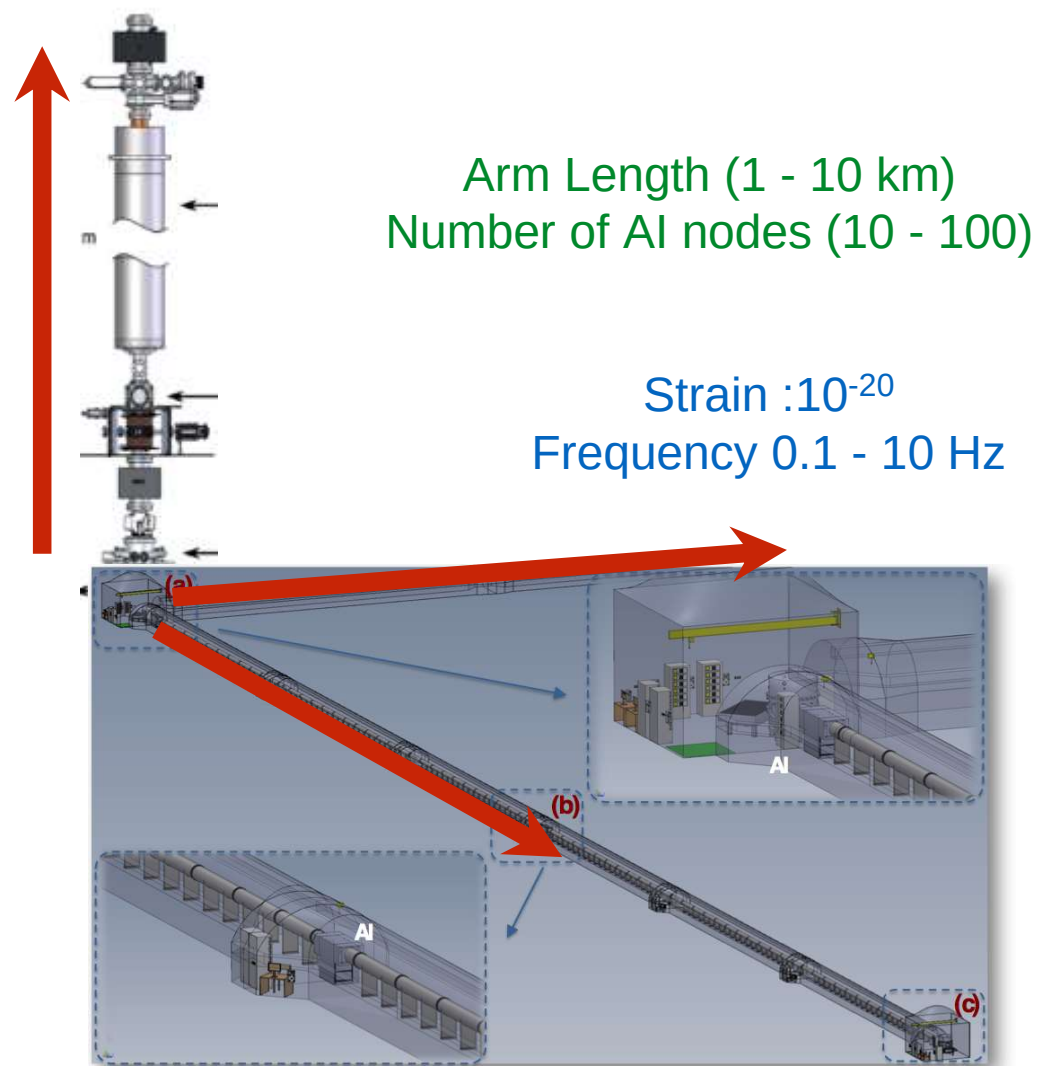
**2017 – Gallery
preparations**

2019

European Laboratory for Gravitation and Atom-interferometric Research (ELGAR)

Sync with other GW observation instruments

3D antenna configuration



“full band analysis”, gravitational noise analysis improvement, joint data management and analysis